Implications of Bell’s Theorem

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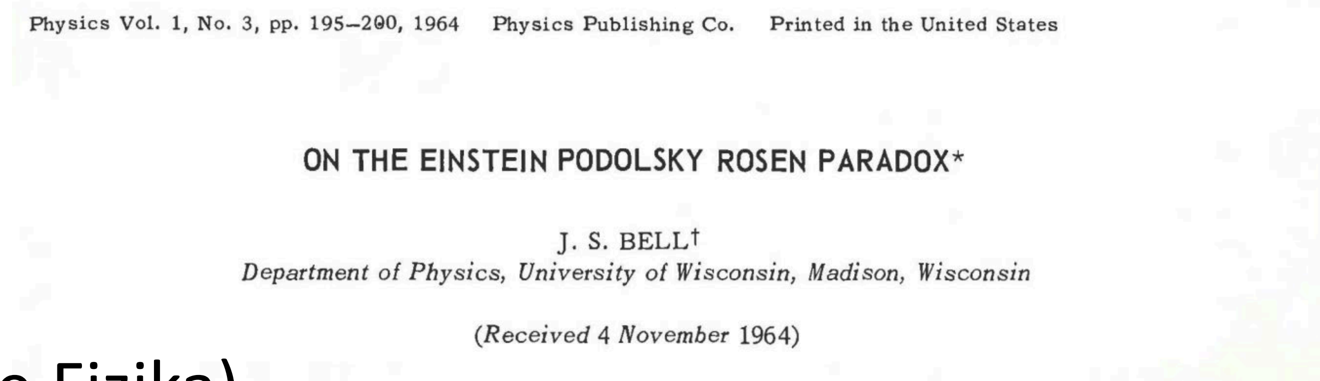
Common Misconceptions of Bell’s Theorem

- Doesn’t apply to QM / Tells us nothing new
- Proves action at a distance
- Rules out Hidden Variable models / Proves QM is Complete
- Doesn’t apply to indeterministic models
- Built on unnecessary assumptions:
  - Assumes Hidden Variables
  - Assumes Realism
  - Assumes Counterfactual Definiteness

Why the confusion?



Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

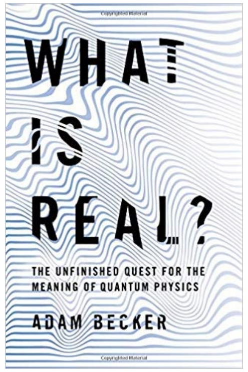
ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

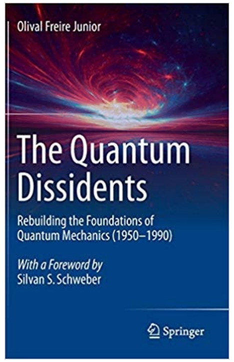
J. S. BELL†

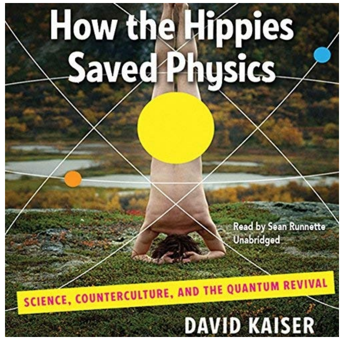
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

- Obscure Journal (Physics Physique Fizika)
- Assumed deep familiarity with EPR Paper (Nested misconceptions.)
  - Originally a two-step argument [EPR, Bell]
- Later, clearer writings by both Einstein and Bell were less studied
- Questioning quantum foundations was then actively discouraged.

