

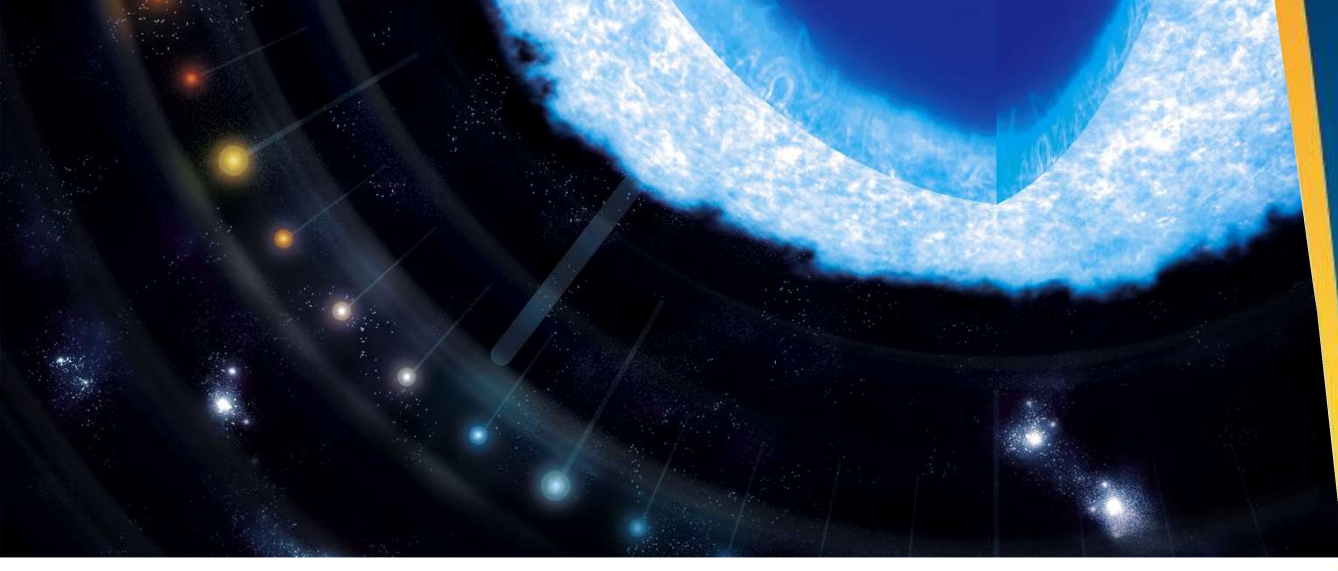
Calculating Melting Curves for Crystallizing Stars

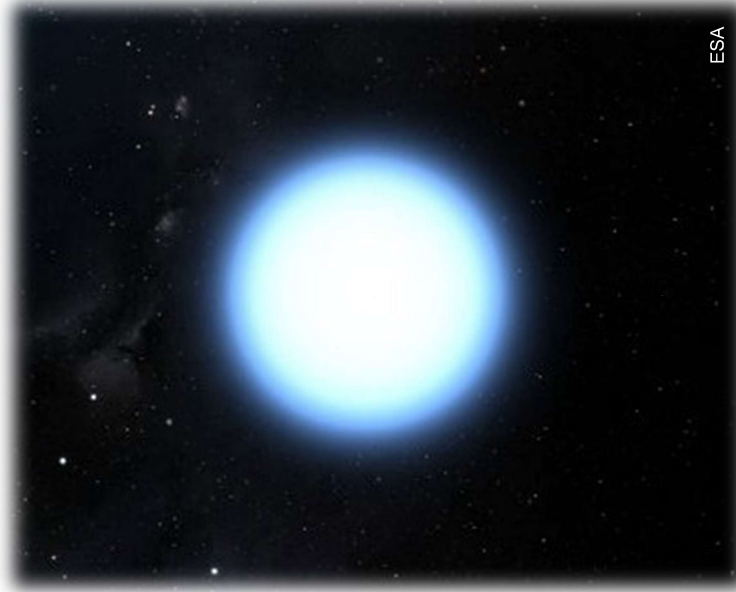
From dense plasma physics to stellar
archeology

Simon Blouin
LANL

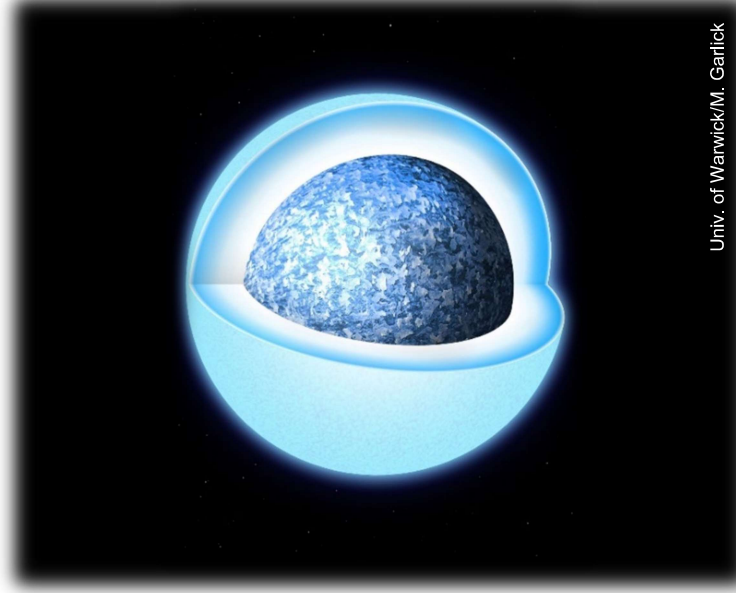
2020-11-05

Slides: LA-UR-20-28803

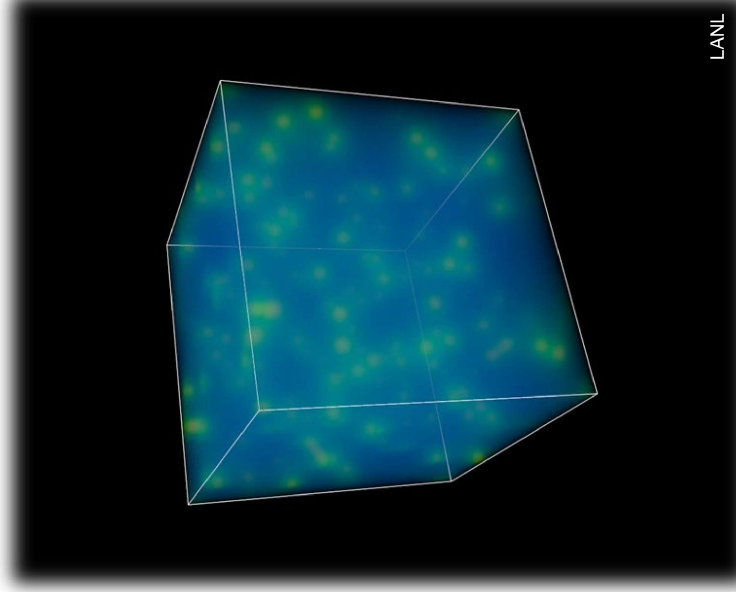




1. What are white dwarfs and why are they interesting?

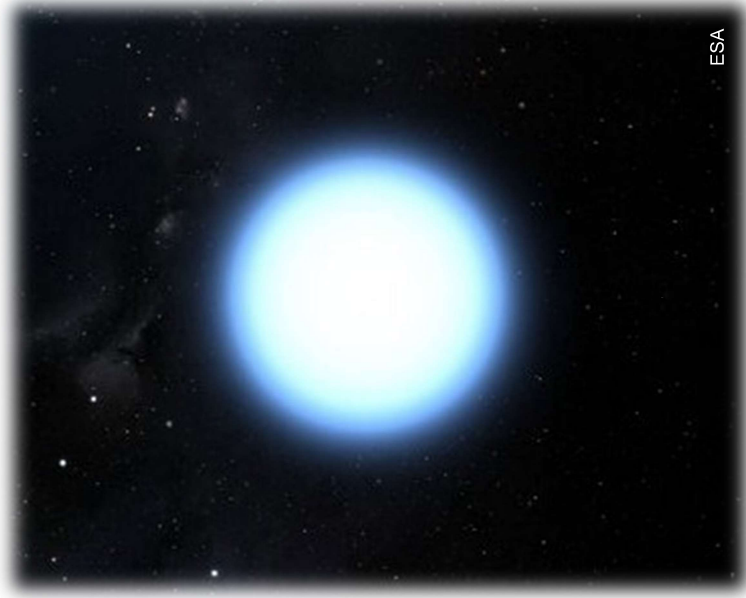


2. Core crystallization



3. Phase diagram calculation

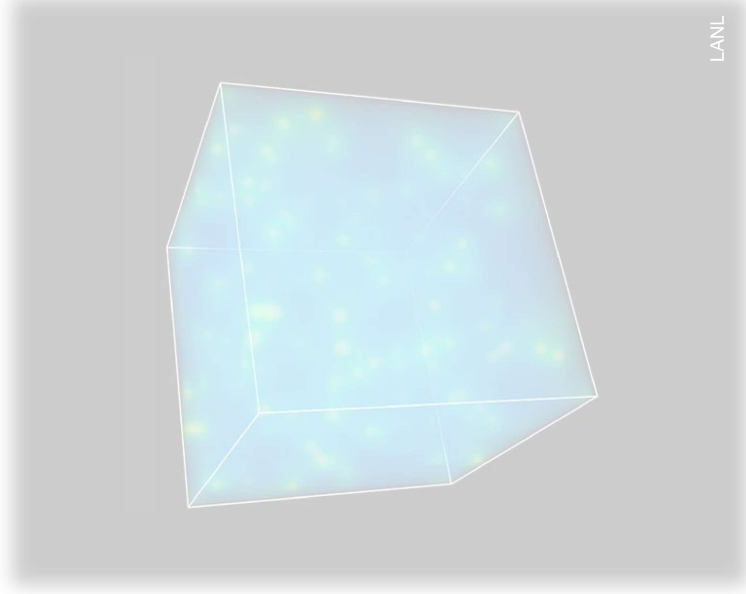




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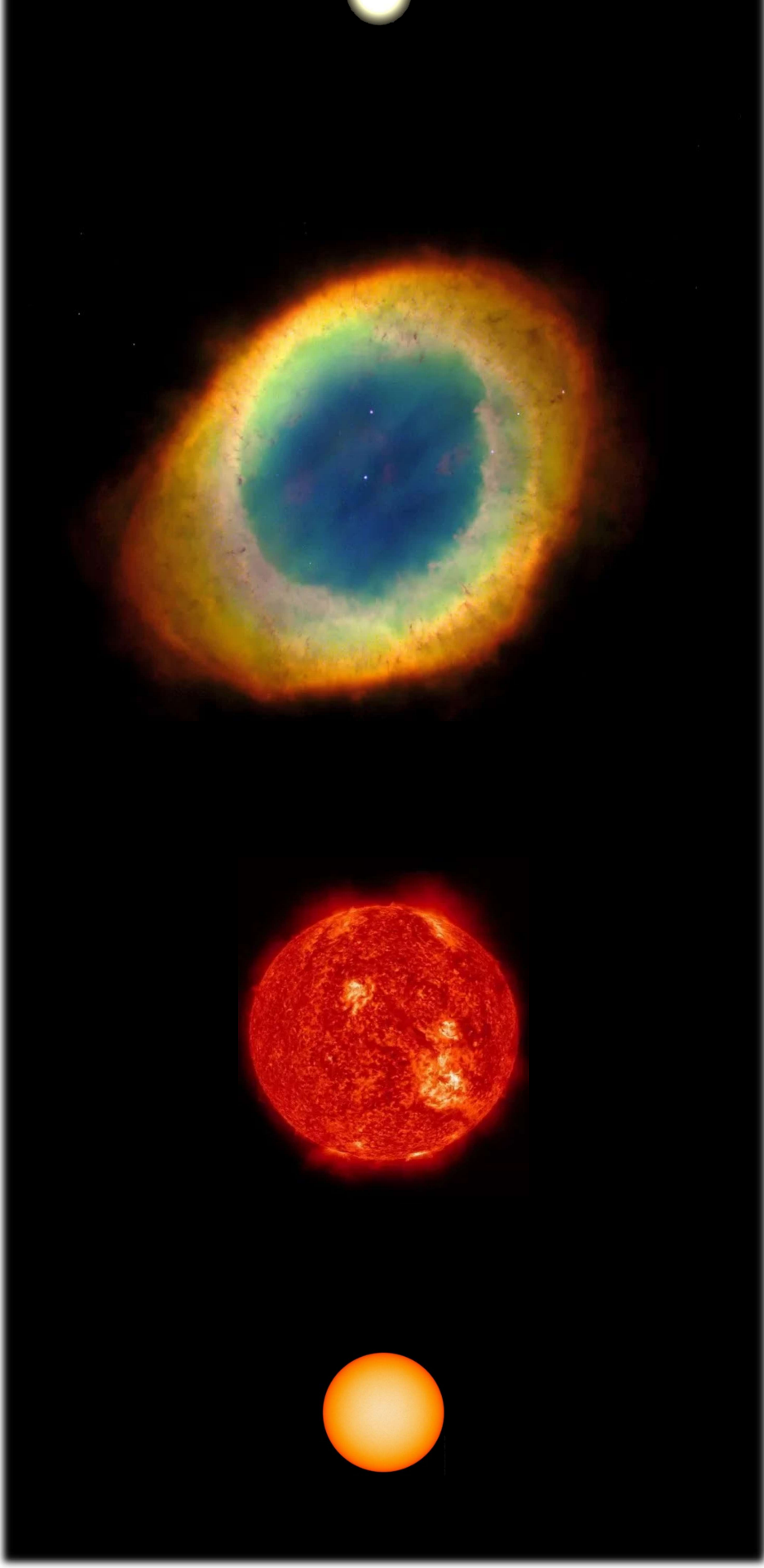
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97% of stars end their lives as white dwarfs

