Kinetic physics of magnetized plasmas and its impact on pulsed power HED experiments

Genia Vogman

Lawrence Livermore National Laboratory

LLNL HED Seminar Jan 24, 2019

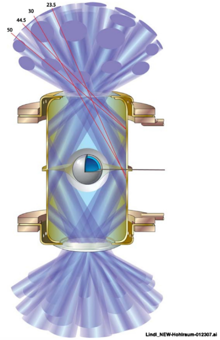
This work was performed under the auspices of the U.S. DOE by LLNL under Contract DE-AC52-07NA27344
and was supported by the LLNL-LDRD Program under Project No. 18-ERD-048.

LLNL-PRES-956594

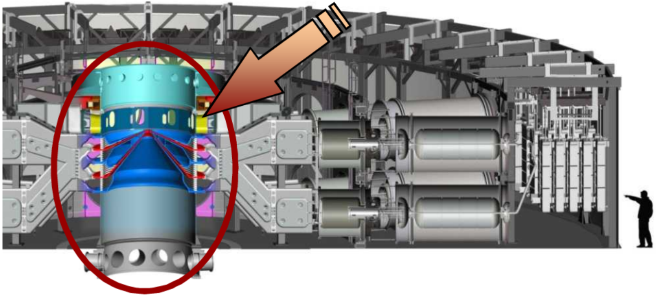
Outline

- Pulsed power experiments and collisionless plasmas
- Kinetic treatment and associated challenges
- Development of machinery to enable computational investigation
- Simulation of sheared flow induced kinetic transport

Collisionless plasmas often limit the high energy density conditions that can be achieved in experiments



- Low density and/or high temperature conditions result in long mean free paths, $\lambda_{mfp} \gg L$
- Kinetic physics, rather than fluid physics, can dictate plasma behavior



Low density plasma formation in power feeds limits performance of pulsed power experiments on Z

Z delivers up to 26 MA of current to a load through magnetically insulated transmission lines, which are designed to prevent high-voltage arcs

Problem: formation of low density plasma in the power feeds

- results in 10–15% current loss
- interferes with load dynamics
- prevents scaling
- undercuts predictive modeling

Figures from M. R. Gomez et al. 2017

Power feed plasmas are not well characterized experimentally

Experimental obstacles

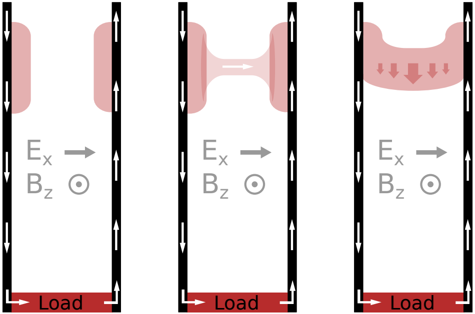
- limited access
- extreme environment

  E-field: 1×10^8 V/m

  B-field: 0-200 T
- mechanism for creating current pathway is unknown

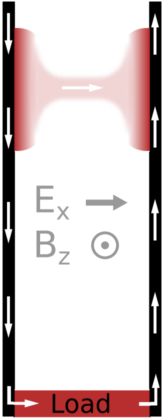
Overarching goals:

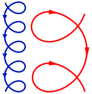
- understand power flow
- ensure load is unaffected
- scale experiment beyond 26 MA



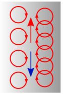
Objectives of this research: shed light on transport mechanisms that can enable parasitic currents

$\boldsymbol{E} \times \boldsymbol{B}$ environment in power feeds features multi-fluid and kinetic physics





$$u_{E\times B} = \frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}$$



$$u_D = -\frac{\nabla p_s \times \boldsymbol{B}}{q_s n_s B^2}$$

Plasma created at electrode surfaces is subject to different transport mechanisms

Effect	MHD	Multi-fluid	Kinetic
$E \times B$ drift	X	X	X
Diamagnetic drift	-	X	X
Macroinstabilities	X	X	X
Microinstabilities	-	-	X
Finite gyromotion	-	-	X

Universal theme: since single-fluid and multi-fluid models do not capture all physics, kinetic treatment is needed to fully understand transport

Kinetic treatment is made challenging by complexity of theory and by computational cost