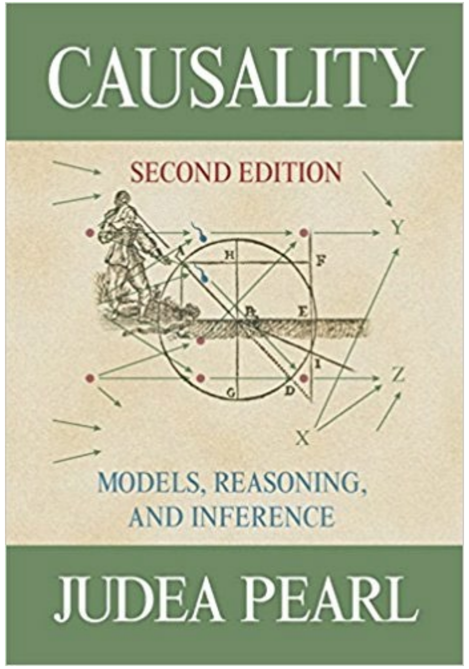


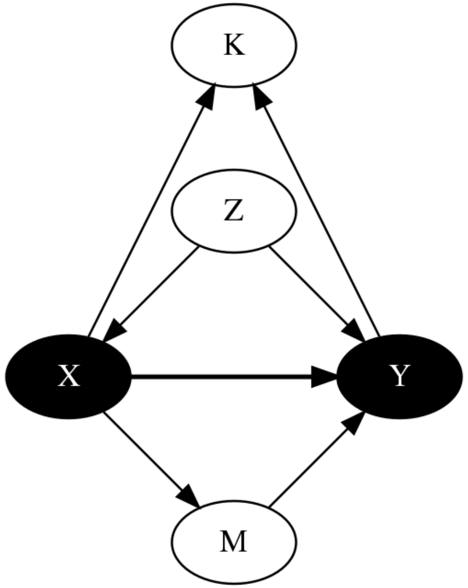


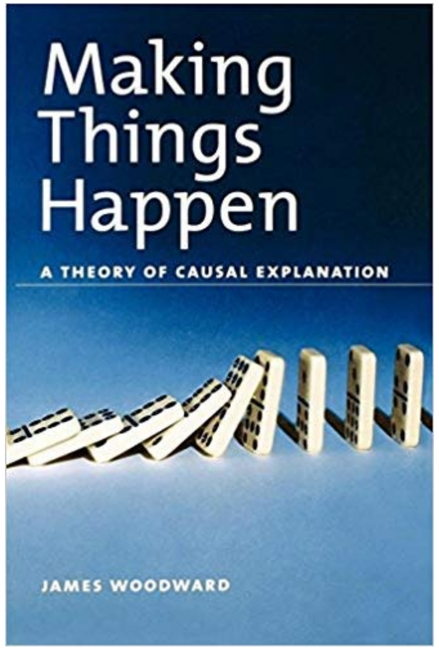


Even Bell missed a few things

- Implicit arrow-of-time assumption, not clearly spelled out
- When once asked about it, confused it with different concern
- Bell’s work preceded modern understanding of causal models





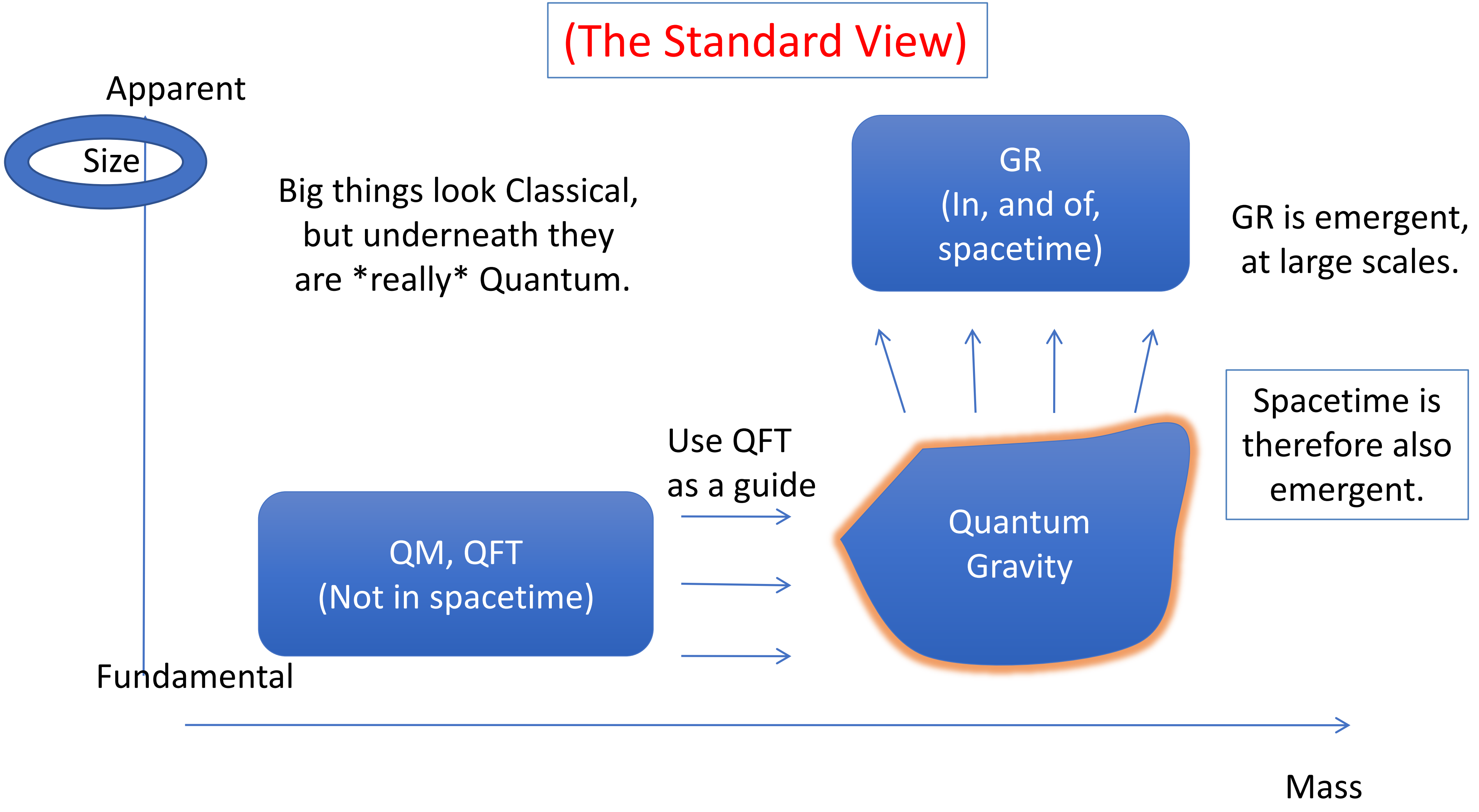


SO WHAT?