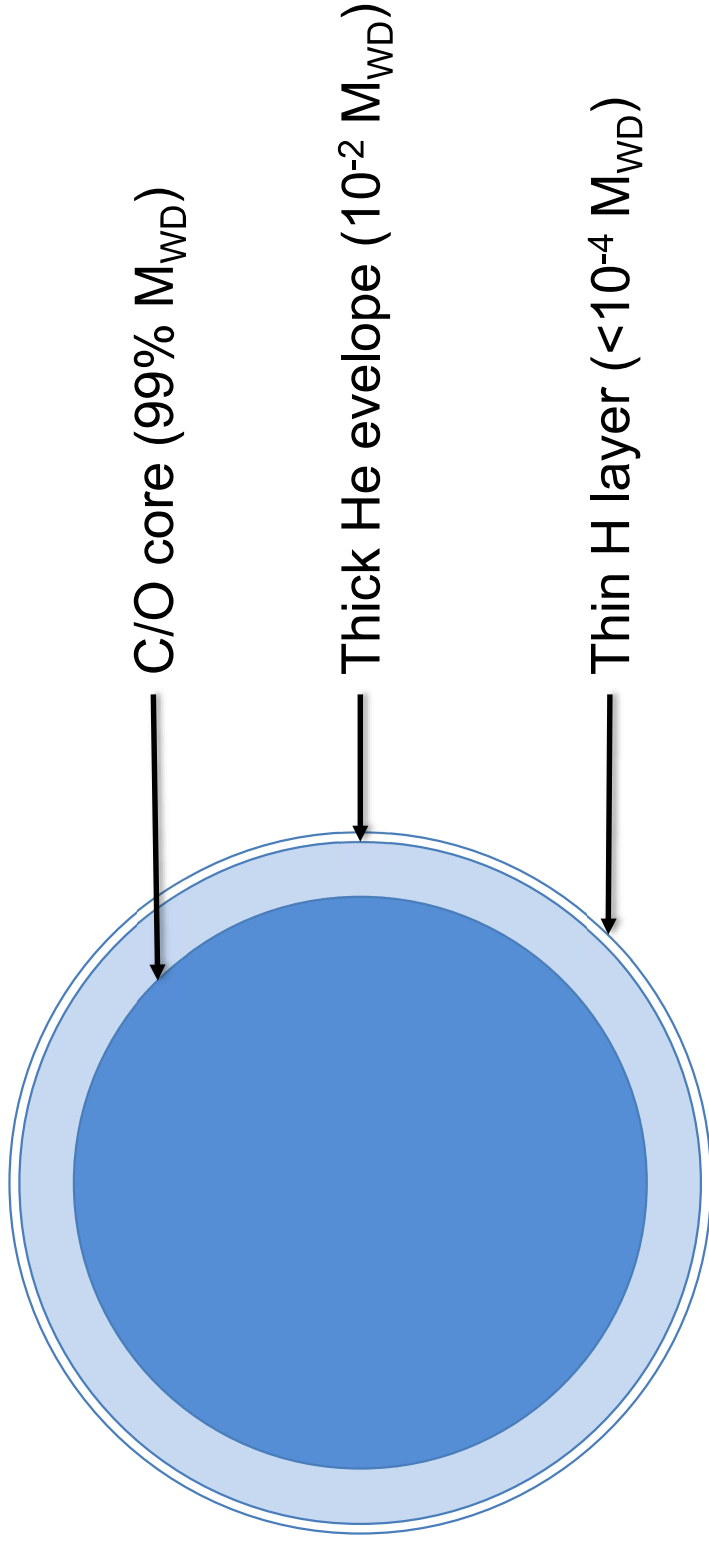
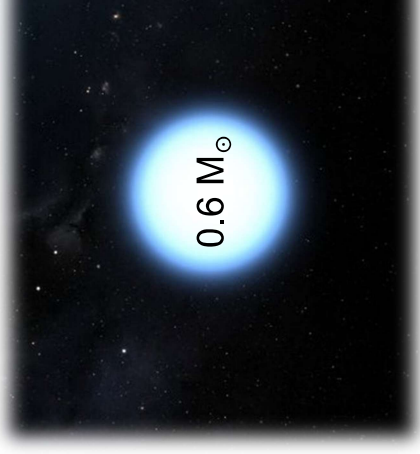


NASA/SDO/HMI; N



Managed by Triad National Security, LLC for the U.S. Department of Energy's NNSA

Anatomy of a white dwarf



$10^6 - 10^8 \text{ g/cm}^3$
 $10^5 - 10^8 \text{ K}$

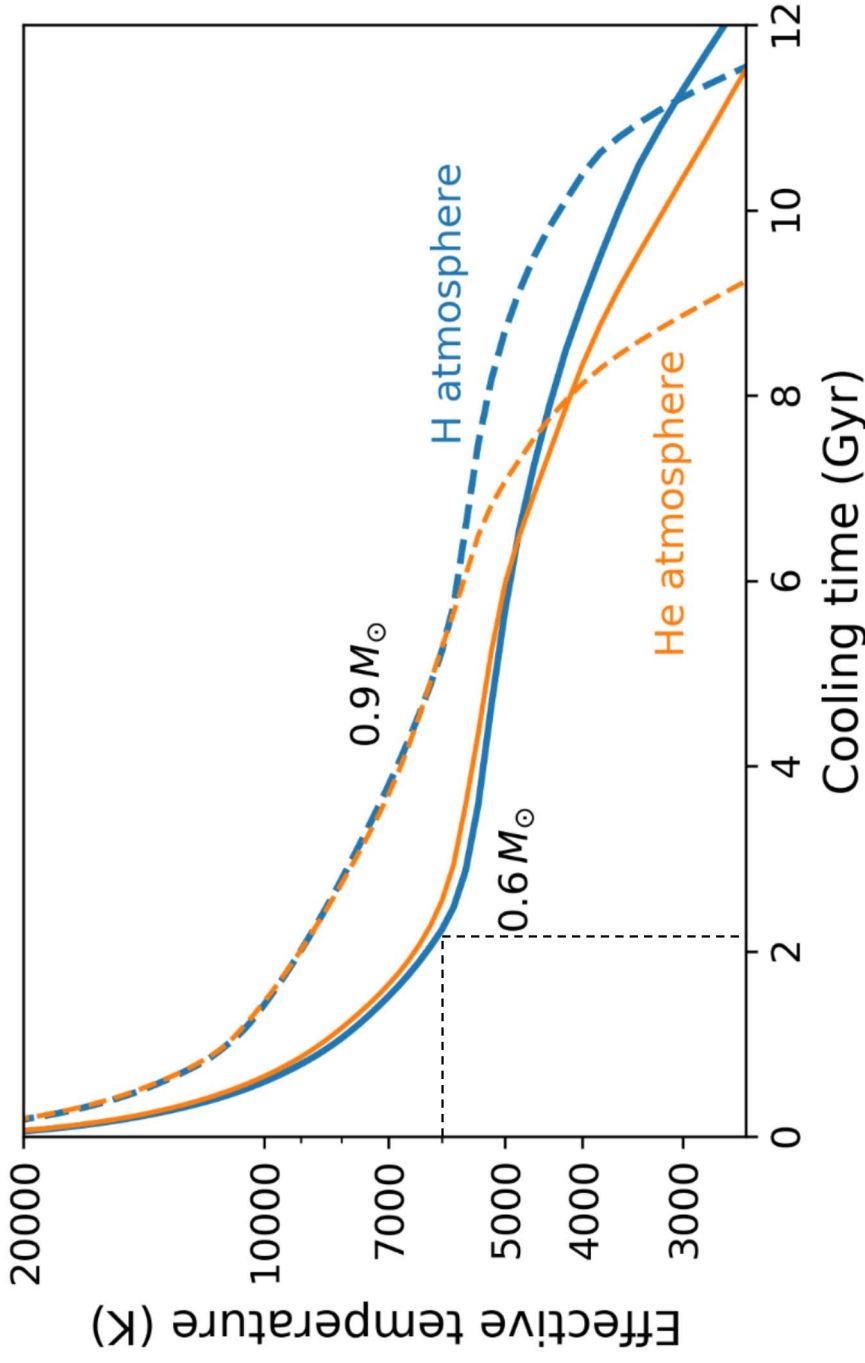
$0 - 2 \text{ g/cm}^3$
 $3000 - 10^5 \text{ K}$

White dwarf cooling

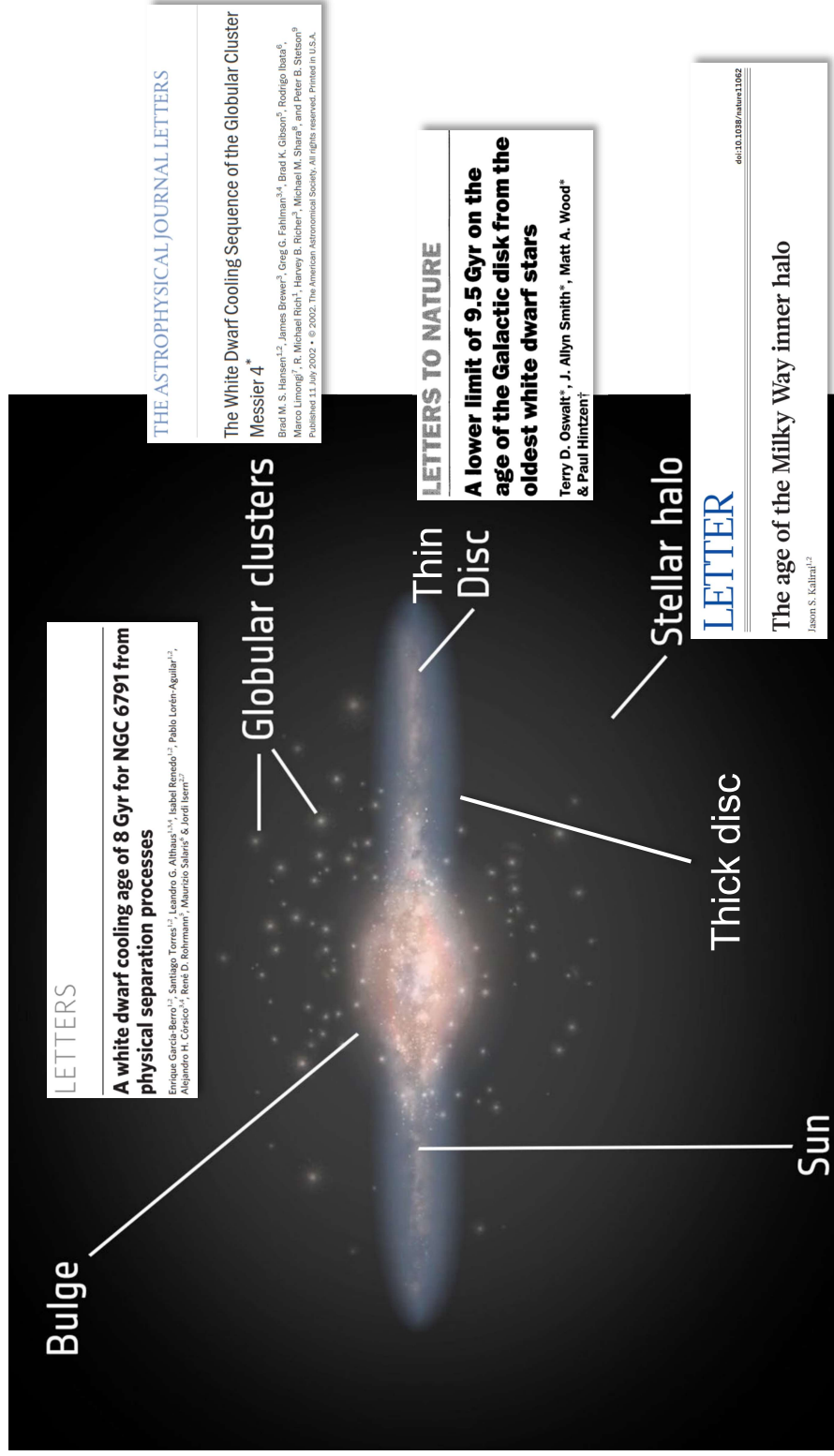
White dwarfs cool down monotonically for their lives

The age of a white dwarf can be obtained from its

- temperature
- mass
- atmospheric composition

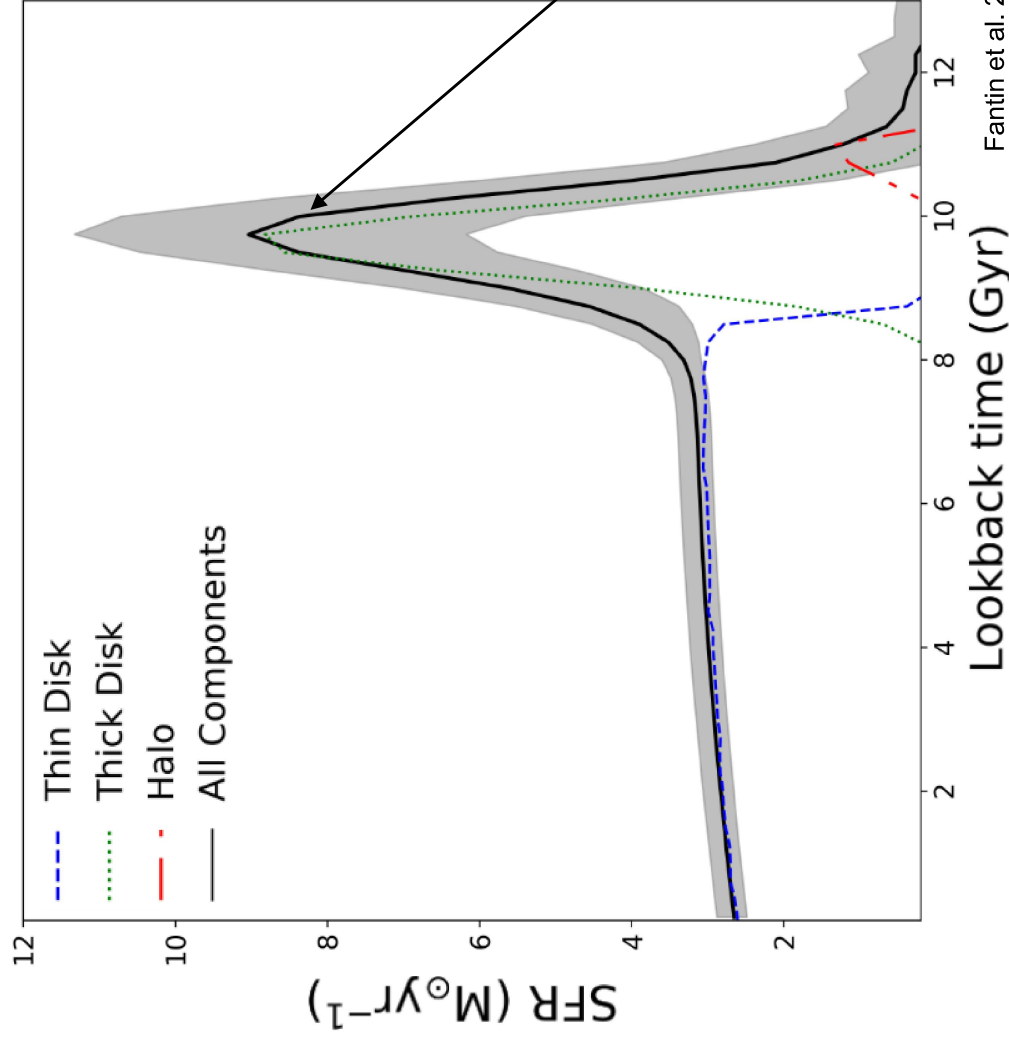


White dwarf cosmochronology



Using this method
has been possible
to measure the
ages of the
major components
of the Milky Way
Galaxy

