Using quantum computers to simulate a toy problem of laser-plasma interactions

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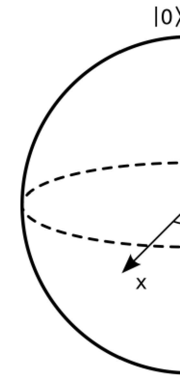
Quantum computing: a promise yet to be fulfilled

- Ideal quantum memory can hold more information
  - Classical computing uses bits: 0 or 1, binary
    Specifying the state of n bits need n numbers, e.g. 101
  - Quantum computing uses qubits: 0 and 1 superpositions
    Specifying the state of n qubits need 2^n numbers, e.g.

$$|\Psi\rangle = c_0|000\rangle + c_1|001\rangle + c_2|010\rangle + c_3|011\rangle$$
$$+ c_4|100\rangle + c_5|101\rangle + c_6|110\rangle + c_7|111\rangle$$



Bloch sphe
superpositio

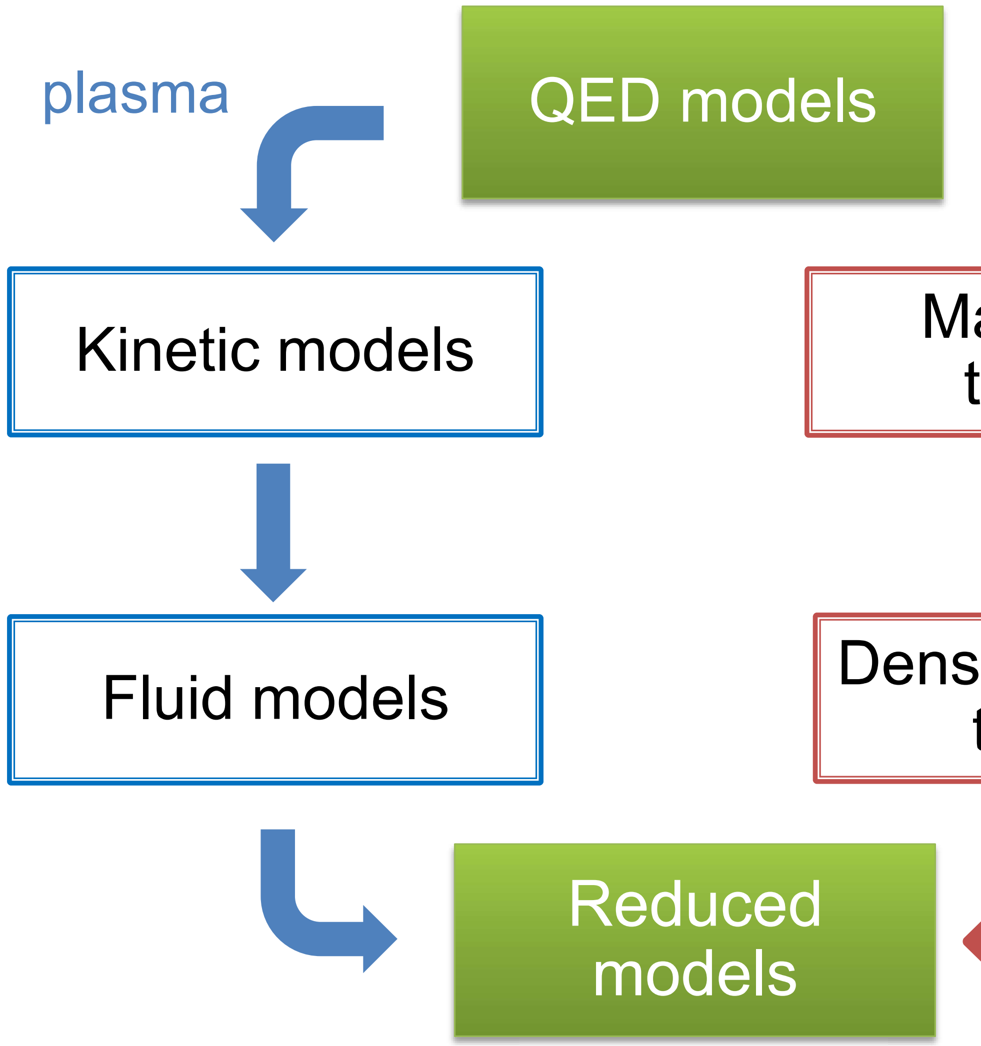
$|\psi\rangle = \cos\frac{\theta}{2}|0$

- Quantum algorithms may require less operations
  - Ideal quantum computers offer unitary operations
    Classical computers rely on irreversible operations
  - Notable quantum algorithms with exponential speedup:
    Quantum Fourier transform, Shor's algorithm for prime factorization,
    Grover's search, quantum random walk, quantum Hamiltonian simulations