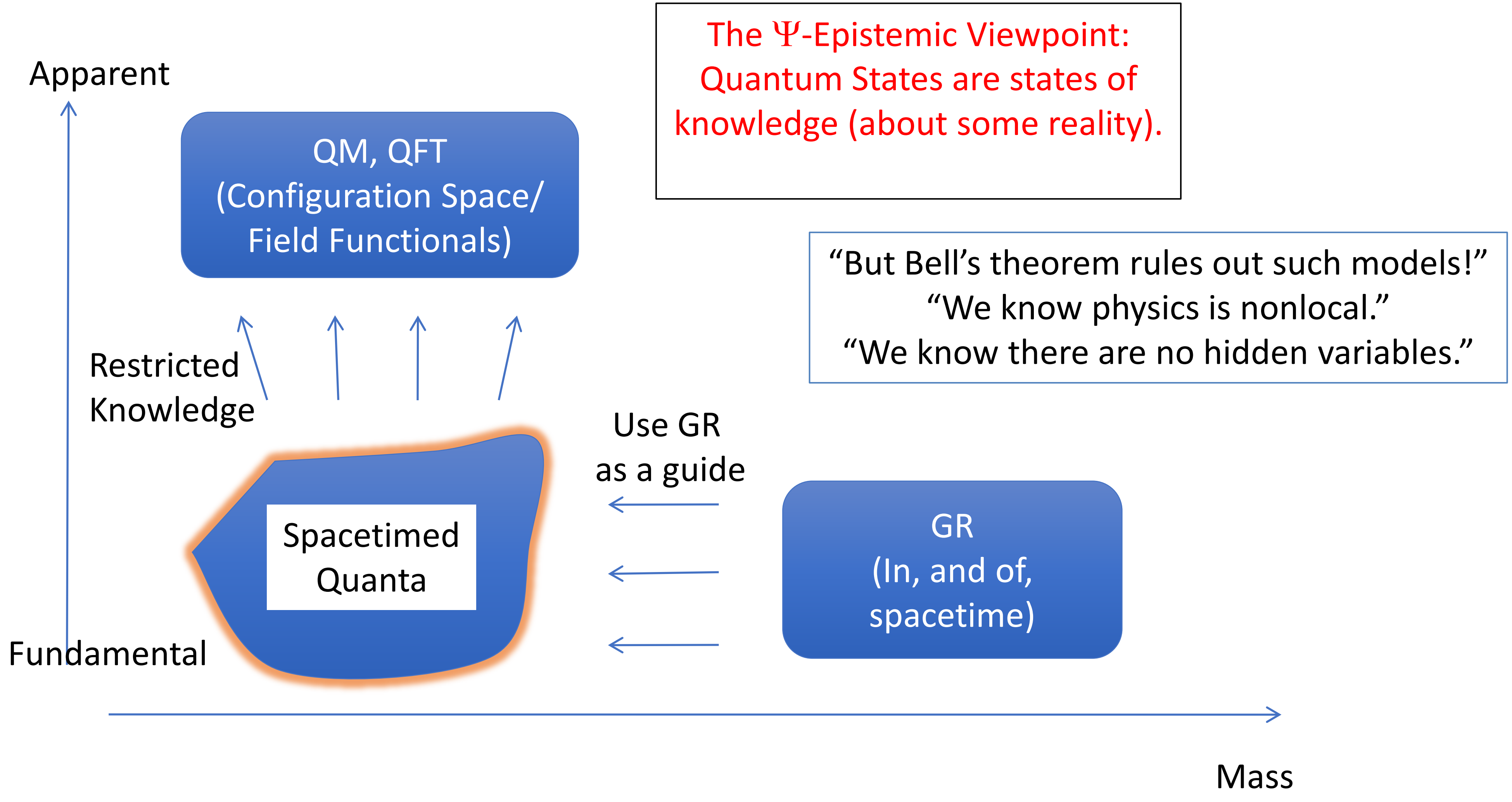
Why should tiny details of Bell’s Theorem matter?

What are the stakes?

Two Unreconciled Pillars of Modern Physics



Might Spacetime be more Fundamental?



Upcoming Rev. Mod. Phys. Paper (Email me for pre-print!)

Colloquium: Constraints on Spacetime-Localized Models from Bell's Theorem

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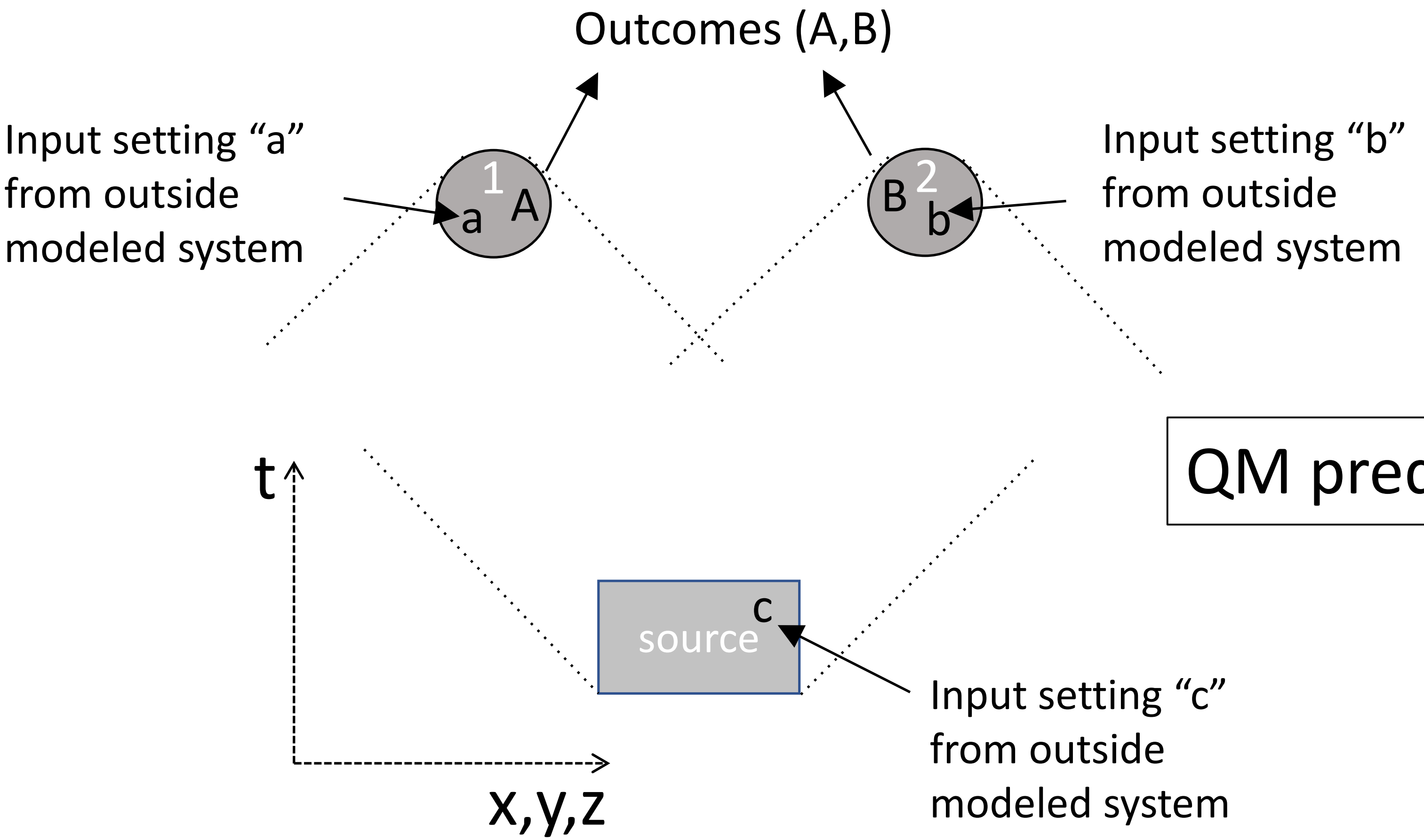
Bell's "Local Beables" and Physics models