Challenges

- Behavior of collisionless, finite-temperature, nonuniform, magnetized plasmas is not well characterized
- Kinetic simulations are computationally costly, making it difficult to explore parameter space

	MHD Simulation	Kinetic Simulation
Coordinates	(x, y)	(x, y, v_x, v_y)
Degrees of freedom	N^2	N^4
# Cores for $N = 128$	1	4096

Aim: exploit modern numerical methods and develop theoretical machinery to address these challenges

Fourth-order Vlasov Continuum Kinetic (VCK) code[1] is used to investigate $\boldsymbol{E} \times \boldsymbol{B}$ power feed environment

- Vlasov and Poisson equations solved directly to evolve ion and electron distribution functions in (x, y, v_x, v_y) phase space

$$0 = \frac{\partial f_s}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_s}{\partial \boldsymbol{x}} + \frac{Z_s}{M_s} (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial f_s}{\partial \boldsymbol{v}}$$

$$-\nabla^2 \phi = \sum_s Z_s \int f_s d\boldsymbol{v}$$

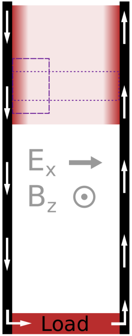
- VCK uses conservative, fourth-order, finite volume discretization, with error $\|f - f^{exact}\| = C\Delta x^4$.
- Convergence offers a mathematical way to check whether simulations give the correct answer.

[1] Vogman et al. *JCP* 2014, Vogman et al. *JCP* 2018

Simulations can access the same regime as experiments, without replicating all parameters

Simplifying assumptions

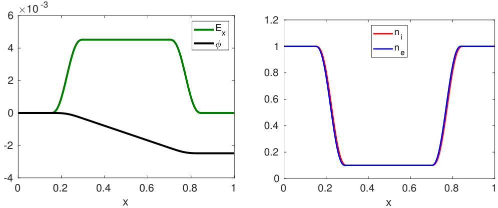
- low-beta electrostatic, magnetostatic plasma
- Cartesian (x, y) geometry
- Boundary conditions are periodic in y and reflecting wall in x
- i^+ and e^- with mass ratio 25



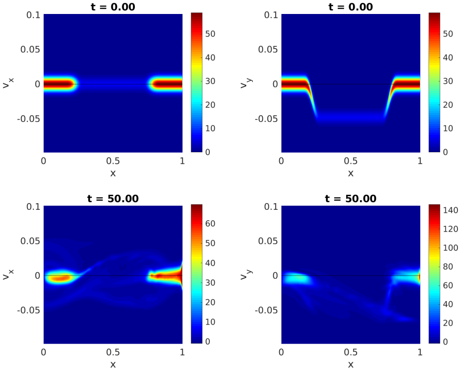
Parameter	Simulations	Experiment
n_0 [m^{-3}]	1e21	$\geq$ 1e21
L [m]	1e-3	1e-2
B_z [T]	200	$0 - 200$
E_x [V/m]	1e8	1e8
T [eV]	5000	$1 - 5000$ (?)

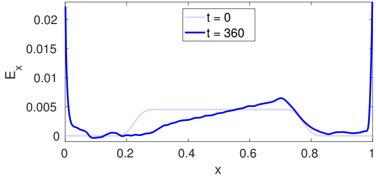
In power feed environment, fluid equilibrium is a poor approximation to a kinetic equilibrium

- $E \times B$ drifting low-density plasma initialized with fluid equilibrium: $q_s n_s(E_x + u_{ys} B_z) = T \nabla n_s$ or $u = -E_x / B_z$

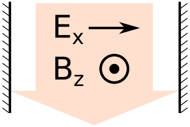


- Sheared flow instabilities are candidate transport mechanism
- Self-consistent kinetic equilibrium needed to study isolated transport effects





1D single-species kinetic equilibrium in $\boldsymbol{E} \times \boldsymbol{B}$ configuration prescribed using constants of motion



- A distribution function written in terms of constants of motion $f(\boldsymbol{p}, \mathcal{E})$ satisfies the equilibrium Vlasov equation

$$p = mv_y + qB_z x \quad \rightarrow \quad x + \frac{v_y}{\Omega}$$

$$\mathcal{E} = \frac{1}{2} m \left(v_x^2 + v_y^2\right) + q\phi \quad \rightarrow \quad \frac{m\left(v_x^2 + v_y^2\right)}{2T} + \frac{q\phi}{T}$$

