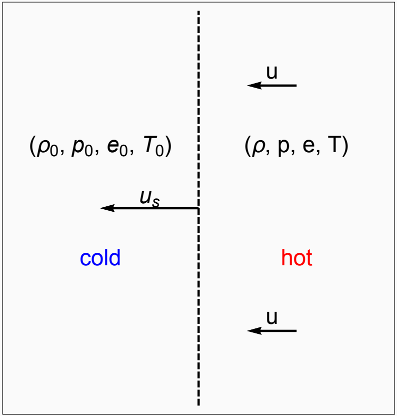
Opacity of Shock-Heated Boron Plasmas

W. R. Johnson and Joseph Nilsen

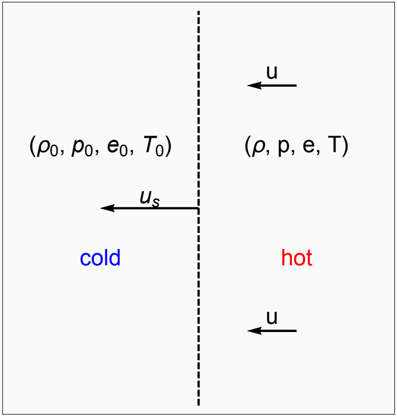
shock waves and opacity

Outline

Shock Waves and the Hugoniot Equation



Shock Waves and the Hugoniot Equation



Conservation Laws

$$\rho_0 u_s = \rho(u_s - u)$$
$$p_0 + \rho_0 u_s^2 = p + \rho(u_s - u)^2$$
$$e_0 + u_s^2/2 = e + (u_s - u)^2/2$$

Rankine[1870] - Hugoniot[1887] Equation

$$H(T,\rho) = (e - e_0) - \frac{1}{2}(p + p_0)(1/\rho_0 - 1/\rho) = 0$$

"Generalized" Thomas-Fermi Theory

Divide the plasma into neutral WS cells, $V_{WS} = \frac{A}{N_A\,\rho}$, each containing a nucleus and Z electrons.

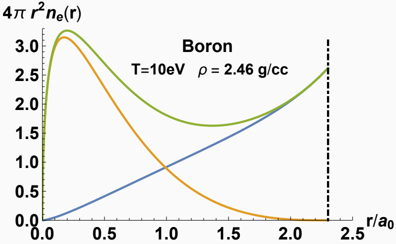
$$V(r) = -\frac{Z}{r} + \int_{r \le R} \frac{n_e(r')}{|\boldsymbol{r} - \boldsymbol{r}'|} d^3 r'$$

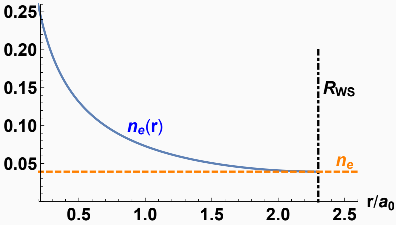
$$n_e(r) = \frac{1}{\pi^2} \int_0^{\infty} \frac{p^2 dp}{1 + \exp[(p^2/2m + V(r) - \mu)/kT]}$$

$$Z = \int_{r \le R} n_e(r) d^3 r$$

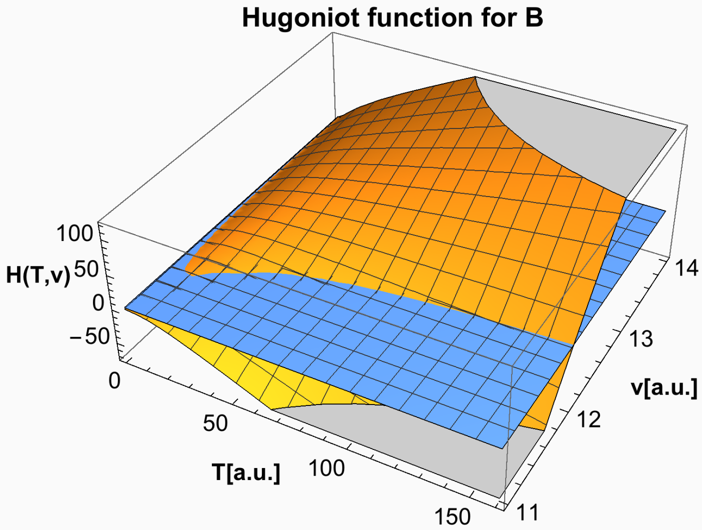
- The above three equations are solved self-consistently to obtain $V(r)$, $n_e(r)$ and μ. Other plasma properties such as electron pressure p and energy e can then be determined.
- The ion contributions to p and e are described using ideal gas theory.