Stellar archeology

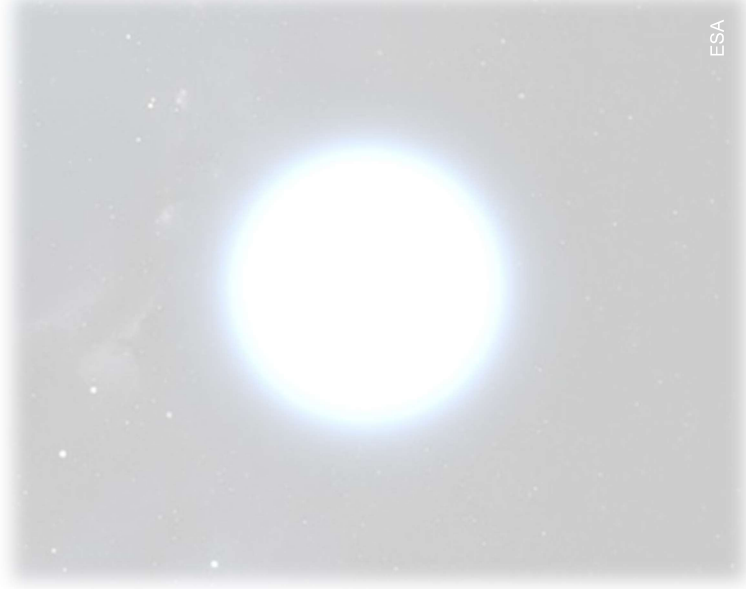


Fantini et al. 2019

White dwarf cosmochronology used to infer **stellar formation** allowing a detailed reconstruction of our Galaxy's history

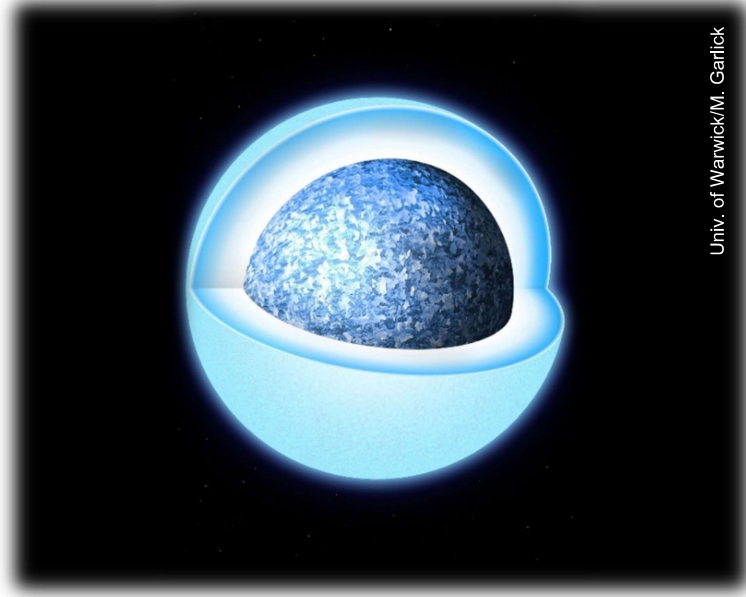
Such applications demand advanced cooling models

Likely merger with a satellite galaxy



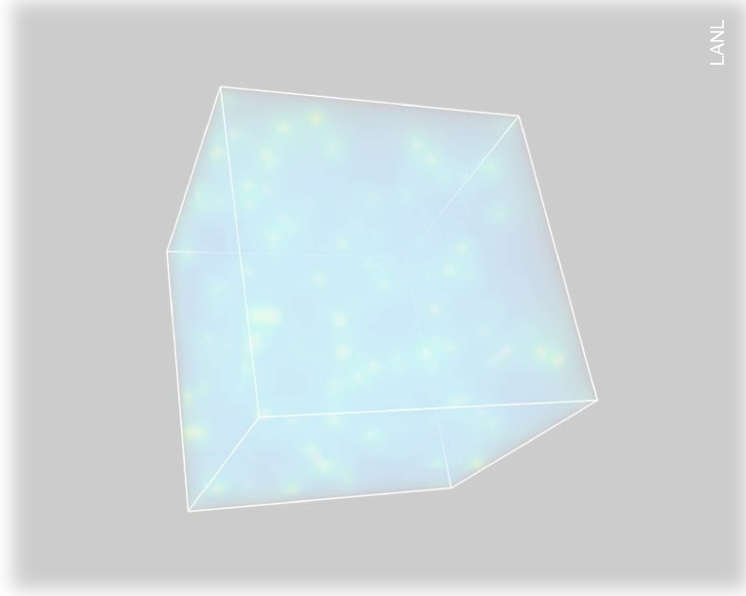
ESA

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Univ. of Warwick/M. Garlick

2. Core crystallization



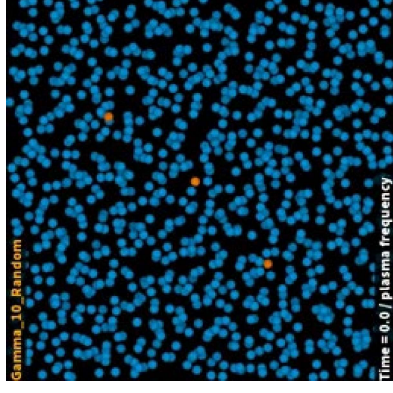
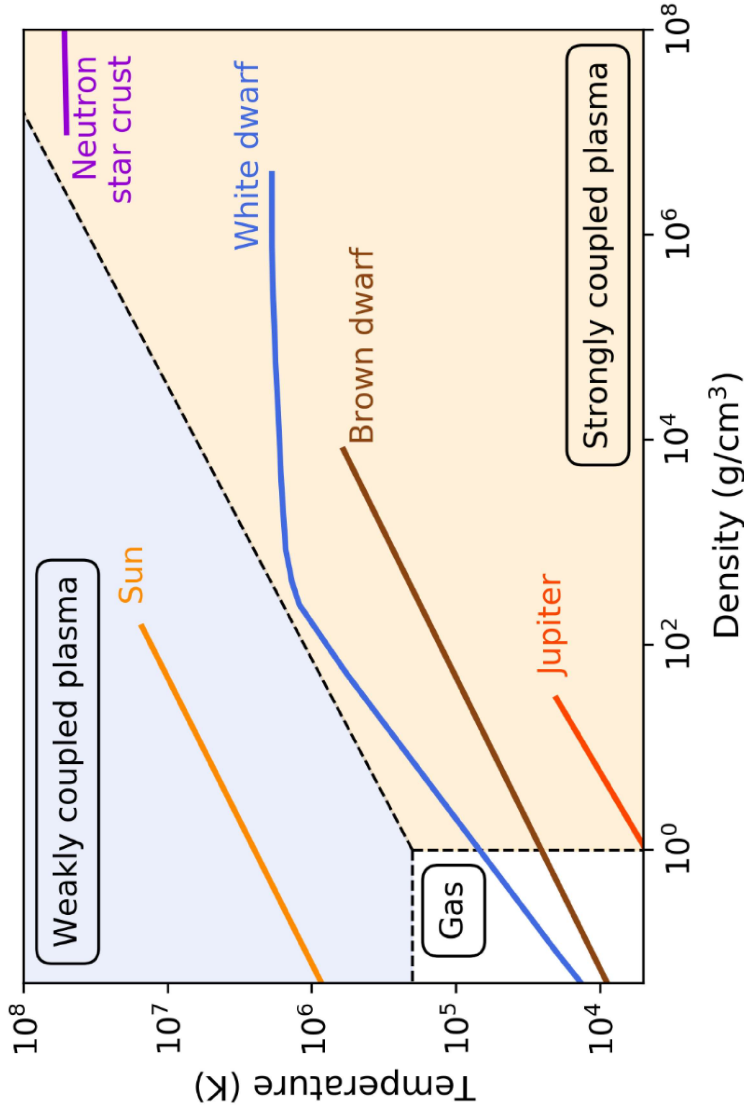
LANL

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White dwarf core crystallization



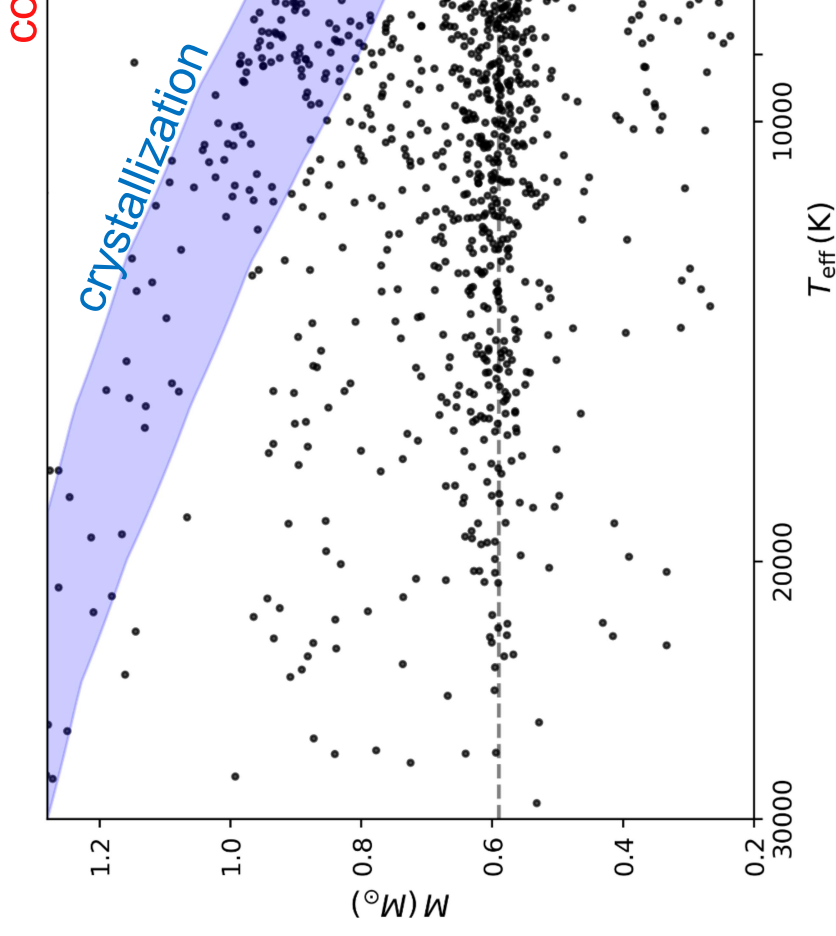
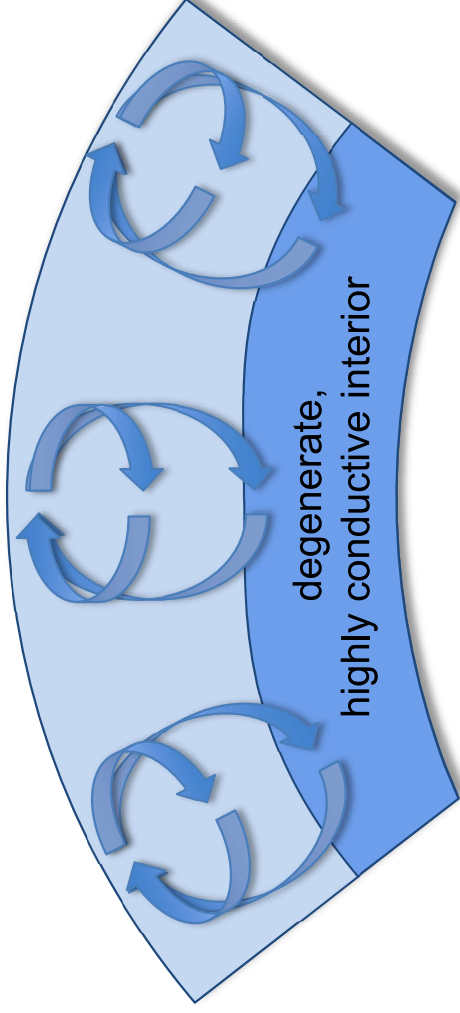
Cooling →

WDs undergo a **liquid** → **solid phase transition** that needs to be accurately modeled to unlock the power of **precision cosmochronology**

Identifying the observational signature of core crystallization

It took 50 years to discover the observational signature of core crystallization

To distinguish the effect of core crystallization from **convective coupling, mass measurements** were needed

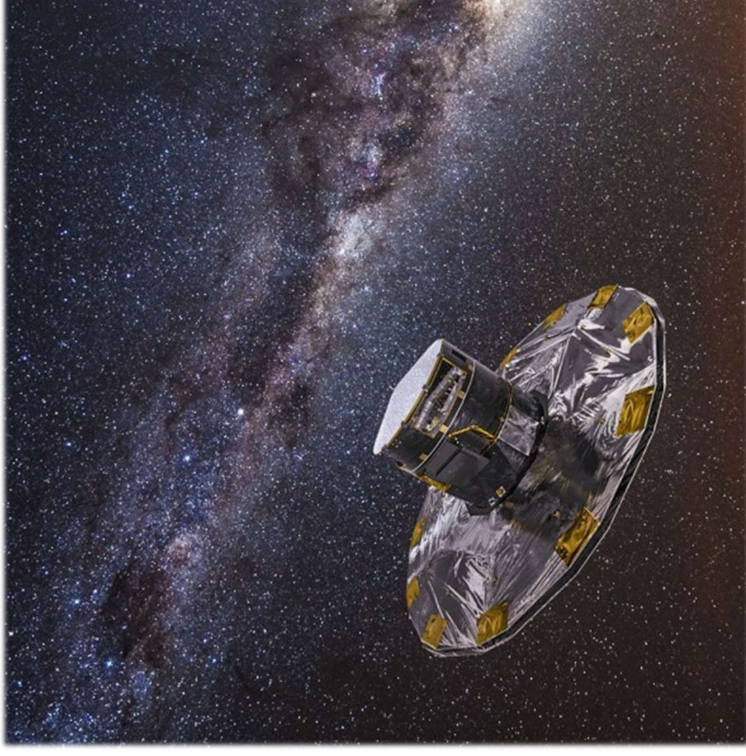
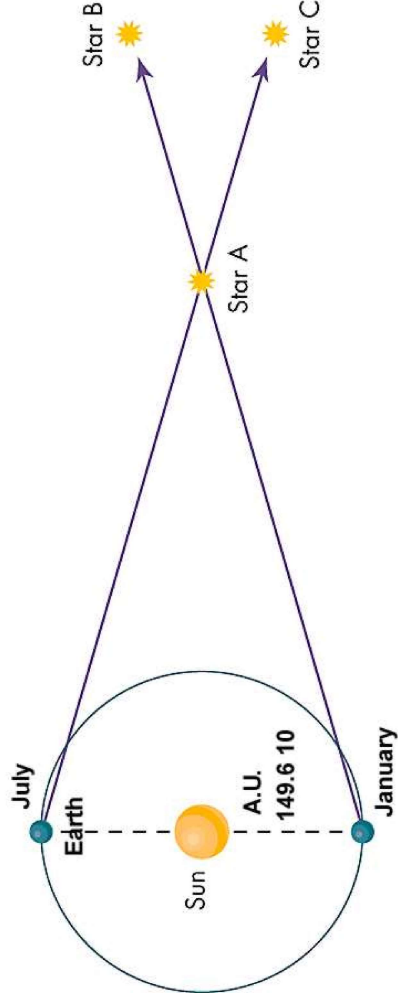