- Canonical momentum p accounts for “smearing” of distribution function due to finite Larmor radius

- For example, $f = \exp(-p) \exp\left(-\mathcal{E}\right)$ is an equilibrium distribution function

Constructing self-consistent two-species kinetic equilibria in $\boldsymbol{E} \times \boldsymbol{B}$ configuration is more complicated

Objectives:

- Express both distribution functions in terms of constants of motion: $f_i(p_i, \mathcal{E}_i)$ and $f_e(p_e, \mathcal{E}_e)$
- Satisfy the Poisson equation

$$-\nabla^2 \phi = Z_i \int f_i d\boldsymbol{v} + Z_e \int f_e d\boldsymbol{v}$$

- Specify sheared flow profiles and density gradients

Approaches from published literature are not applicable

- Small Larmor radius limit
- $E = 0$ or $\nabla \cdot \boldsymbol{E} = 0$
- Single-species
- Poisson equation is not satisfied, not self-consistent
- Density is uniform
- Domain is periodic

Building machinery for constructing equilibria using relationship between canonical momentum dependence and species number density

Define distribution function with separable canonical momentum and energy dependence

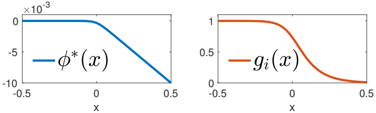
$$f_s = N_s\left(x + \frac{v_y}{\Omega_s}\right)\frac{M_s}{2\pi T_s}\exp\left(-\frac{M_s\left(v_x^2+v_y^2\right)}{2T_s} - \frac{Z_s\phi}{T_s}\right), \qquad n_s = \int f_s d\boldsymbol{v}$$

Expanding $N_s\left(x + \frac{v_y}{\Omega_s}\right)$ as a Taylor series and evaluating zeroth velocity moment yields analytic relationship between density and canonical momentum term

$$n_s(x) = \exp\left(-\frac{Z_s\phi(x)}{T_s}\right)\left[N_s(x) + \frac{r_{Ls}^2}{2}N_s''(x) + \frac{3r_{Ls}^4}{4!}N_s''''(x) + \cdots\right]$$

Method 1: equilibrium construction using analytic functions, provided plasma is sufficiently well magnetized

1. Pick potential profile ϕ^* and ion density $g_i(x)$ to model power feed. Stipulate that $g_e = g_i + \frac{\partial^2 \phi^*}{\partial x^2}$



2. Choose special form of canonical momentum dependence

$$N_s(p_s) = \left[g_s(X) \exp\left(\frac{Z_s \phi^*(X)}{T_s}\right) - \frac{r_{Ls}^2}{2} \frac{\partial^2}{\partial X^2} \left(g_s(X) \exp\left(\frac{Z_s \phi^*(X)}{T_s}\right)\right)\right]_{X=p_s}$$

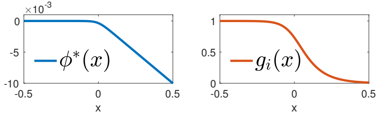
3. Construct equilibrium distribution functions such that $n_i = g_i + \mathcal{O}(r_{Li}^4)$

$$f_i = \left[g_i(X) \exp\left(\frac{Z_i \phi^*(X)}{T_i}\right) - \frac{r_{Li}^2}{2} \frac{\partial^2}{\partial X^2} \left(g_i(X) \exp\left(\frac{Z_i \phi^*(X)}{T_i}\right)\right)\right]_{X=p_i} \times \frac{M_i}{2\pi T_i} \exp\left(-\frac{M_i(v_x^2 + v_y^2)}{2T_i} - \frac{Z_i \phi^*(x)}{T_i}\right)$$

$$f_e = \left[g_e(X) \exp\left(\frac{Z_e \phi^*(X)}{T_e}\right) - \frac{r_{Le}^2}{2} \frac{\partial^2}{\partial X^2} \left(g_e(X) \exp\left(\frac{Z_e \phi^*(X)}{T_e}\right)\right)\right]_{X=p_e} \times \frac{M_e}{2\pi T_e} \exp\left(-\frac{M_e(v_x^2 + v_y^2)}{2T_e} - \frac{Z_e \phi^*(x)}{T_e}\right).$$