Thomas-Fermi Electron Densities

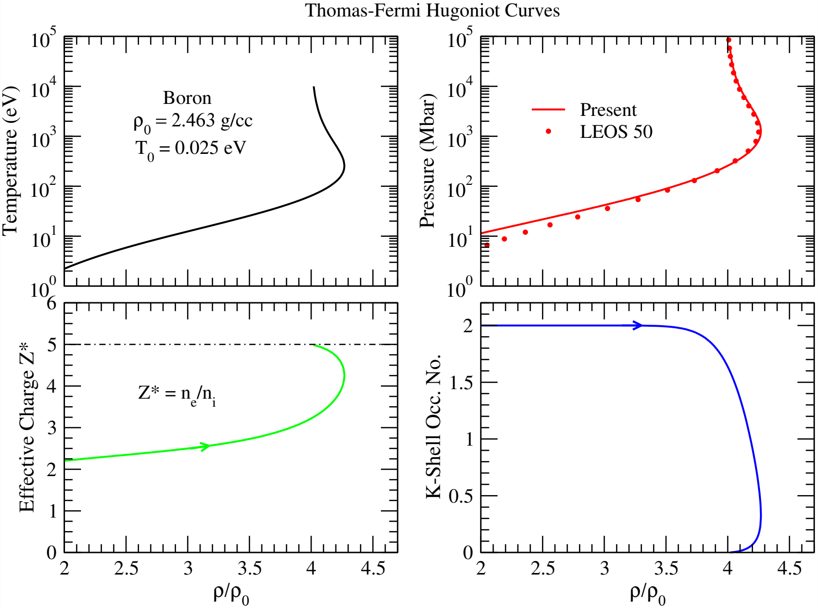




Hugoniot Function for B T_0=.025eV and $\rho_0 = 2.463$ g/cc.



Solution curves

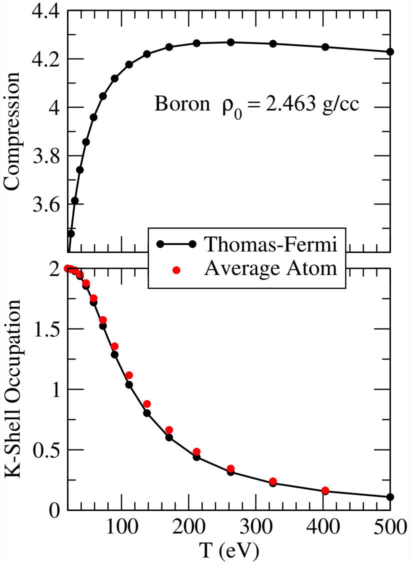


Average-Atom modification of Thomas Fermi Theory

$P_{nl}(r)$ and $P_{\epsilon l}(r)$ are solutions of the radial Schrödinger equation.
$P_{\epsilon l}(r)$ is normalized on the energy scale.

$$\begin{aligned}
V(r) = & -\frac{Z}{r} + \int_{r\leq R} \frac{n_e(r')}{|\boldsymbol{r}-\boldsymbol{r}'|} d^3r' + V_{xc}(r) \\
n_e(r) = & \frac{1}{4\pi r^2}\left[\sum_{nl} \frac{2(2l+1)}{1+\exp[(\epsilon_{nl}-\mu)/kT]} P_{nl}^2(r)\right. \\
& \left. + \sum_{l=0}^{\infty}\int_0^{\infty} d\epsilon \frac{2(2l+1)}{1+\exp[(\epsilon-\mu)/kT]} P_{\epsilon l}^2(r)\right] \\
Z = & \int_{r\leq R} n_e(r) d^3r
\end{aligned}$$

Average-Atom vs. Thomas Fermi Occupation Numbers



Opacity: Atomic Scattering Factor f_2

B. I. Henke *et al.*, At. Data & Nucl. Data Tables, **54**, 181 (1993)

Scattering from a single ion

$$e^{ikz} \rightarrow e^{ikz} + \frac{r_0 e^{ikr}}{r} f(\theta)\cos(\phi)$$

Scattering from many ions

$$e^{ikz} \rightarrow e^{inkz}$$

$$\begin{aligned} n &= 1 + \frac{r_0\lambda^2}{2\pi} n_i f(0) \quad \text{(Huygens)} \\ f(0) &= f_1 - i f_2 \\ f_2 &= \frac{\sigma_{bb}(\omega) + \sigma_{bf}(\omega) + \sigma_{ff}(\omega)}{2 r_0 \lambda} \\ I(z) &= I_0 \exp(-2 r_0 \lambda n_i f_2 z) \\ &= I_0 \exp(-n_i \sigma_{bf} z) \end{aligned}$$