How the Gaia satellite provided mass measurements

ESA's Gaia satellite has increased by a factor of 10 the number of known white dwarfs (now ~250,000)

Launched in 2013, first major data release in 2018

Gaia measures **stellar parallaxes** with an unprecedented accuracy



How the Gaia satellite provided mass measurements

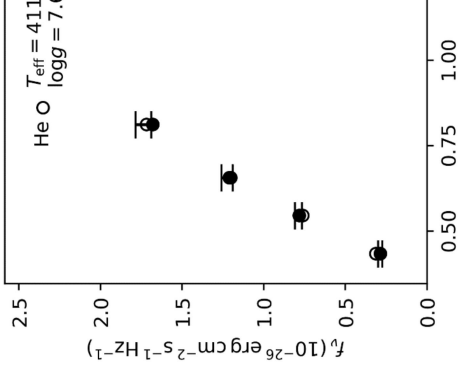
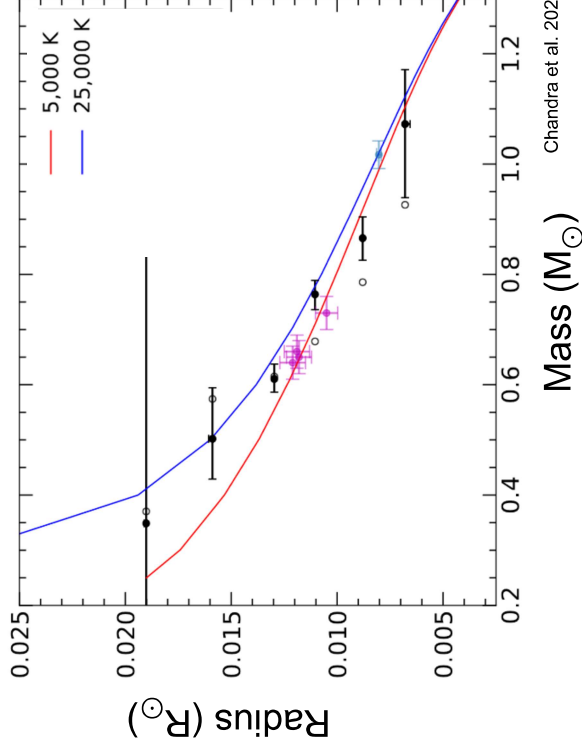
Gaia measurements



$$\text{Observed flux} = \frac{\pi R^2}{D^2} \times \text{Surface flux}$$

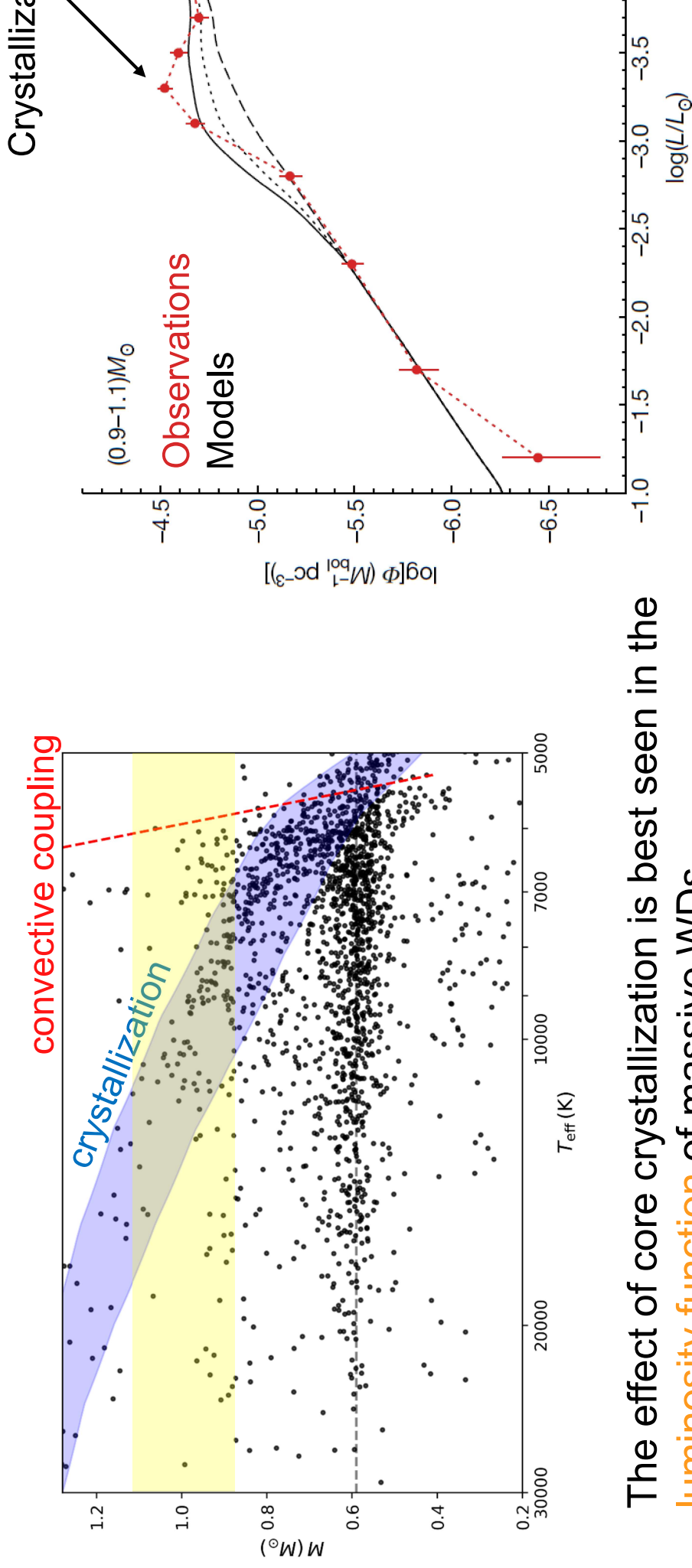
Model at

WDs follow a well-defined mass-radius relationship

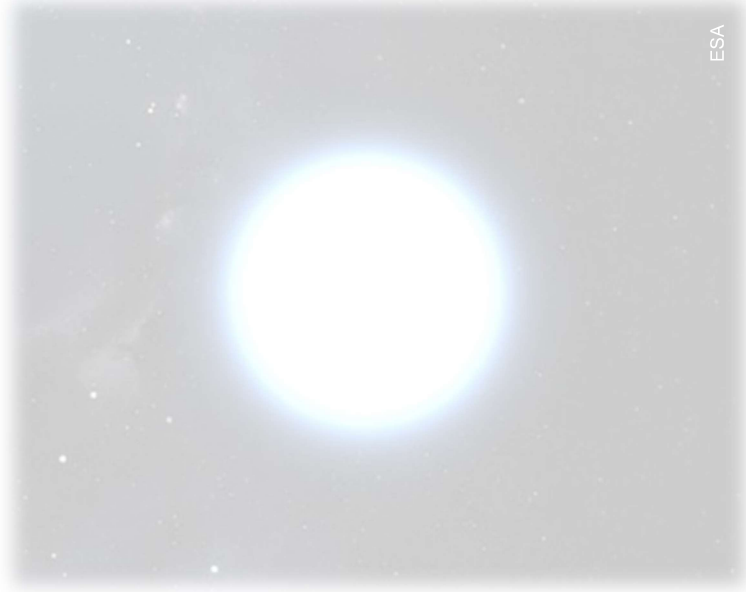


In the pre-Gaia era, there was a degeneracy between R and D for most white dwarfs

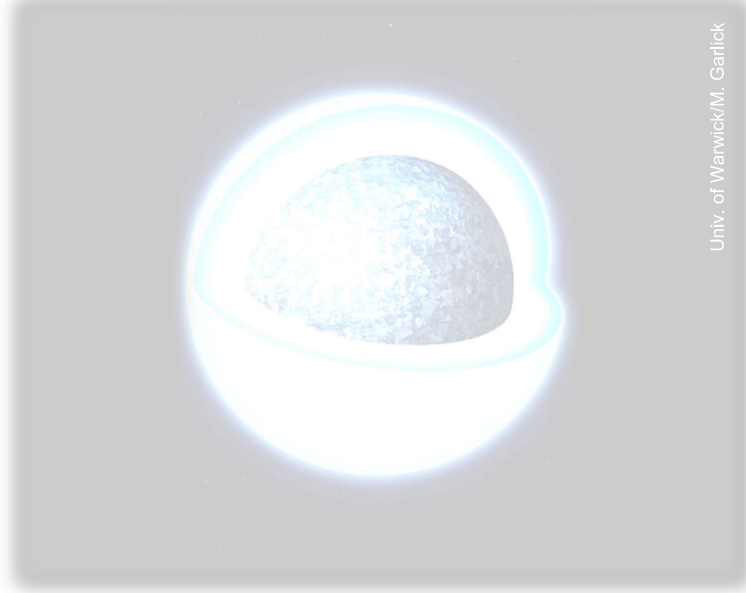
The signature of core crystallization



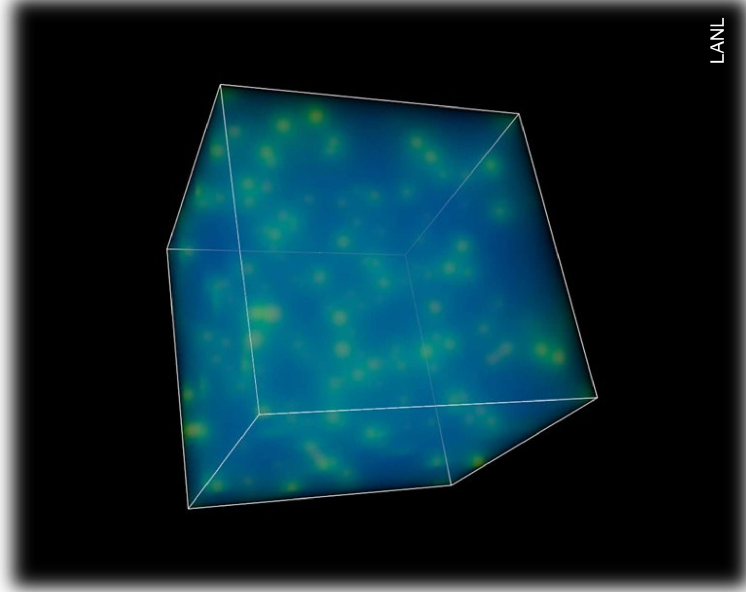
The effect of core crystallization is best seen in the
luminosity function of massive WDs



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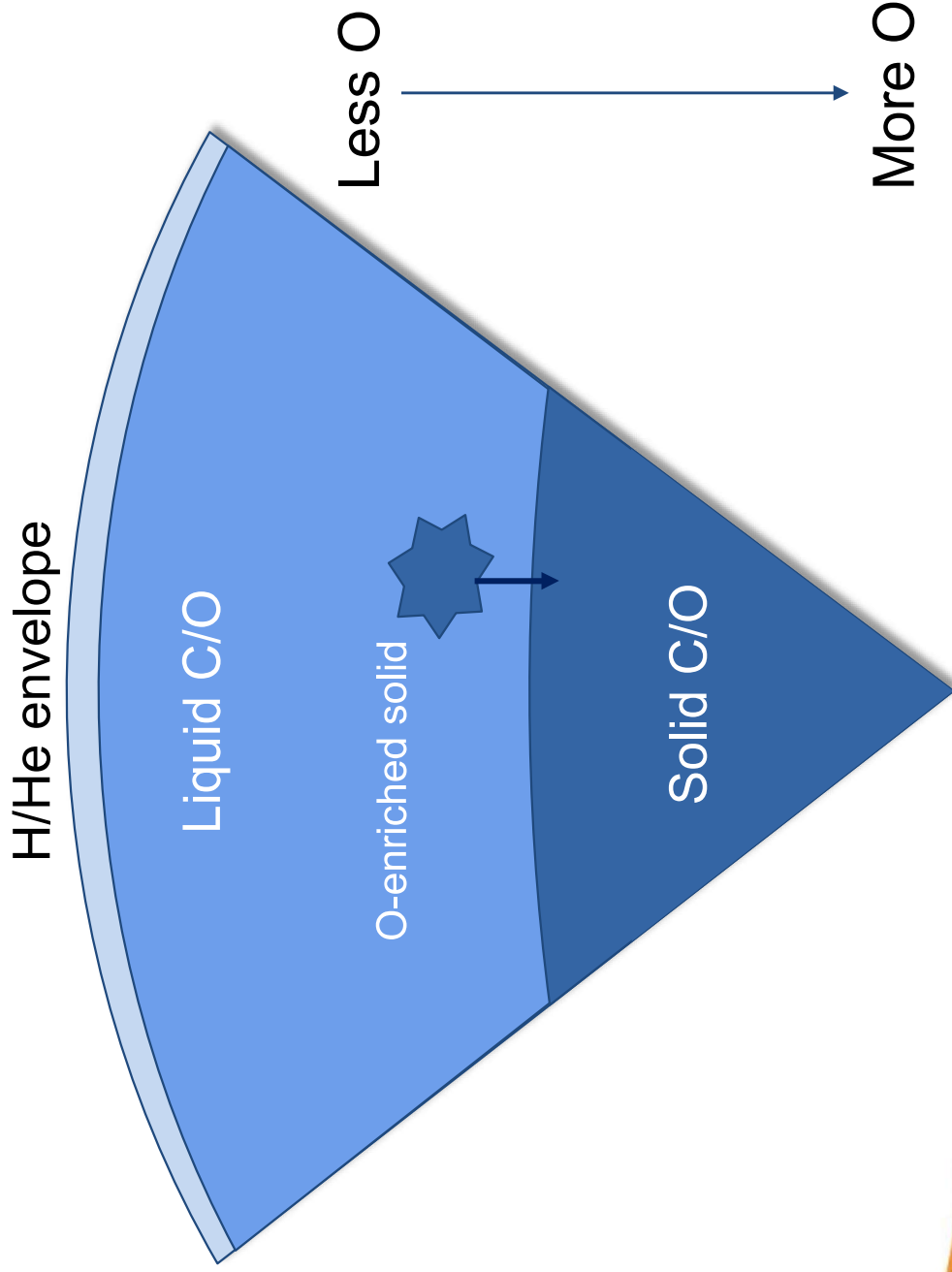
2. Core crystallization