Method 2: generalized equilibrium construction based on numerical solution of nonlinear differential equation

1. Select potential profile ϕ^* and ion density profile $g_i(x)$



2. Construct auxiliary ion and electron distribution functions that are close to equilibrium

$$f_i^{aux} = g_i\left(x + \frac{v_y}{\Omega_i}\right) \exp\left(\frac{Z_i\phi^*\left(x + \frac{v_y}{\Omega_i}\right)}{T_i}\right) \times \frac{M_i}{2\pi T_i} \exp\left(-\frac{M_i(v_x^2 + v_y^2)}{2T} - \frac{Z_i\phi^*(x)}{T_i}\right), \qquad n_i^{aux} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_i^{aux}\, dv_x dv_y$$

$$f_e^{aux} = \left[\left(Z_i n_i^{aux,fit}(X) + \frac{\partial^2\phi^*(X)}{\partial X^2}\right) \exp\left(\frac{Z_e\phi^*(X)}{T_e}\right)\right]_{X=pe} \times \frac{M_e}{2\pi T_e} \exp\left(-\frac{M_e(v_x^2 + v_y^2)}{2T_e} - \frac{Z_e\phi^*(x)}{T_e}\right), \qquad n_e^{aux} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_e^{aux}\, dv_x dv_y$$

3. Formulate and solve nonlinear Poisson equation for the equilbrium potential ϕ

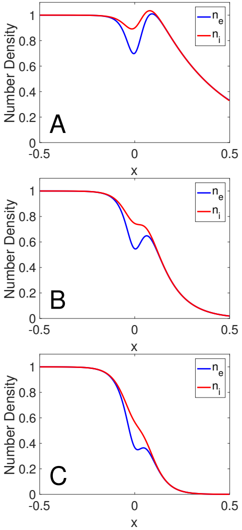
$$-\frac{\partial^2\phi}{\partial x^2} = Z_i n_i^{aux} \exp\left(\frac{Z_i(\phi^* - \phi)}{T_i}\right) + Z_e n_e^{aux} \exp\left(\frac{Z_e(\phi^* - \phi)}{T_e}\right)$$

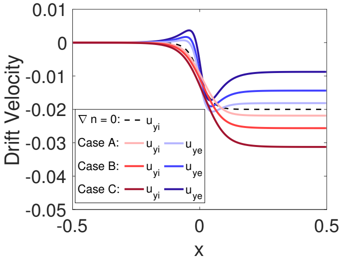
4. Exact equilibrium distribution function $f_s = f_s^{aux} \exp\left(\frac{Z_s(\phi^* - \phi)}{T_s}\right)$

Equilibrium construction methods[2] expand the scope of configurations that can be studied

Now we can construct equilibria in which we have

- customizable density profiles
- spatially varying electric fields
- finite Larmor motion for ions and electrons
- nonneutral plasmas
- non-periodic boundary conditions
- diamagnetic drift