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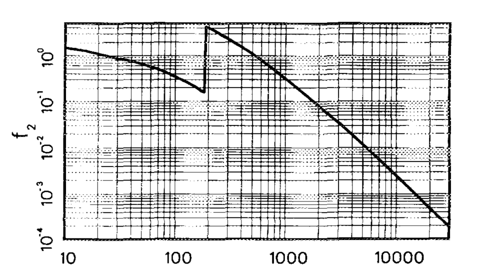
(cold) Boron



ω(eV)

Average-Atom Evaluation of $\sigma_{bf}(\omega)$ and $f_2(\omega)$

Atomic $\sigma_{bf}(\omega)$: J. J. Yeh & I. Lindau, ADNDT, **32**, 1 (1985).

$$\sigma_{bf}(\omega) = \frac{8\pi^2}{3}\alpha\omega|D|^2$$

$$D = \int_0^\infty P_{\epsilon 1}(r)\, r\, P_{1s}(r) dr$$

$$\sigma_{bf}(\omega) \to \frac{\text{occ.}}{2}\sigma_{bf}(\omega)$$

$$f_2(\omega) = \frac{\sigma_{bf}(\omega)}{2r_0\lambda}$$

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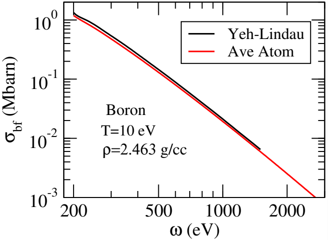
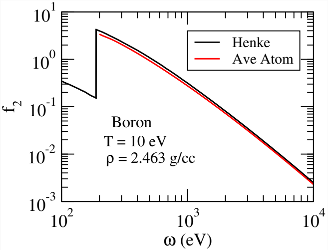
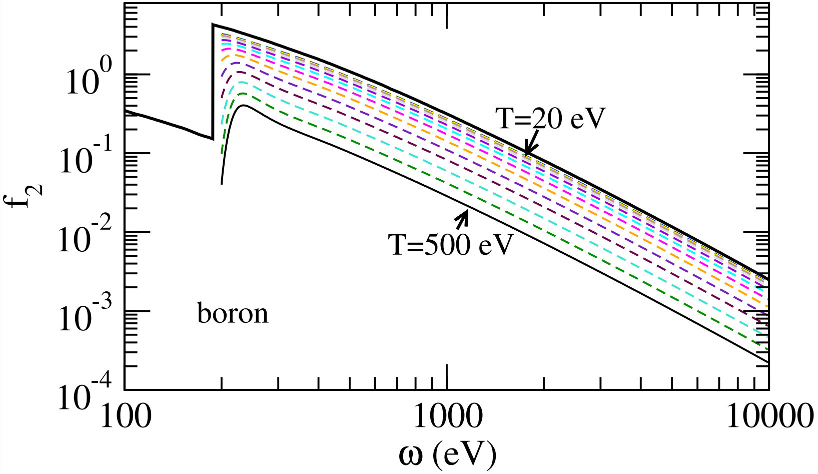




Opacity: $f_2(\omega)$ at points along Boron Hugoniot



Opacity: Mass Attenuation Coefficient μ/ρ

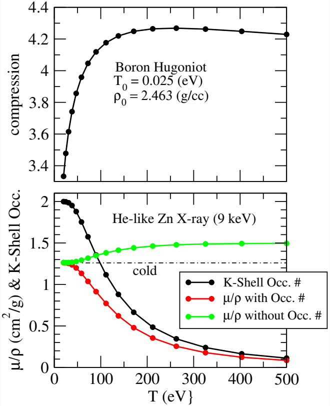
J. H. Hubbell and S. M. Seltzer,
physics.nist.gov/PhysRefData/XrayMassCoef/cover.html