- Can't talk about "locality" for parameters outside of spacetime
- Must distinguish spacetime-localized parameters $Q(\boldsymbol{x}, t)$
  - e.g. Classical E and B fields, $g^{\mu\nu}(x^{\alpha})$, energy density, etc. (Events!)
  - Can also use non-spacetime-based parameters (total energy, $|\Psi >$, etc.)
  - Wavefunctions live in Configuration space, $\Psi(x_1, x_2, \ldots, x_N$,t)!
- Model parameters can be "inputs" (settings), "outputs" (eigenvalues)
  - Additional parameters might be neither. (hidden/unmeasureable).
  - A model should define its non-input parameters (Q), inputs (I), outputs (O).
  - Models are like functions, yielding joint probabilities $P_I(Q)$.
- QM is essentially a model P_I(eigenvalues)
  - Here I includes preparation settings and measurement settings/basis.

Spacetime Geometry of Entanglement Expt.

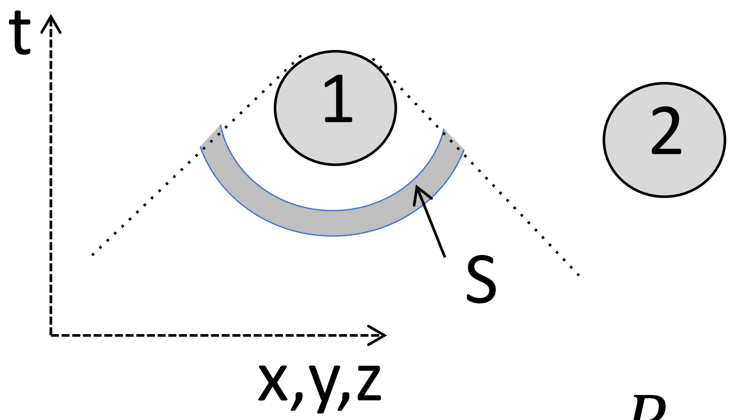
(Most interesting when regions "1" and "2" are spacelike separated.)



QM predicts $P_{a,b,c}(A,B)$

How to define “locality”?

BSA=“Bell’s Screening Assumption”



$$P_{I1,I2}(Q_1|Q_2, Q_S) = P_{I1}(Q_1|Q_S)$$

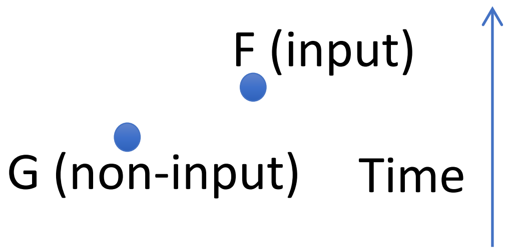
When predicting parameters in “1”, parameters in “2” are REDUNDANT, given parameters in ”S”.

You cannot get Bell's Theorem from BSA alone!