3. Phase diagram calculation



Carbon/oxygen phase separation



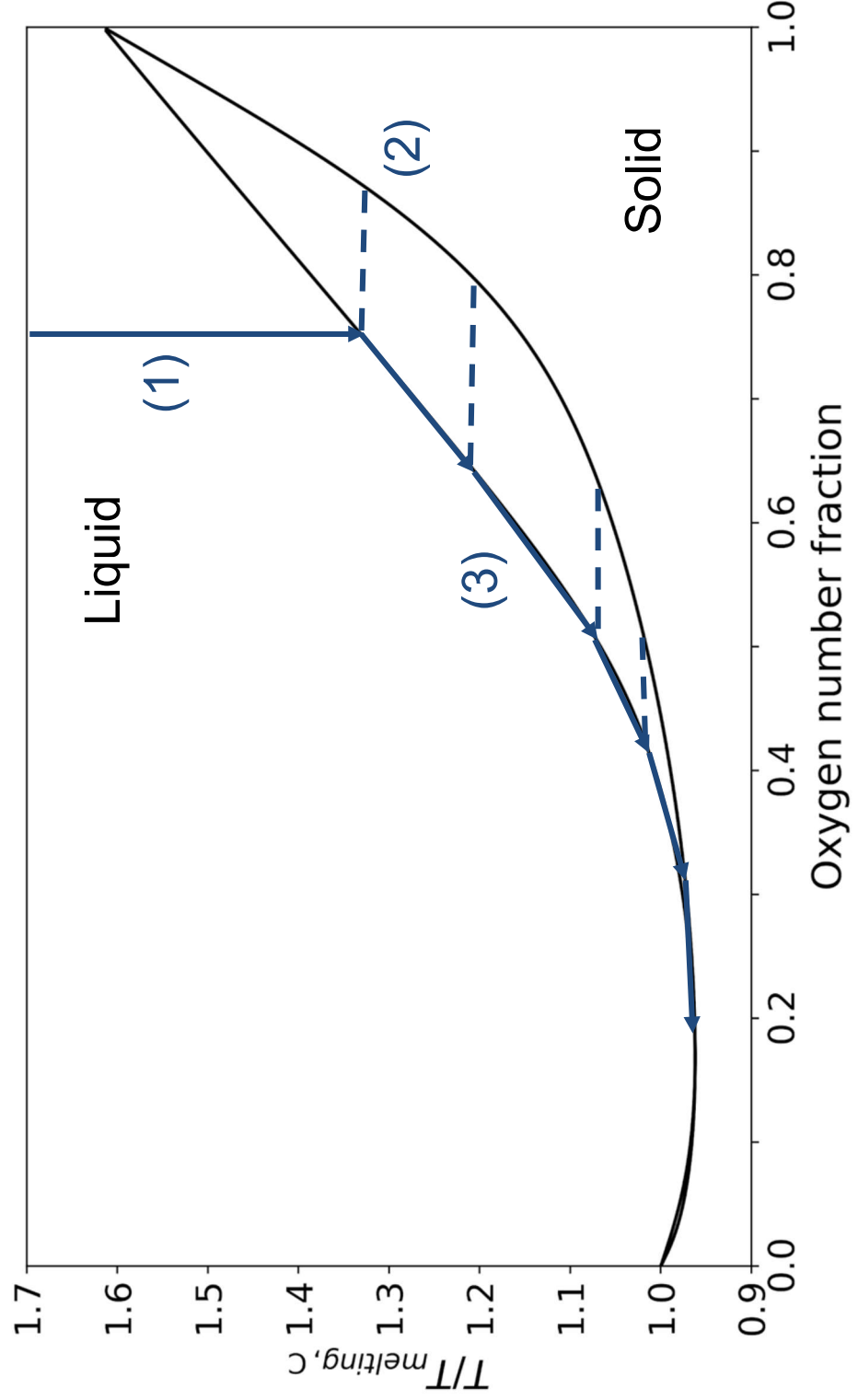
The crystallization from **upward** as the core cools.

The **new crystals** form at the solid-liquid interface and grow **downward** compared to the liquid.

Gradually, a concentration gradient of oxygen is built, thereby **releasing** **gravitational energy**.

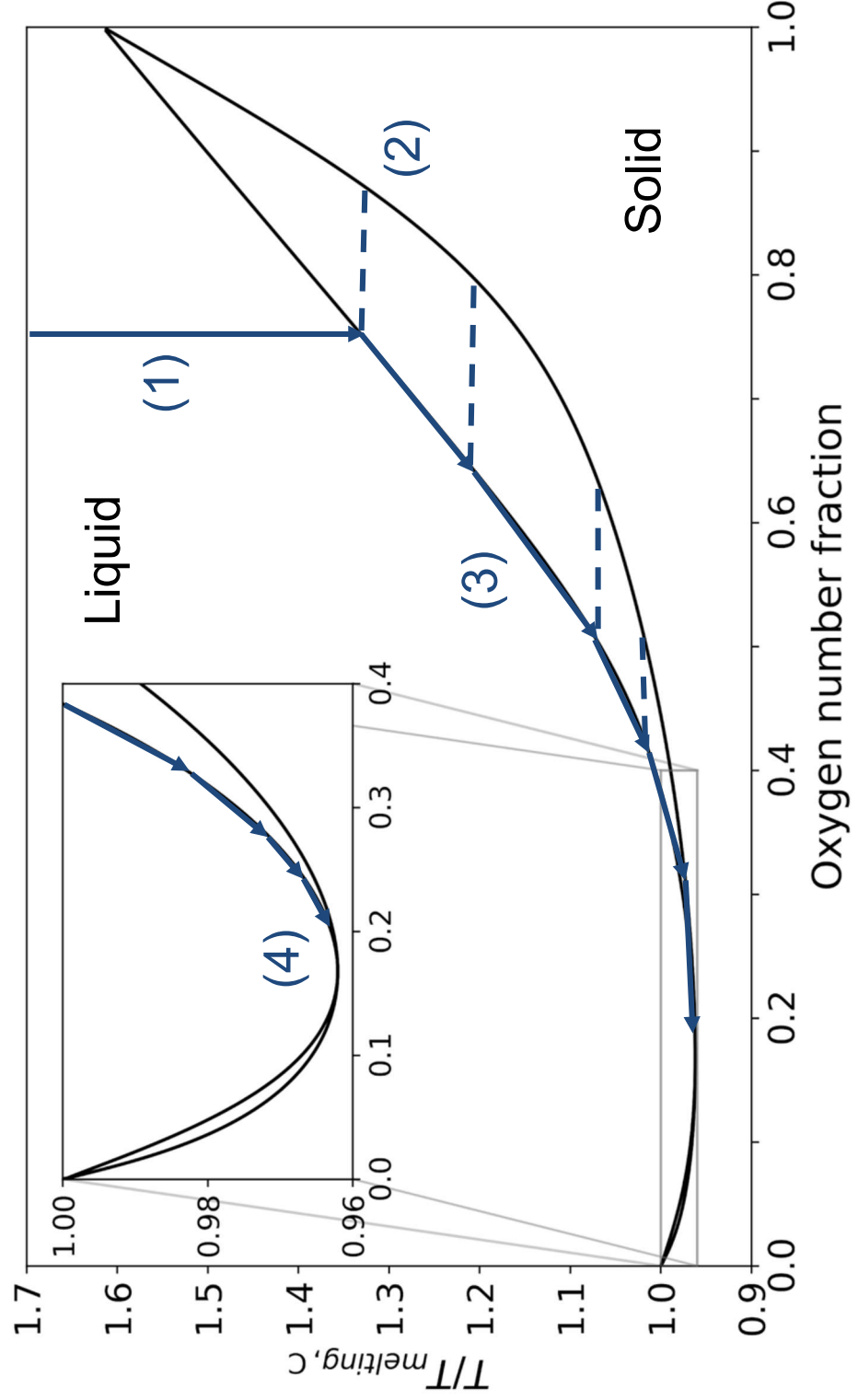
The carbon/oxygen phase diagram

1. The liquid cools **constant concentration** reaches the liquidus
2. The liquid coexists **that contains more**
3. As the crystallization **progresses, the liquid becomes** **impoverished in O**



The carbon/oxygen phase diagram

1. The liquid cools **constant concentration** reaches the liquidus
2. The liquid coexists **that contains more**
3. As the crystallization **progresses, the liquid becomes** **impoverished in O**
4. This process ends **when the azeotropic**

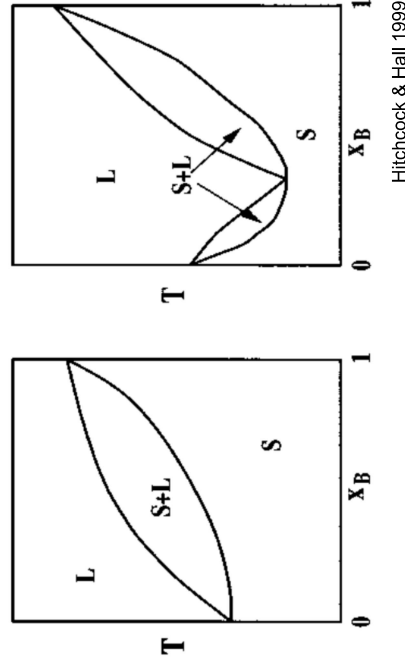


Previous attempts at computing the C/O phase diagram

Option 1: Analytical free energy models

Need to interpolate sparse simulation data, so results are **sensitive to arbitrary choices** made when building the interpolation functions

Even the *qualitative* shape of the phase diagram is affected by those choices (DeWitt et al. 1996)



$$f_{\text{mix}} = x_1 f_{\text{OCP}}(\Gamma_1) + x_2 f_{\text{OCP}}$$

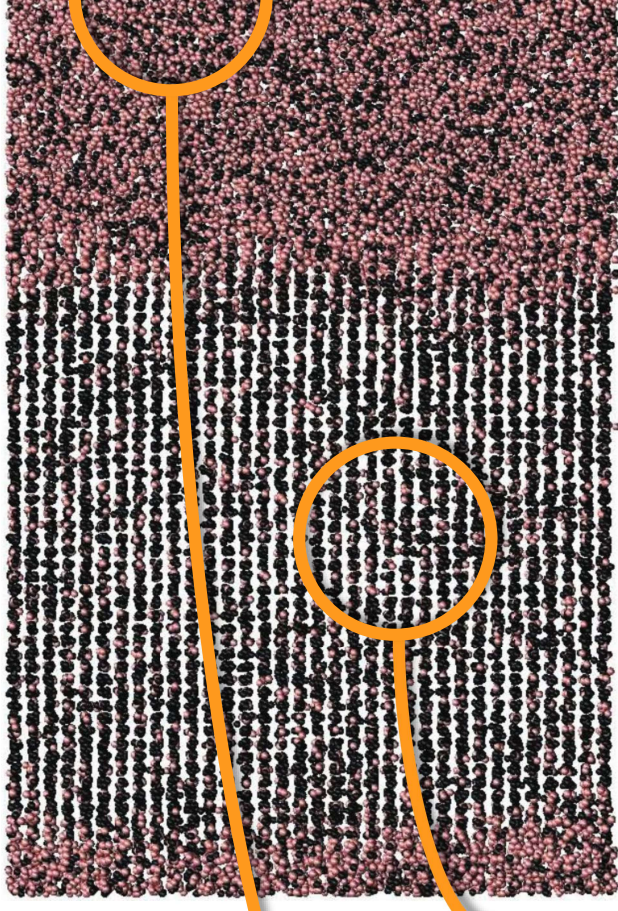
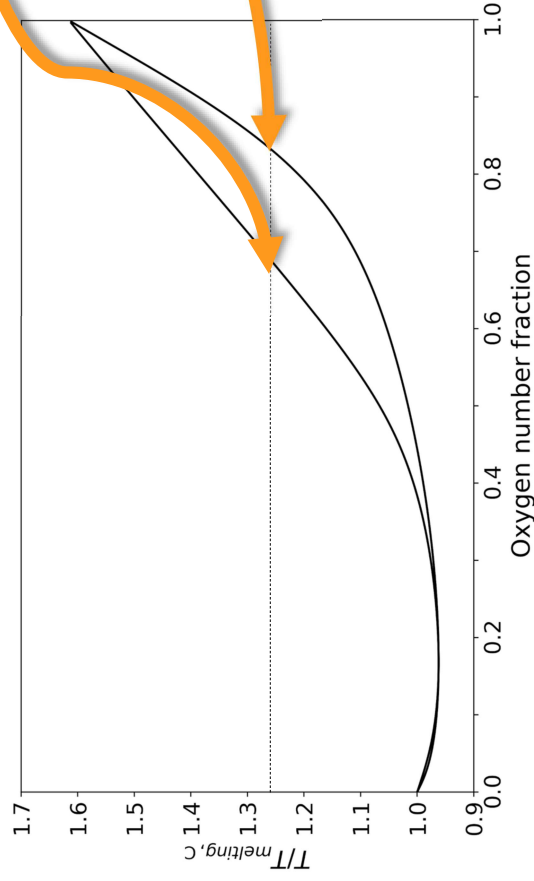
fitted to MC calculation