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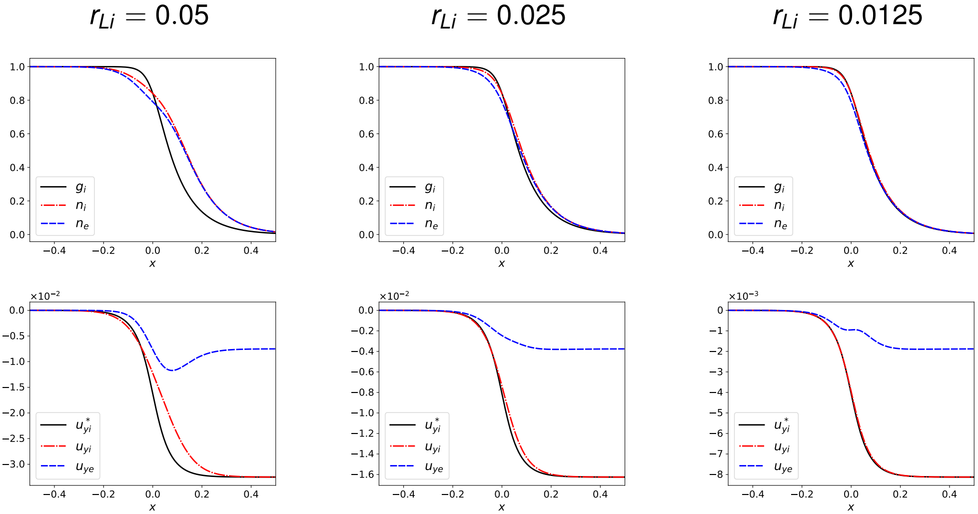
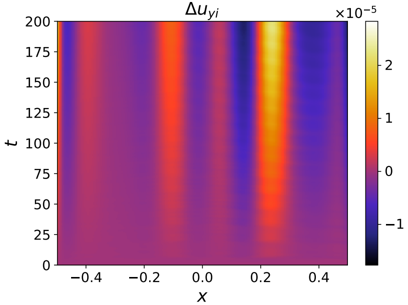
[2]Vogman et al. submitted to *PoP* 2019

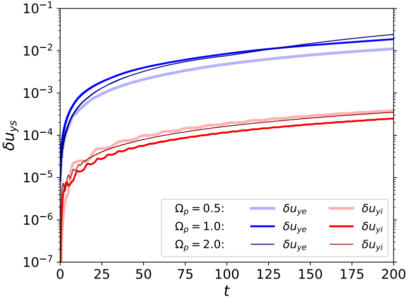
Self-consistent two-species kinetic equilibria for different levels of magnetization



Time-dependent VCK code can faithfully preserve kinetic equilibria

In simulations, ion equilibrium is preserved to within $< 0.04\%$ of initial condition $\implies$ code can capture complex equilibria with high fidelity. $\frac{\Delta f}{\Delta t} = -\nabla \cdot \boldsymbol{F} + C\Delta x^4$



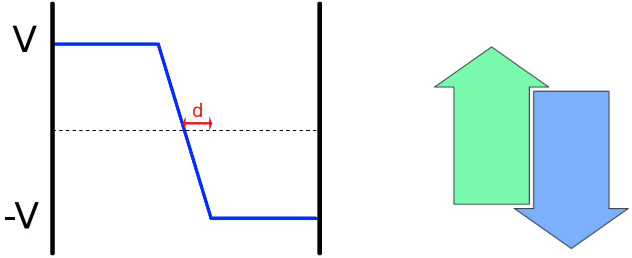


Initial kinetic simulations of power feed environment showed

- ☑ Self-consistent kinetic equilibrium needed to study isolated physics
- ☐ Sheared flow instabilities are candidate transport mechanism

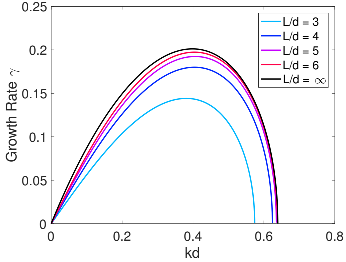
Single-fluid incompressible hydrodynamic theory of Kelvin-Helmholtz instability [Chandrasekhar 1961]

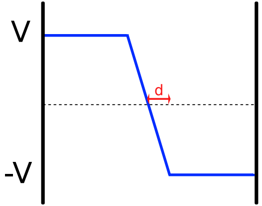
- Hydrodynamics is consistent with MHD theory in low-β limit
- Finite-width shear layer in uniform density fluid



- Linearize continuity and momentum equations to obtain eigenmode ODE for u_{1x}
- Apply continuity conditions, boundary conditions, jump conditions to obtain $\omega(k)$

Single-fluid incompressible hydrodynamic theory of Kelvin-Helmholtz instability: dispersion relation





Infinite domain

$$\omega^2 = \frac{V^2}{4d^2}\left(1 - 4kd + 4k^2d^2 - e^{-4kd}\right)$$

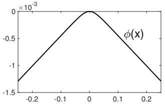
Finite domain

$$\omega^2 = \frac{V^2}{4d^2} \cdot \frac{(e^{2k(d-L)} - 1 + 2kd)^2 - e^{-4kL}(e^{2k(L-d)} - 1 - 2kd)^2}{1 - e^{-4kL}}$$