NFID=“No Future Input Dependence**”**

$$P_{I,F}(G) = P_I(G)$$

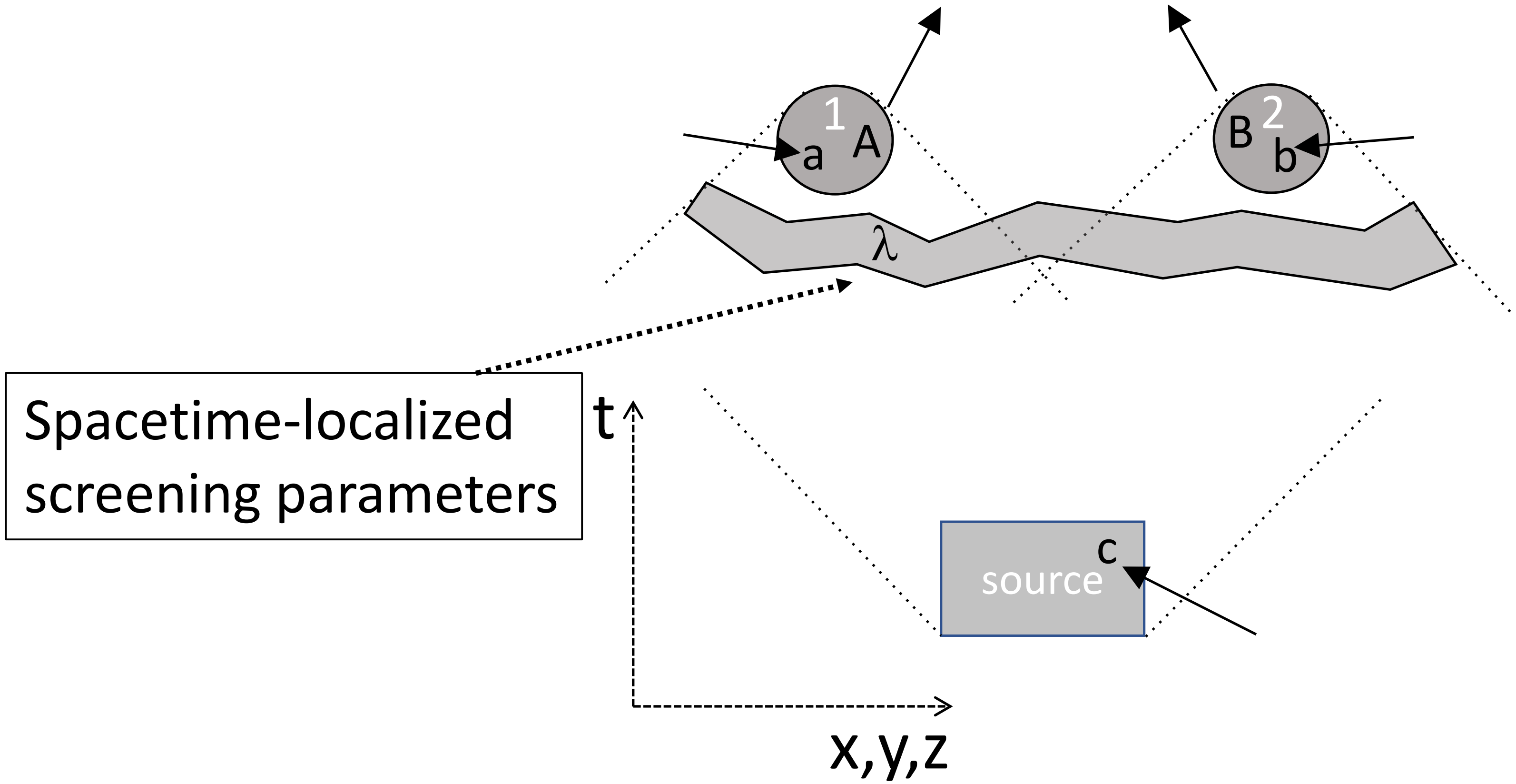
For all inputs F in the future of G



BSA + **NFID** = “Local Causality” (**LC**) => Bell's Theorem

Bell’s Theorem:

There is no **LC** model $P_{a,b,c}(A, B, \lambda)$ in agreement with QM.



Importance of **NFID**:

It’s trivial to encode any given correlation via common parameters λ.

We just can’t encode all ***possible*** correlations if $P_{a,b,c}(\lambda) = P_c(\lambda)$.

Non-**LC** Reformulations of QM are possible!

Categorization of QM-Agreeing Models:

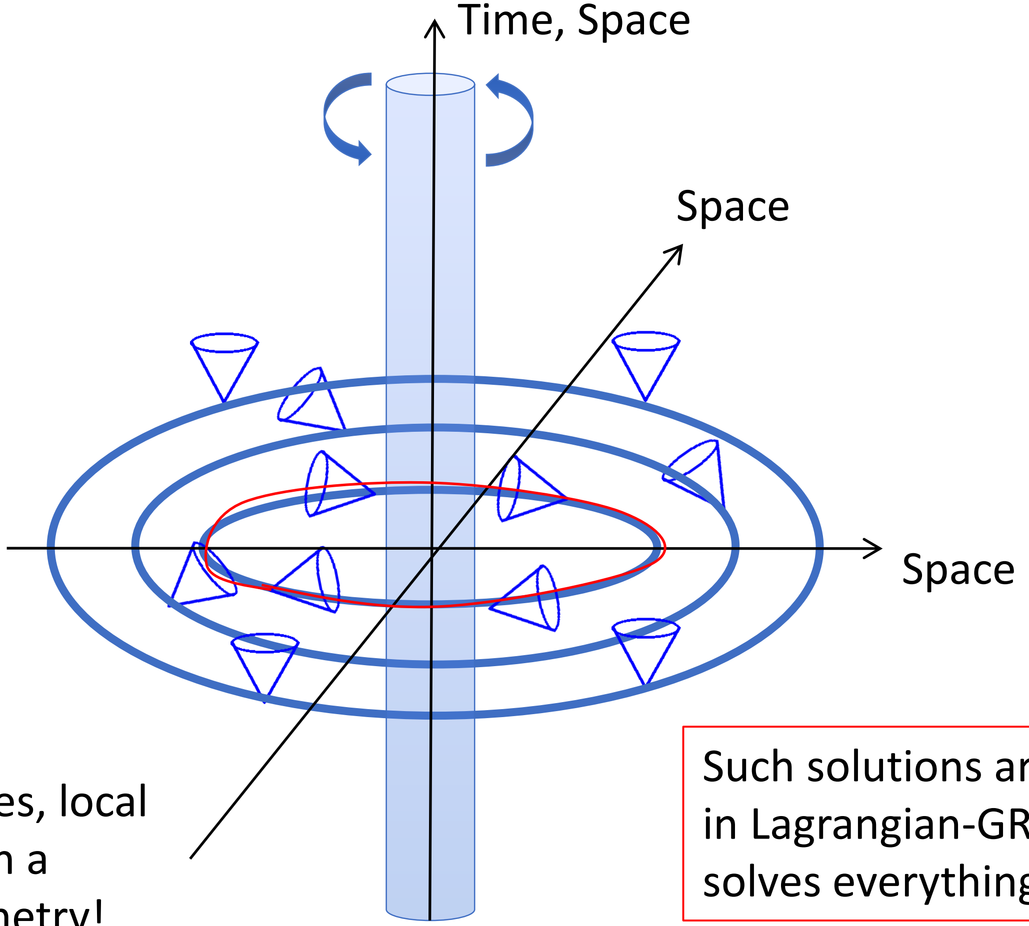
- Type-**I**: Violate BSA (nonlocal)
  - Action at a distance via non-spacetime localized parameters (like QM itself!)
  - Faster-than-light physical influences
- Type-**II**: Violate NFID (future inputs matter)
- Type-**III**: Violate both BSA and NFID