Previous attempts at computing the C/O phase dia

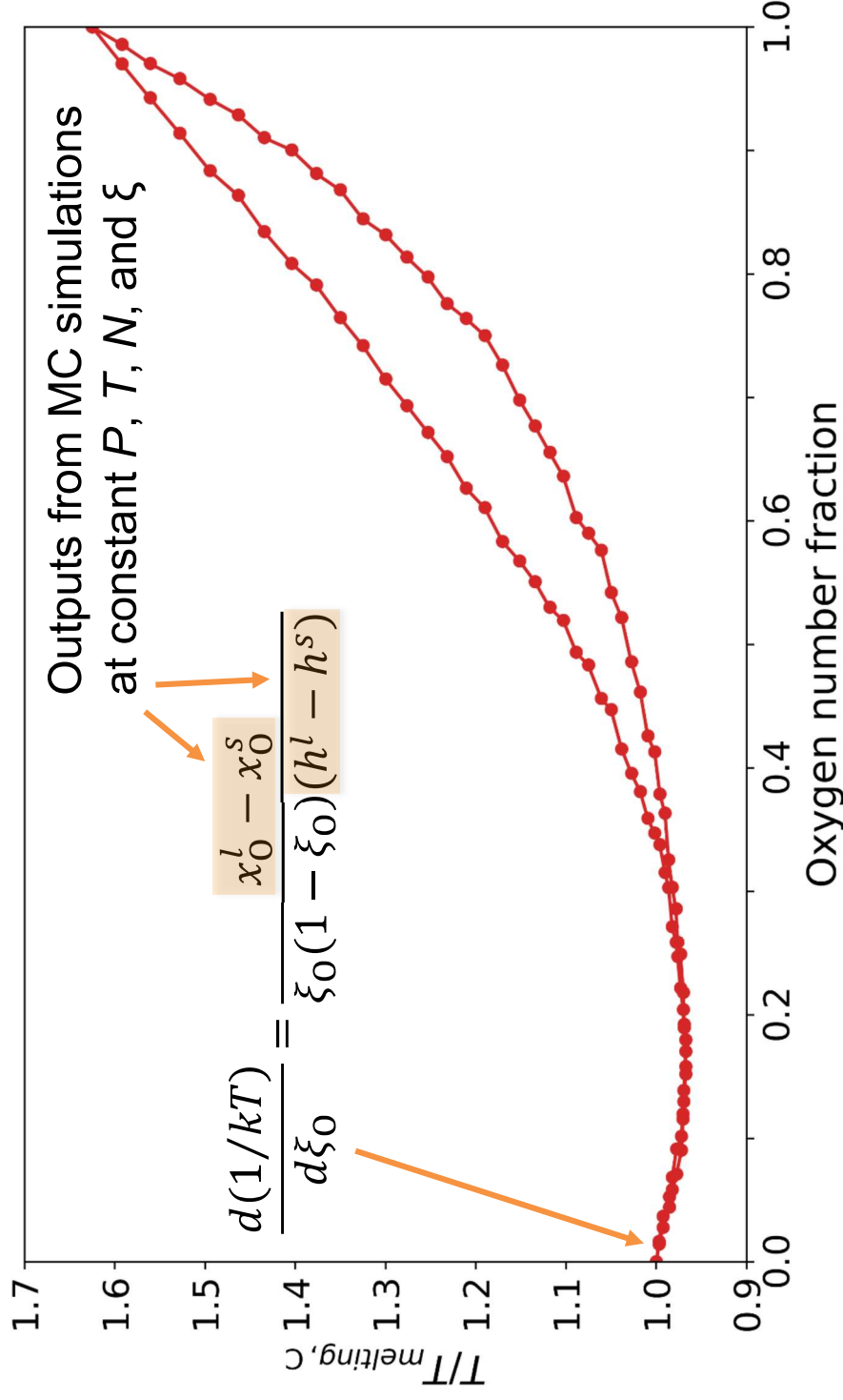
Option 2: Two-phase MD simulations

A solid-liquid interface is simulated and ions are allowed to diffuse

Very **costly** and sensitive to **finite-size effects**



The Gibbs-Duhem integration technique



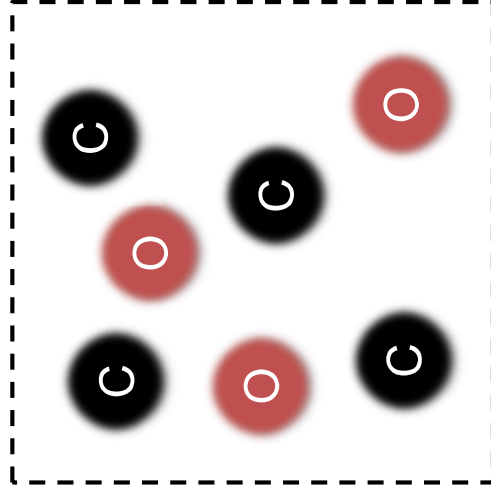
Technique previously used for neutral mixtures; we have extended it to plasmas

Relies on the Monte Carlo simulation and the Clapeyron equation

$$\frac{dP}{dT} = \frac{\Delta H}{T \Delta V}$$

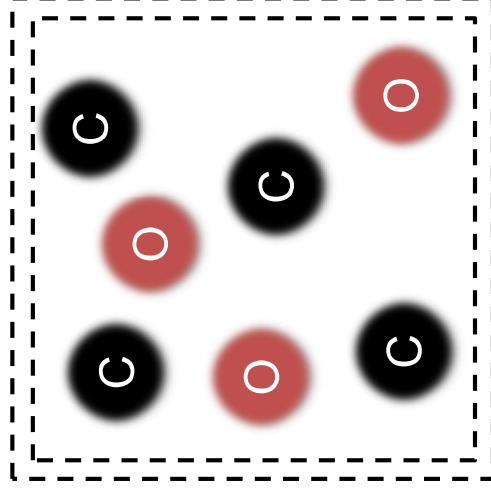
Integration along the liquid-vapor coexistence curve in the fugacity space

Semigrand canonical MC simulations

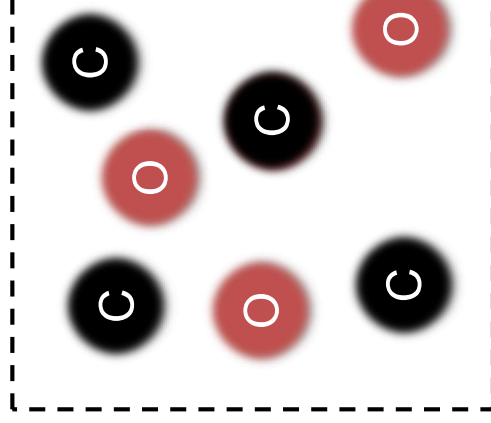


1. Ion displacement

Limited for solid (bcc) phase



2. Volume change



3. Identity change

The volume and the number of ions of each species **are not fixed** (but P and N_{tot})

Those steps insure that the energy is minimized and that the targeted pressure and fugacity fraction (ξ_0) are reached

Semigrand canonical MC simulations

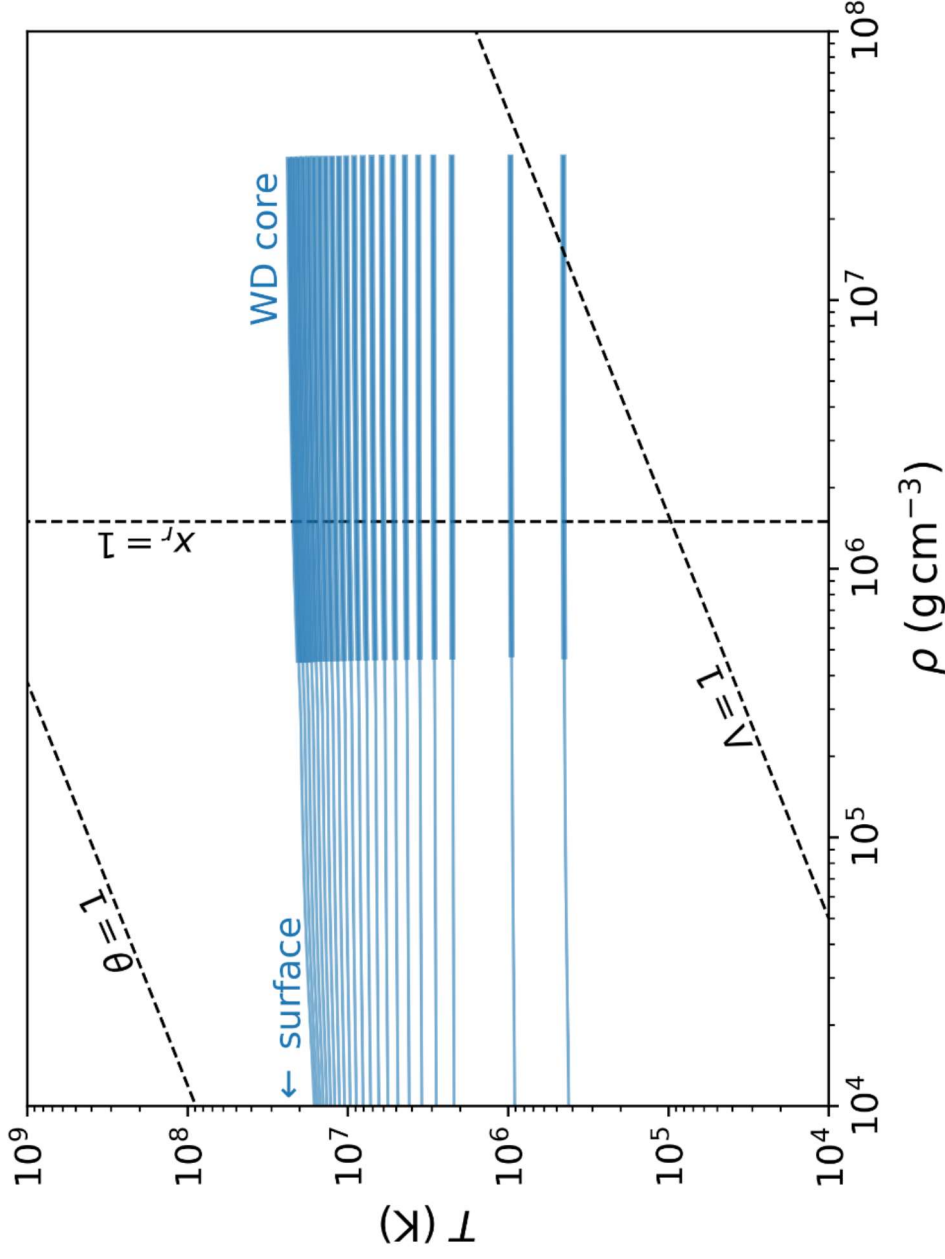
The choice of the type of MC move is randomly chosen at each iteration: **particle displacement** or **volume change** or **identity change**

The overall acceptance probability of a move is $\min(1, e^\Lambda)$, where

$$\Lambda = \underbrace{-\beta(U^{\text{trial}} - U^{\text{old}})}_{\text{orange box}} - \beta P(V^{\text{trial}} - V^{\text{old}}) + N \ln \frac{V^{\text{trial}}}{V^{\text{old}}} + \underbrace{m \ln \frac{\xi_0}{1 - \xi_0}}_{\text{red box}}$$

$m = \pm 1$

Physics included in our MC simulations



The plasma is **completely ionized**

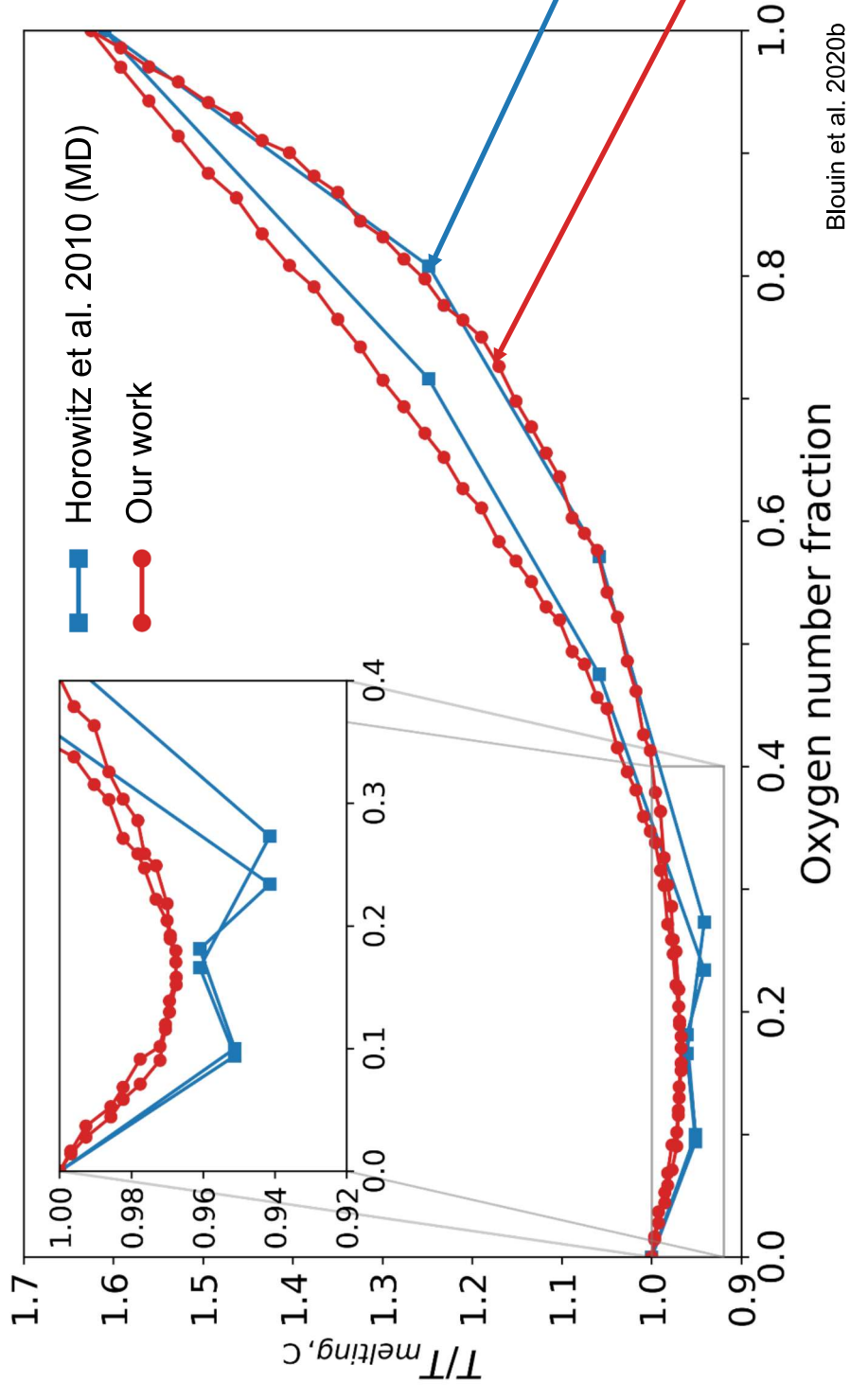
The degenerate, partially relativistic **jellium** is explicitly included in the simulation