- Growth rate scales with shear V/d
- kd: wavenumber relative to shear layer half-width determines growth rate
- kL: walls provide stabilization
- Not included: finite Larmor motion, compressibility

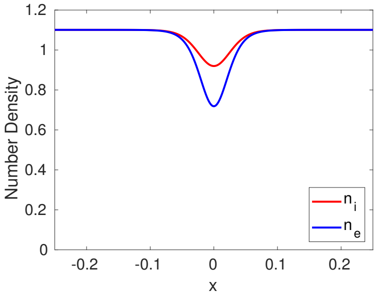
Combining kinetic equilibrium construction with single fluid analysis: kinetic Kelvin-Helmholtz instability in MHD limit

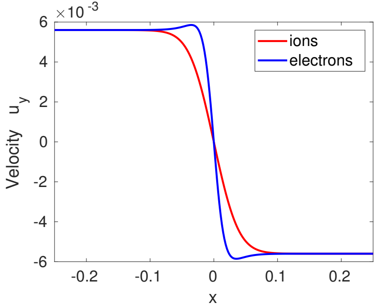
Self-consistent kinetic equilibrium initialization $f(p, \mathcal{E})$, where sheared flow is facilitated by electric field



$$\phi = \phi_0 \log \left[\cosh\left(\frac{x}{d}\right)\right]$$

$$f_i = \frac{M_i}{2\pi T_i} \exp\left(-\frac{M_i(v_x^2 + v_y^2)}{2T_i} - \frac{Z_i\phi(x)}{T_i}\right) \exp\left(\frac{Z_i\phi\left(x + \frac{v_y}{\Omega_i}\right)}{T_i}\right)$$



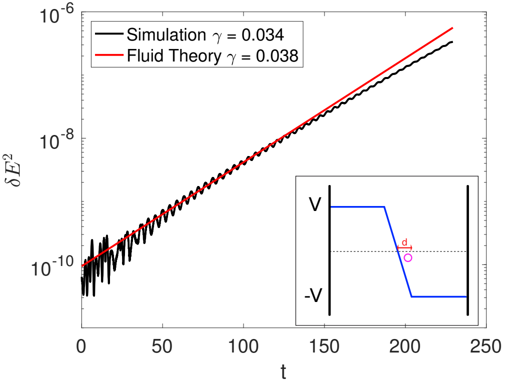


Kinetic simulation successfully captures KH instability & agrees with single-fluid theory prediction for $r_{Li}/d = 0.45$

- Metric for instability growth:

$$\delta E^2 = \int \left[(E_x - E_{x0})^2 + (E_y - E_{y0})^2 \right] dxdy$$

- Finite Larmor motion leads to smaller growth rate of Kelvin-Helmholtz instability in kinetic simulations



Shear layer analysis of KH instability extended to two-fluid plasma to account for diamagnetic drift

What happens when plasma is not uniform?

Isothermal two-fluid linear theory analysis

- Adiabatic theory for a 2D strongly-magnetized plasma
- Assumptions: electron inertia is negligible, plasma is quasineutral, electrostatic, magnetostatic
- low-β limit $\implies$ magnetostatics is a sensible assumption

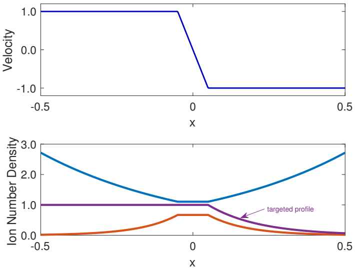
Equations	Unknowns
Ion continuity	n_{i1}
Ion momentum $\hat{x}$	u_{i1x}
Ion momentum $\hat{y}$	u_{i1y}
Electron momentum $\hat{x}$	u_{e1x}
Electron momentum $\hat{y}$	u_{e1y}
$\nabla \cdot \boldsymbol{J} = 0$	ϕ_1

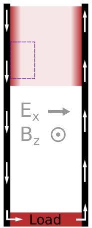
Two-fluid Kelvin-Helmholtz dispersion relation sheds light on effect of density gradients