Any of these models could have localized intermediate parameters λ.

Why are Type-II models not more popular?

- Remember what's at stake: Spacetime itself!
- Time-travel paradoxes from $P_{a,b,c}(\lambda) \neq P_c(\lambda)$?
  - There is no protocol for measuring λ; it is "hidden"!
  - Not retro-signaling, no deviations from predictions of QM.
- "Second Law" arguments?
  - Macroscopic / partial-knowledge, not fundamental
  - Can get second law from asymmetric boundary conditions alone
- Physics never uses future inputs; it's not how physics "works".
  - Really?

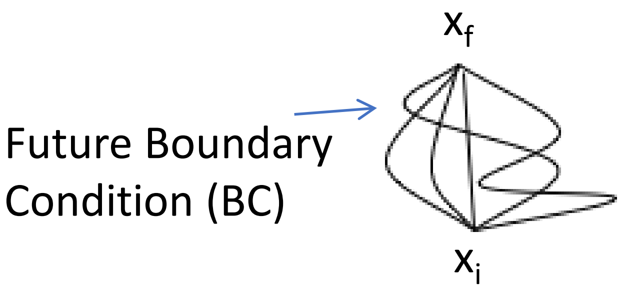
Exhibit A: Closed Time-Like Curves in GR



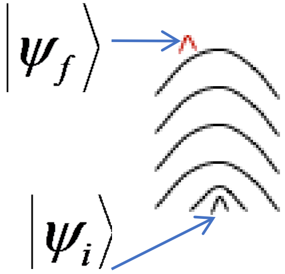
In closed timelike curves, local variables do depend on a future spacetime geometry!

Such solutions are only present in Lagrangian-GR, where one solves everything ``all at once''.

Exhibit B: Generic Action Principles



vs.



Features of Action Principles:

1) Future Boundary Conditions are essential.
2) Solved "all-at-once".
3) Histories in 4D spacetime (no need for 3D foliation)

Are future boundaries just a mathematical trick for generating dynamical equations?

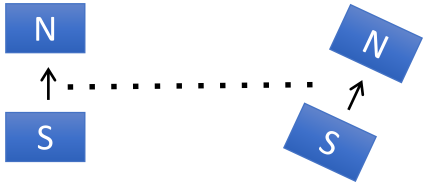
Or might they correspond to controllable constraints on the prior system?

Exhibit C: Time-Symmetry

Time-Symmetry implies measurement is just a time-reverse of preparation.
(Only difference is that external agents can't control the outcome of a measurement but can control the input to a preparation.)

Suppose a spin-1/2 particle passes through two Stern-Gerlach devices:

Preparation angle → (Free choice of setting)



← Measurement angle (Free choice of setting)

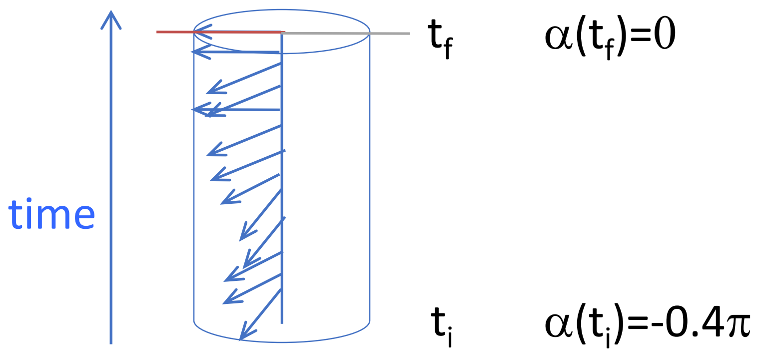
Any intermediate time-symmetric account would naturally violate NFID.

Of course; future constraints can't "change" the past beables.
(Safest to analyze histories "all at once".)

FID Example: The Schulman Anzatz

Proposed by L.S. Schulman
(Time's Arrows and Quantum Measurement, 1997; *Entropy* **14**, p.665 2012)