The ion-ion interactions are **screened**, partially relativistic electrons,

$$V(r) = \frac{q^2}{r} \exp(-kr)$$

Not included: quantum behavior

Our new C/O phase diagram

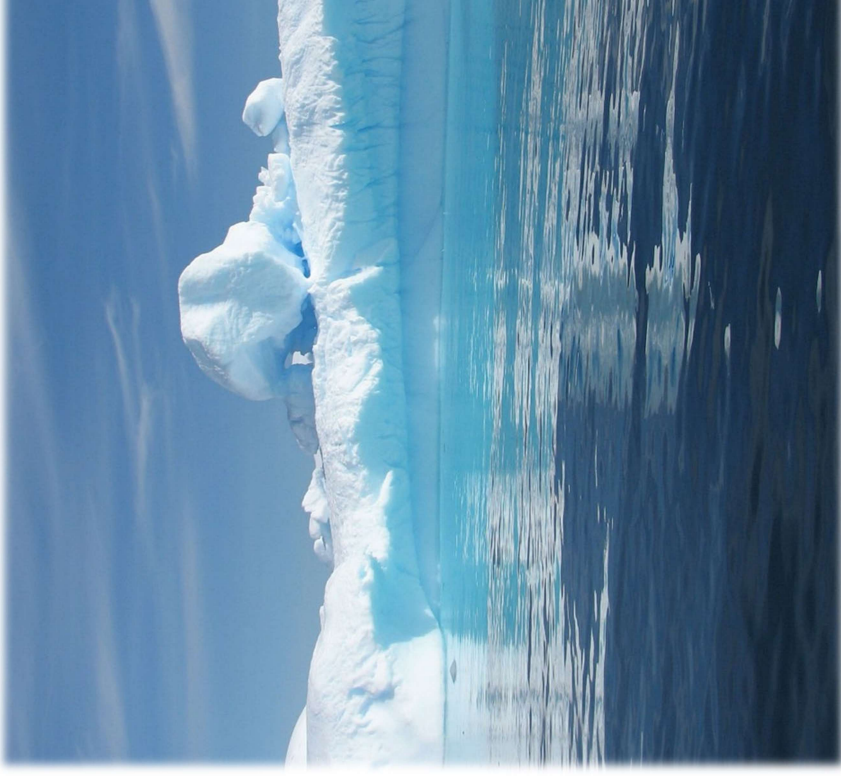


Allows a very accurate sampling of the phase fraction of the competing methods

Major advantages of the Gibbs-Duhem technique

All calculations are done in the relevant statistical ensemble: phase transitions take place at **constant P** , not constant V

We have **access to all thermodynamic properties** at the phase transition (e.g., latent heat) at no additional cost (not the case with MD)

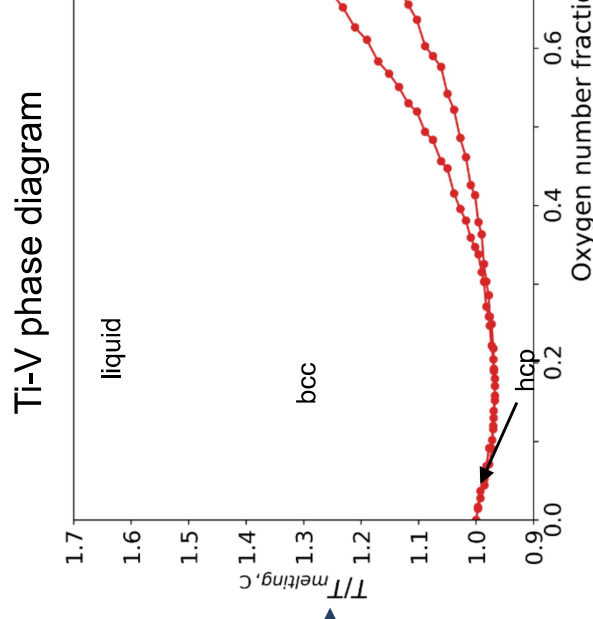
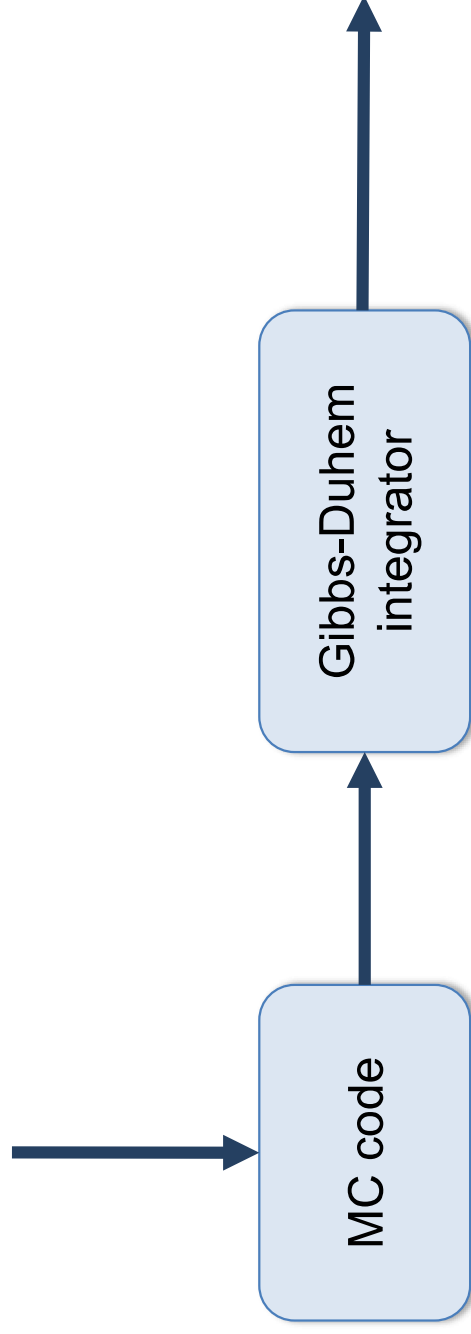


Future applications

This new simulation capability can easily be used to obtain phase diagrams of non-aqueous systems (e.g., alloys)

We are working on adding the option of computing phase diagrams for ternary mixtures/alloys

$$V(\mathbf{r}) = \frac{q^2}{r} \exp(-\kappa r / a)$$



Ghosh 2002

Future applications

This new simulation capability can easily be used to obtain phase diagrams of non-astrophysical systems (e.g., alloys)

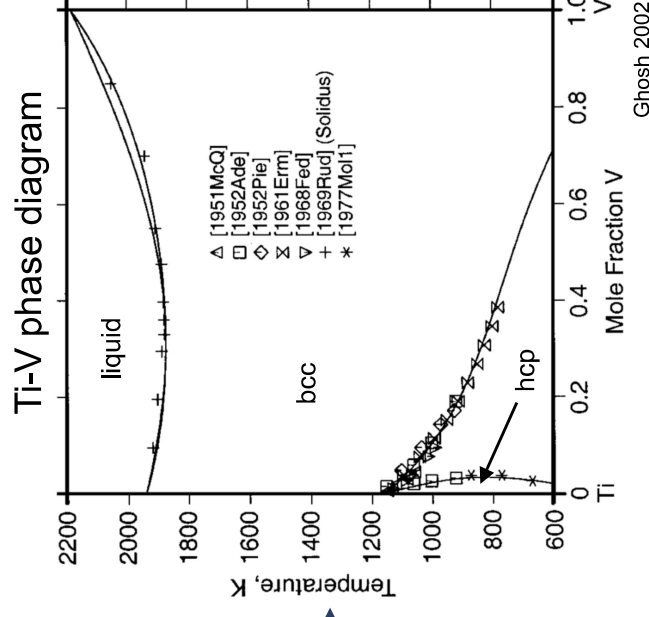


MC code

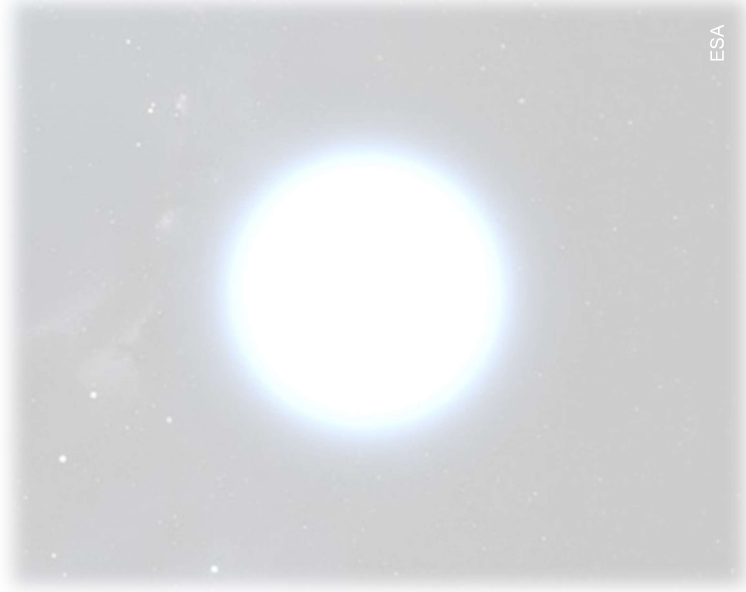
MC code

Gibbs-Duhem integrator

Gibbs-Duhem integrator

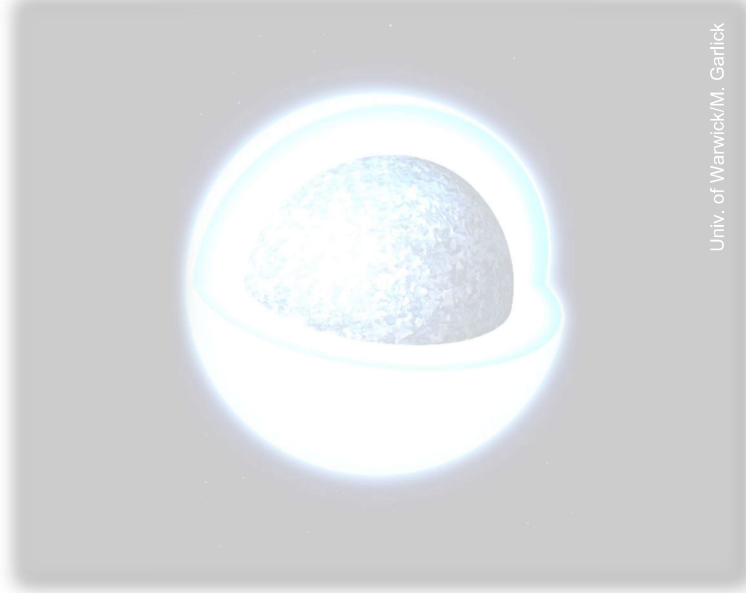


Ghosh 2002



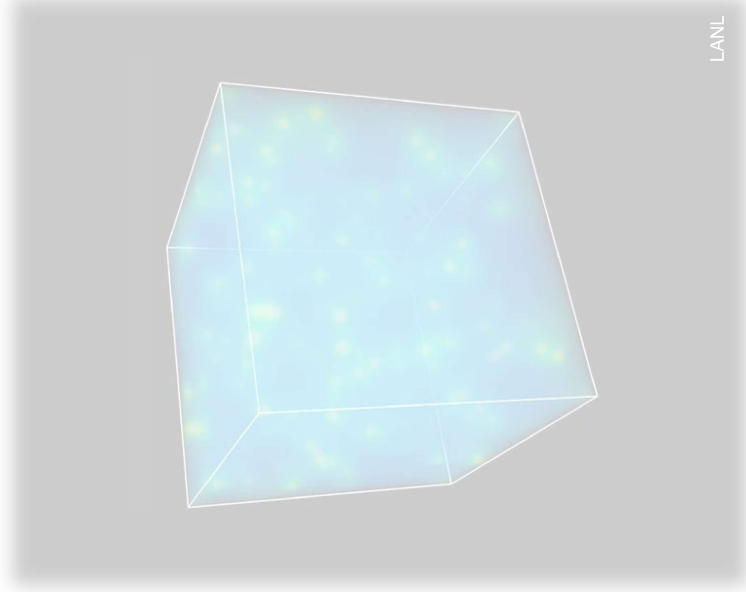
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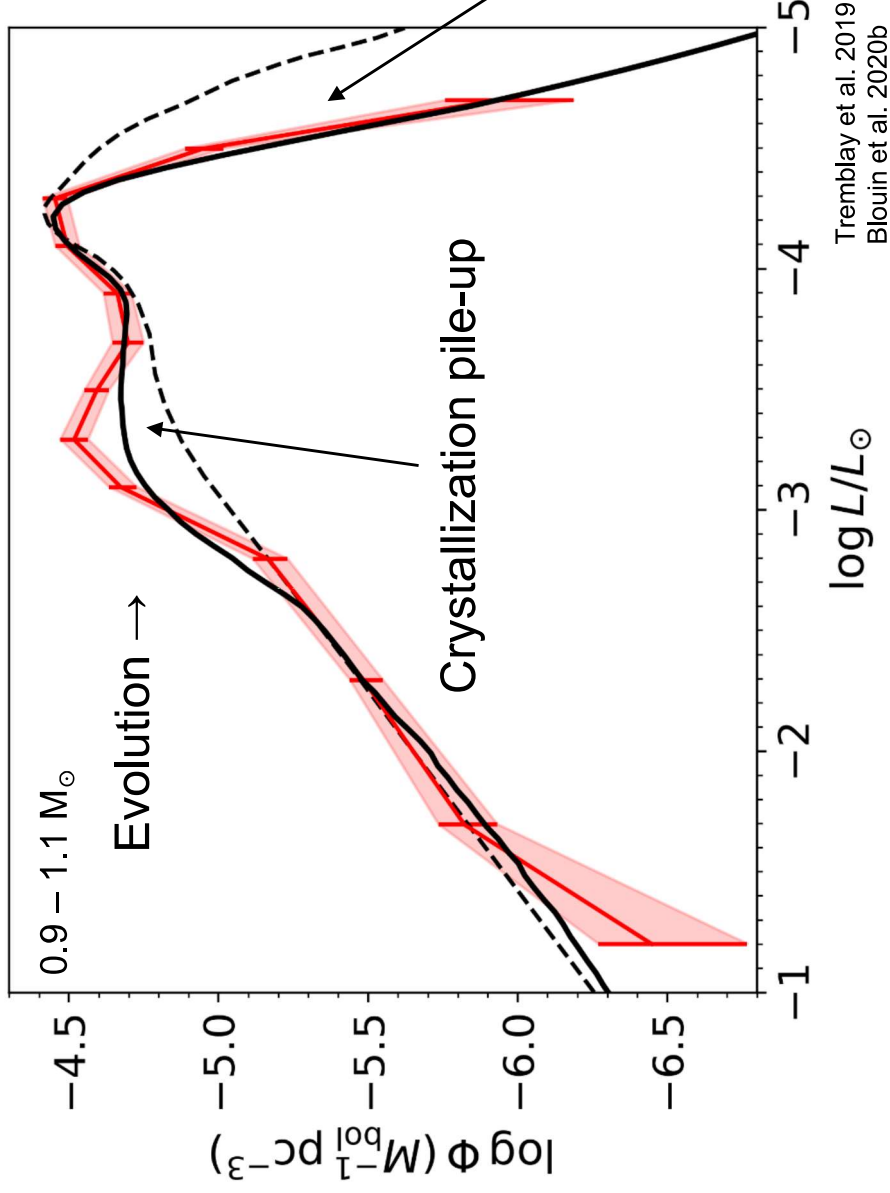


4. C o

Comparison to observational data

Core crystallization leaves a signature on the luminosity function of WDs, allowing a **direct comparison** to observations

The underestimation of the crystallization pile-up may be the role of a 3rd minor species,