Eigenmode equation encapsulates response of a non-uniform sheared-flow plasma subject to perturbations

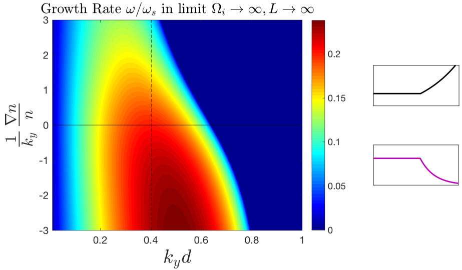
$$0 = n_0 \hat{u}_{i1x} k_y^2 (k_y u_{i0y} - \omega) + \frac{\partial}{\partial x} \left[\frac{-(k_y u_{i0y} - \omega) n_0 \frac{\partial \hat{u}_{i1x}}{\partial x} + n_0 \hat{u}_{i1x} \frac{\partial u_{i0y}}{\partial x} k_y}{1 - \frac{m_i}{q_i B_{z0} k_y} (k_y u_{i0y} - \omega) \frac{1}{n_0} \frac{\partial n_0}{\partial x}} \right]$$

If $\nabla n / n$ is piecewise constant then we can obtain an analytic dispersion relation and solve for $\omega(k_y)$ numerically

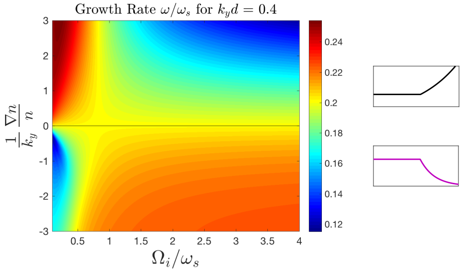




Two-fluid Kelvin-Helmholtz dispersion relation captures effect of density gradients and diamagnetic drift

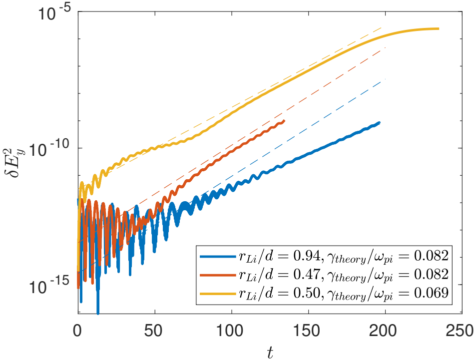


- Growth rate relative to shear $\omega_s = V/d$ is determined by density gradient scale length.
- Sharp density gradient near the shear layer ($\nabla n < 0$) results in more unstable configuration.



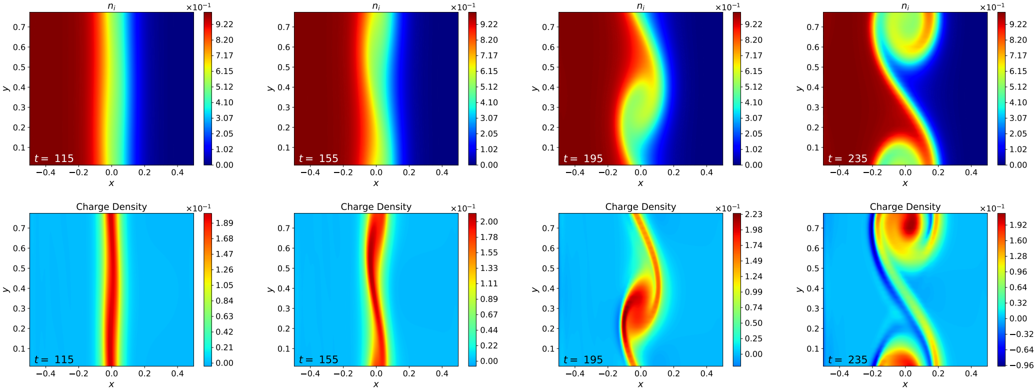
- Growth rate is affected by magnetization.
- For sufficiently small Ω_i fluid theory does not accurately capture plasma behavior.
- Growth rates used to inform initial conditions in kinetic simulations.