Single Ion Plasmas

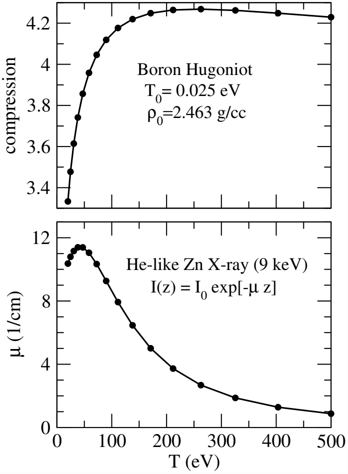
$$I(z) = I_0 \exp(-n_i \sigma_{bf} z)$$
$$= I_0 \exp(-\mu z)$$
$$\mu/\rho = \frac{N_A}{A} \sigma_{bf}$$

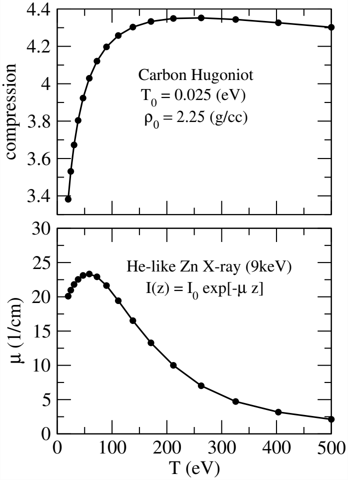
Multi-ion Plasmas

$$\mu/\rho = \sum_i x_i \, (\mu/\rho)_i$$



Optical Depth: Single-Ion Plasmas





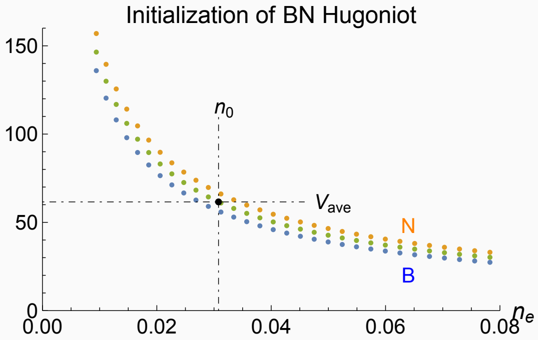
Multi-Ion Plasmas

Ion concentrations x_1 and x_2, $x_1 + x_2 = 1$

$$\frac{x_1 A_1 + x_2 A_2}{N_A \rho} = x_1 \frac{A_1}{N_A \rho_1} + x_2 \frac{A_2}{N_A \rho_2}$$

$$n_e(2) = n_e(1)$$

Solve for ρ_1 and ρ_2



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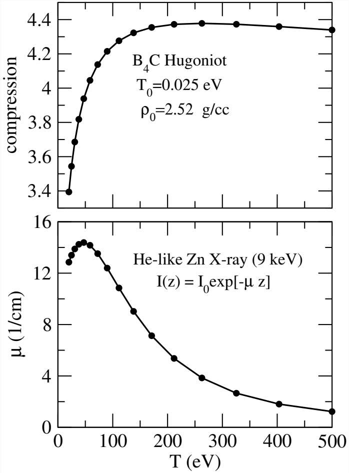

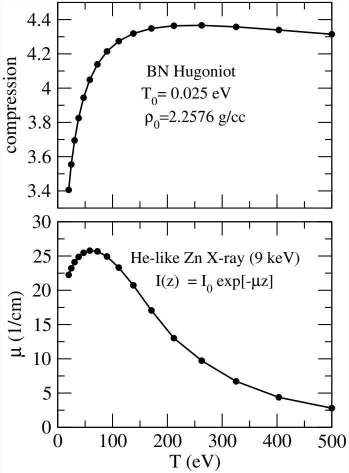

Conclusion

- At relatively low temperatures (10-20 eV), average-atom calculations of f_2 and μ/ρ for boron plasmas agree well with previously tabulated cold matter values for $\omega < 10$ keV.
- $f_2(\omega)$ and $\mu/\rho(\omega)$ decrease rapidly in the temperature range 20-500 eV where the K-shell occupation falls from 2 to 0.1.
- The opacity is also sensitive to temperature along the Hugoniot through the temperature dependence of the photoionization cross section (18% for a 9keV photon at $T = 500$ eV)

Acknowledgment:
Thanks to Phil Sterne, Brian Wilson and Heather Whitley for helpful advice.

Some Related Work

- J-M Yuan, Chinese Phys. Letts. **19**, 1459 (2002)
- N. Shaffer, N. Ferris *et al*, HEDP **23**, 31 (2017)
- M. B. Trzhaskovskaya and V. K. Nikulin, HEDP **26**, 1 (2018)
- R. T. Piron and T. Blinski, Contrib. Plasma Phys. **58**, 30 (2018)
- This work: to be published HEDP – arxiv:1810.00244