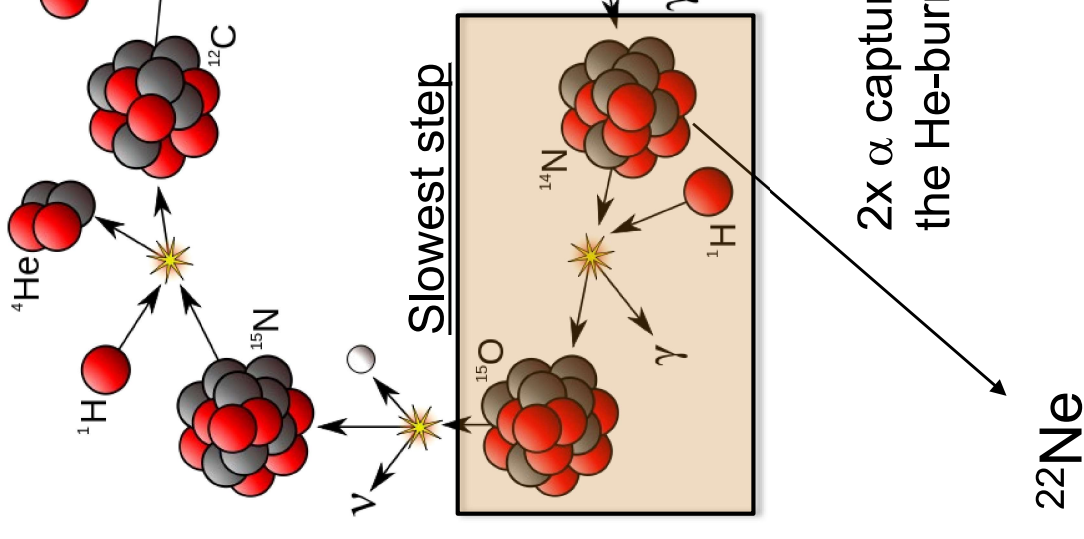


^{22}Ne in white dwarfs

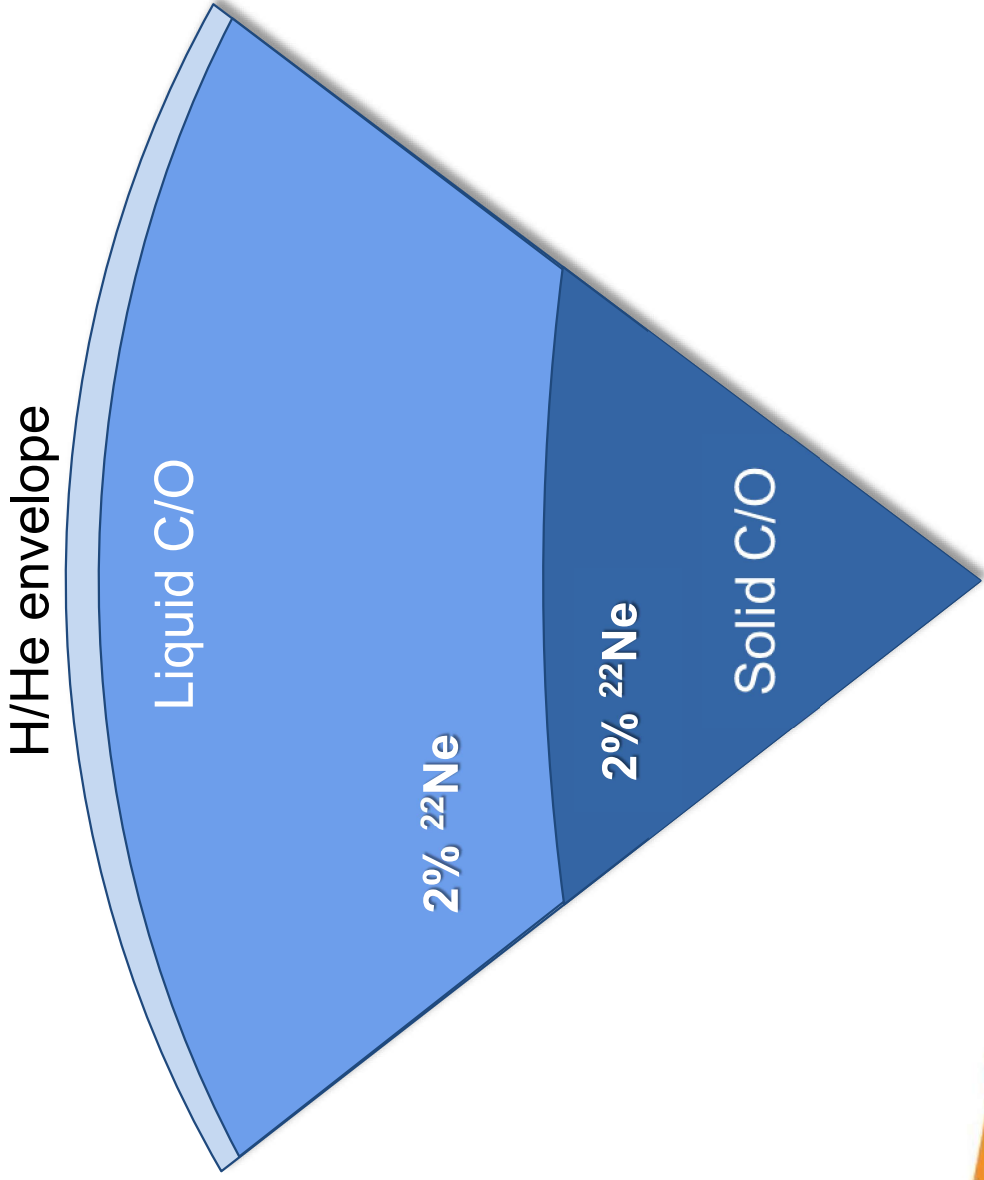
^{22}Ne makes up about 1-2% of a WD's mass

It is produced from the leftovers of the H-burning phase

Because it has 2 excess neutrons ($Z=10$, $A=22$), the sedimentation of a given quantity of ^{22}Ne releases ~25 times more gravitational energy than the sedimentation of the same amount of ^{16}O



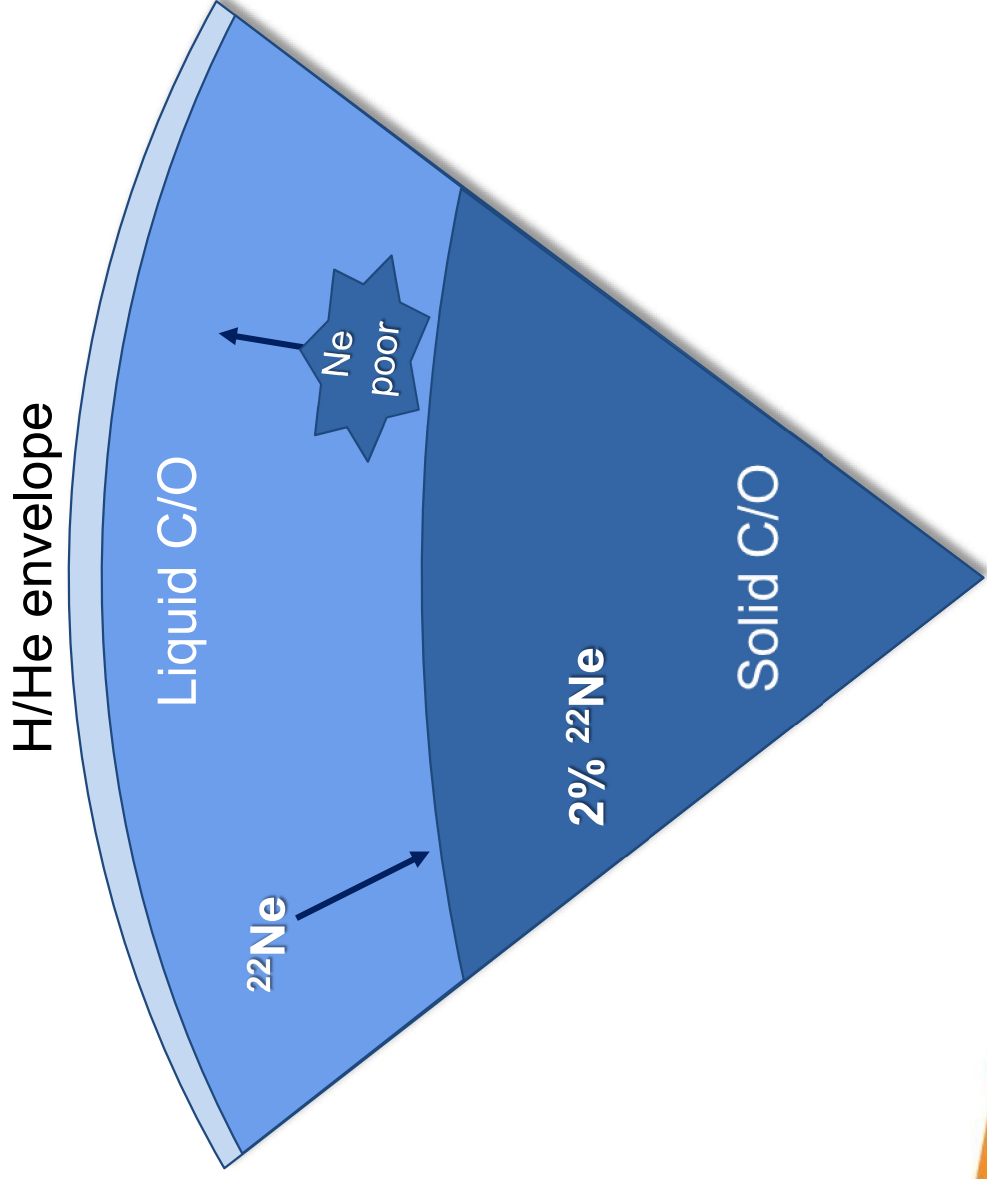
What may be going on with ^{22}Ne



Preliminary calculations of the C/O diagram suggest that:

1. At first, there is no ^{22}Ne phase

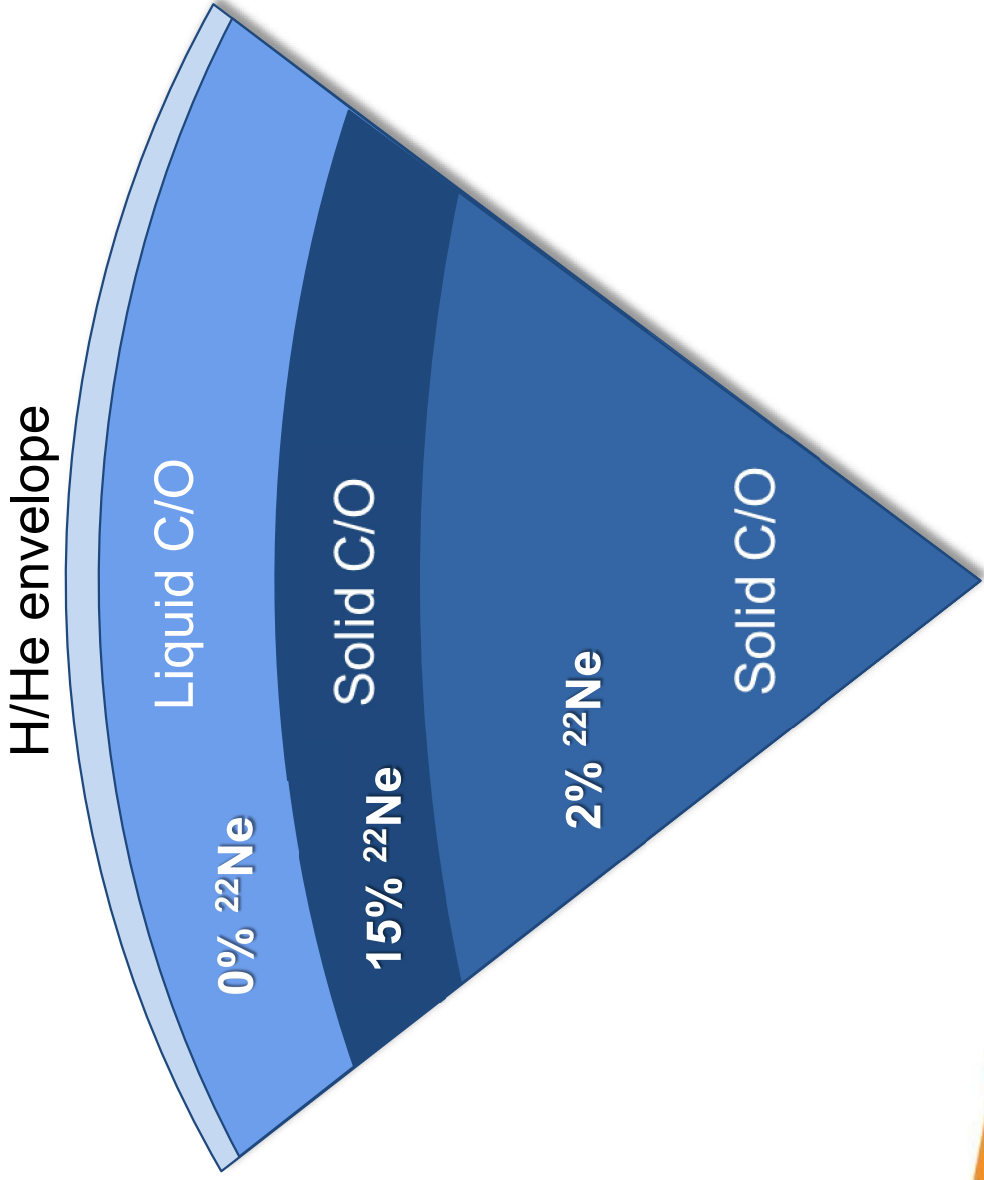
What may be going on with ^{22}Ne



Preliminary calculations of the C/O diagram suggest that:

1. At first, there is no ^{22}Ne phase separation.
2. Later, when approaching the C/O azeotrope, the solid is depleted in ^{22}Ne compared to the liquid.
3. The ^{22}Ne -poor solid floats, and the ^{22}Ne -rich liquid to the crystalline phase.

What may be going on with ^{22}Ne



Preliminary calculations of the C/O diagram suggest that:

1. At first, there is no ^{22}Ne phase.
2. Later, when approaching the azeotrope, the solid is depleted compared to the liquid.
3. The ^{22}Ne -poor solid floats, leaving ^{22}Ne -rich liquid to the crystallization front.
4. Eventually, the azeotrope is reached, and all the remaining ^{22}Ne is frozen into the solid, resulting in a ^{22}Ne -rich layer, resulting in a cooling delay.

Summary

White dwarfs offer unique opportunities to study the **history of our Galaxy**

New observational datasets challenge our understanding of white dwarfs and **call for refined physics models**

We have developed an efficient and physically accurate technique to **model the phase transition undergone by white dwarfs**

This new technique can now be applied to other systems

Thank you! Questions?