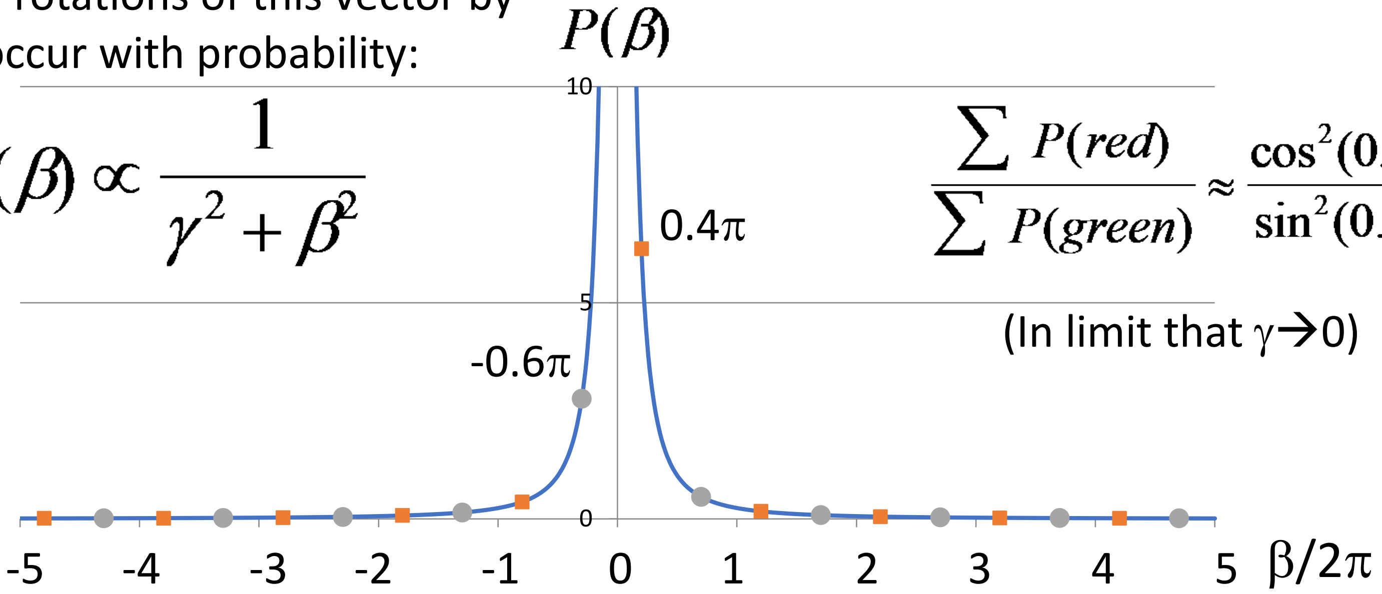
Possible final outcomes



Spin-1/2 particles have a well-defined spin vector aligned with all measurements (imposed as BCs).

Between two measurements, anomalous net-rotations of this vector by some angle β occur with probability:

$$P(\beta) \propto \frac{1}{\gamma^2 + \beta^2}$$

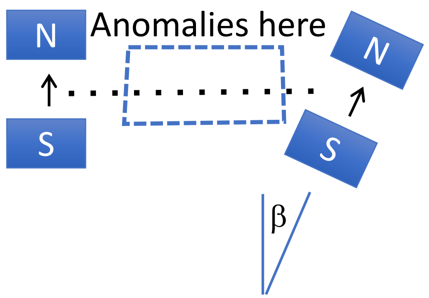


$$\frac{\sum P(red)}{\sum P(green)} \approx \frac{\cos^2(0.2\pi)}{\sin^2(0.2\pi)}$$

(In limit that $\gamma \to 0$)

Single-Particle Case

(Alice chooses spin-up input)



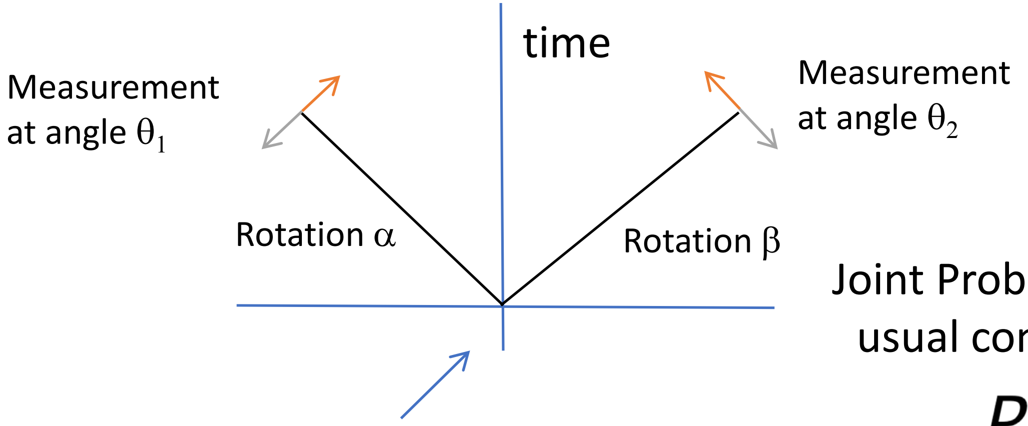
If Bob chooses a relative angle β,
the joint probability of a net rotation is

$$P(\beta + 2n\pi) = \cos^2(\beta/2)$$
$$P(\beta + \pi + 2n\pi) = \sin^2(\beta/2)$$

- Only remaining time-asymmetry is agency. (Bob can't choose outcome.)
- Probabilities are associated with entire histories. (Joint, not conditional.)
- Even the one-particle model violates NFID!

  - A different future angle setting by Bob (β) determines the probability that a (hidden) anomalous rotation has occurred in the past:

Trivial Extension to Singlet Entanglement



Opposite (but hidden) spin
states at preparation; θ and $\theta+\pi$.
(Bell/singlet state, constrained via conservation law)

Each particle might have an anomalous rotation.

Joint Probability of entire history is simply usual combination of joint probabilities:

$$P(\alpha,\beta) = P(\alpha)P(\beta)$$

Histories with either α=0 or β=0 are now overwhelmingly probable.