Equilibrium construction machinery combined with two-fluid analysis enables study of Kelvin-Helmholtz instability



- Kinetic simulations successfully capture Kelvin-Helmholtz instability as described by two-fluid theory
- For small r_{Li}/d simulation growth rate agrees with two-fluid theory
- Large Larmor orbits have stabilizing effect, as expected

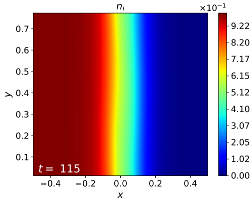
Nonlinear evolution exhibits eddy feature characteristic of Kelvin-Helmholtz instability

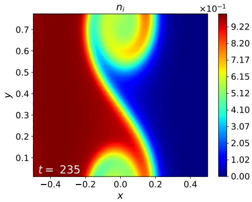


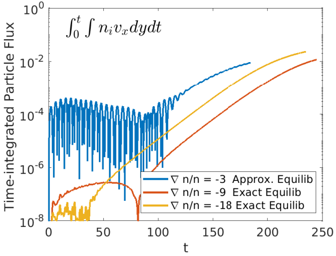
- Kinetic features: temperature anisotropy, finite Larmor motion, non-Maxwellian distribution functions

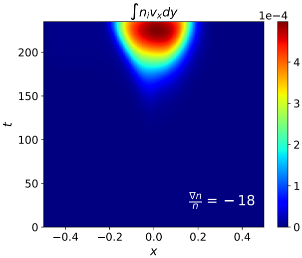
Vlasov-Poisson simulations show that Kelvin-Helmholtz instability drives transport of plasma across shear layer

- X-directed particle flux across shear layer grows exponentially in time, and continues to grow after instability saturates
- Local x-directed particle flux depends on the size of the Kelvin-Helmholtz eddy

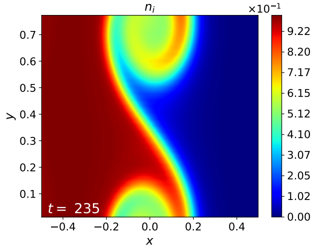


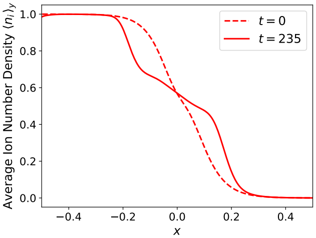






Kelvin-Helmholtz-driven transport leads to plasma density becoming more uniform, creating current pathway

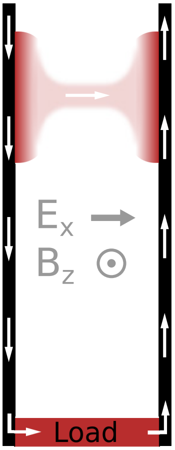




Density gradient decreases toward zero as high density plasma moves into low density region

Implication for pulsed power:

Kelvin-Helmholtz-driven transport of plasma across shear layers can produce current pathways and contribute to parasitic currents in power feeds



Leveraging machinery to study more generalized kinetic physics and assess its impact on experiments

Computational leads

- Study other transport mechanisms, e.g. lower-hybrid drift instability
- Devise transport model for MHD simulations
- Incorporate more generalized physics, e.g. nonneutral plasmas
- Platform for studying interface between kinetic and fluid physics

Tie to experiments

- Gain insight on what to look for in experiments
- Devise mitigation strategies
- Simulation-informed design of power feeds

Summary and Conclusions

- 4D (x, y, v_x, v_y) Vlasov-Poisson simulations are used to study $\boldsymbol{E} \times \boldsymbol{B}$ environment in power feeds to understand cause of parasitic currents in pulsed power HED experiments. Simulations show that sheared flow is an important source of free energy and kinetic equilibria are needed.
- Computational study of isolated physics is enabled by
  - High-order accurate Vlasov-Poisson solver
  - Machinery for constructing customizable self-consistent two-species kinetic equilibria
  - Extension of Kelvin-Helmholtz linear theory analysis to include two-fluid physics
- Simulations are successfully verified against fluid theory and enable investigation of particle transport in nonuniform magnetized plasmas with drifts, velocity shear, and finite Larmor motion.
- Kinetic Kelvin-Helmholtz instability is identified as a driver of plasma transport and candidate mechanism for creating current pathways in power feed.
- Future work: leverage simulation capabilities to improve understanding and performance of pulsed power experiments