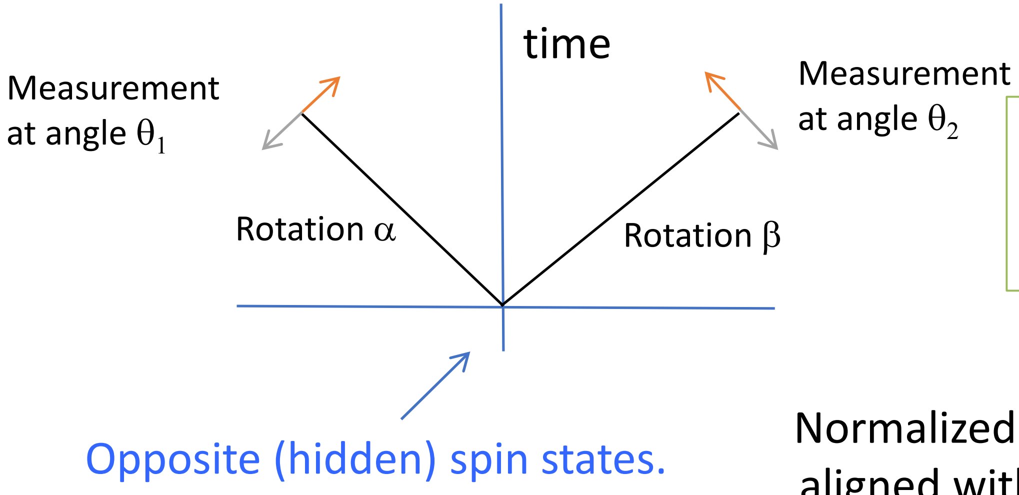
$$P(\alpha,\beta) \approx \left(\frac{1}{\gamma^2+\alpha^2}\right)\left(\frac{1}{\gamma^2+\beta^2}\right)$$

If α, β both >>γ: $P \approx \frac{1}{\alpha^2\beta^2}$

But if α<<γ : $P \approx \frac{1}{\alpha^2\gamma^2} >> \frac{1}{\alpha^2\beta^2}$

Obvious FID influence on hidden state: $\theta=\theta_1$ or $\theta=\theta_2$ (mod π)

Yields the actual joint probabilities!



One particle will need no anomaly. The other particle will need to rotate by $\theta_2 - \theta_1 = \Delta\theta \pmod{\pi}$)

Normalized probability of both outcomes aligned with both measurement settings:

$$\frac{P(\Delta\theta+\pi)}{2P(\Delta\theta)+2P(\Delta\theta+\pi)} = \frac{1}{2}\sin^2\left(\frac{\Delta\theta}{2}\right)$$

Violates Bell inequality exactly as does a singlet state, but conforms to BSA.
(this is accomplished via a violation of NFID)

Generalizable to any two-qubit maximally entangled state.

What did Bell think of NFID?

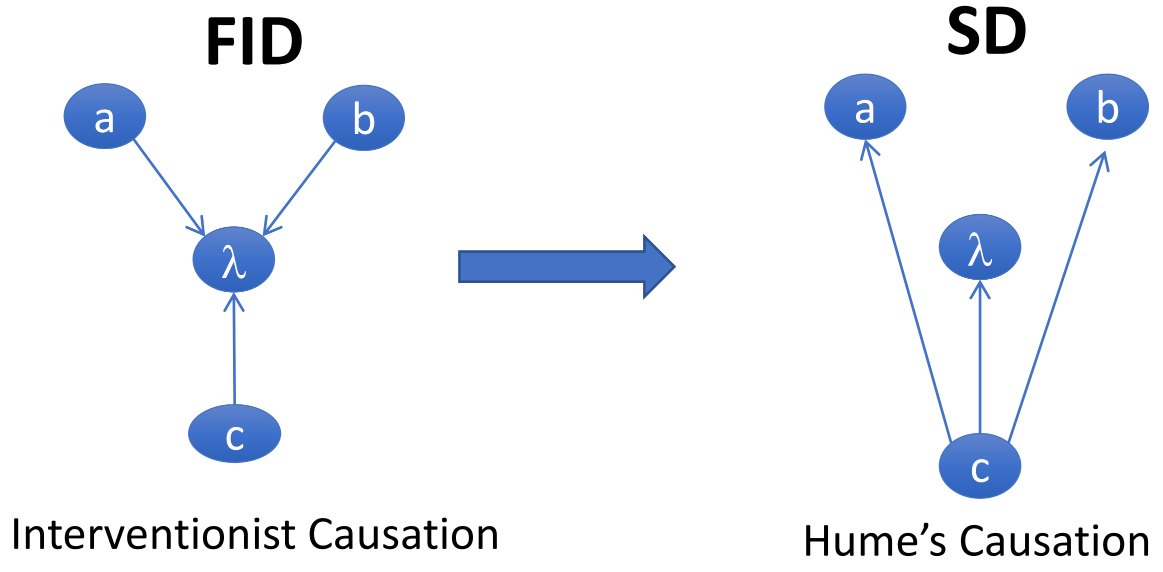
- Knew about mathematical importance of $P_{a,b,c}(\lambda) = P_c(\lambda)$.
- Defended special nature of inputs in text, but conflated in the math:

$$P(\lambda|a,b,c) = P(\lambda|c)$$

- Allows Bayesian Inversion to probabilities of *settings*!

$$P(a,b|\lambda) = P(a,b)$$

- This can be violated via "superdeterminism" (**SD**) (a different causal model)
- Bell conflated **NFID** with issues of free settings.



Take Away Messages

- Bell's Theorem doesn't assume or rule out localized hidden parameters.
- It doesn't even rule out "locality" (**BSA**) if one violates **NFID**.
- Unlike superdeterminism, this would agree with QM's inputs
- FID models are worthy of more study and development
  - A dozen such models already in the literature, in past decade
  - Would support popular "Ψ-epistemic'' viewpoint
  - Might even save the structure of conventional spacetime!