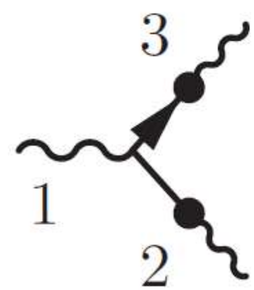
Idealized quantum algorithms require error correction, not yet operational

QC may help condensed matter. How about plasmas

- Quantum computing (QC) is believed to offer advantages for condensed matter problems via quantum Hamiltonian simulations
- Plasma problems usually classical and nonlinear. But hierarchy of plasma models analogous to condensed matter models
- Plasma and condensed matter are directly connected at the level of QED and reduced models



Example reduced model: three-wave interactions

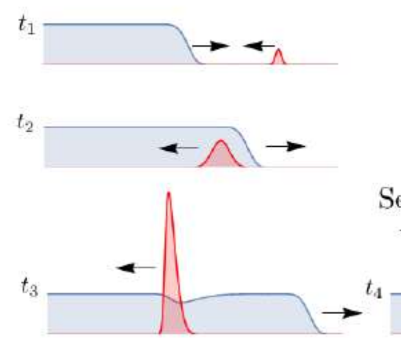


- Interesting problems are usually nonlinear
  - Lowest order: cubic couplings, common in nonlinear media
  - Examples: laser-plasma interactions, turbulence, nonlinear optics, lattice QED …
  - Classical resonant interactions described by three-wave envelope equations

$$d_t A_1 = g A_2 A_3, \quad d_t A_2 = -g^* A_1 A_3^\dagger, \quad d_t A_3 = -g^* A_1 A_2^\dagger$$

$$d_t = \partial_t + \mathbf{v}_j \cdot \nabla \,, \qquad \mathbf{v}_j = \partial\omega_j / \partial\mathbf{k}_j$$

Coupling coefficient



  - Quantized version $[A_j, A_l^\dagger] = \delta_{jl}$

    Interaction Hamiltonian $H_I = i g A_1^\dagger A_2 A_3 - i g^* A_1 A_2^\dagger A_3^\dagger$

- Quantum hardware usually lacks native cubic couplings: nonnativ

  Can we program cubic interactions on general-purpose quantum computers

Simulating nonnative interaction is challenging

- Standard approach to quantum Hamiltonian simulation
  - Hardware Hamiltonian: determined by device architecture

$$H_0 = \sum_{k=1}^{m} H_k$$

  - Hamiltonian of the physical system: likely different from H_0

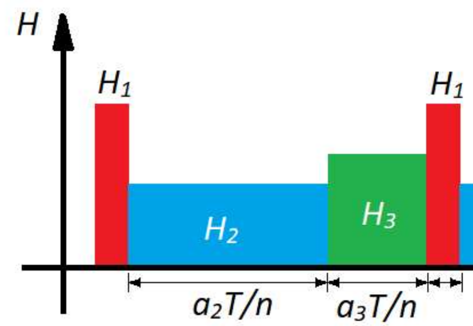
$$H = \sum_{k=1}^{m} a_k H_k$$

  - Lie-Trotter-Suzuki approximation when terms in H are natively available

$$\exp(-iHT) = \lim_{n\to\infty} [\prod_{k=1}^{m} U_k(a_k T/n)]^n$$

$$U_k(t) = \exp(-iH_k t)$$

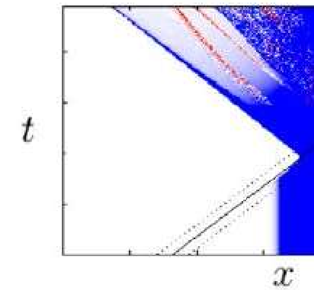
require native H_k



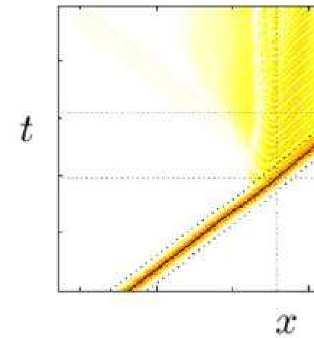
  - What if H contains terms that are nonnative?
    Implement general unitary is exponentially expensive!

It's difficult! OK, let's say we can do it. Does it help?

- Example application: laser-plasma interactions
  - Plasma parameters evolve under laser drives
  - Laser propagation/scattering affected by plasma conditions



- Simulating real-time dynamics: expensive!
  - Plasma states $|\mathbf{g}(t+\Delta t/2)\rangle = \mathcal{A}(t)|\mathbf{g}(t-\Delta t/2)\rangle$
  - Laser states $|\mathbf{a}(t+\Delta t)\rangle = \mathcal{G}(t+\Delta t/2)|\mathbf{a}(t)\rangle$



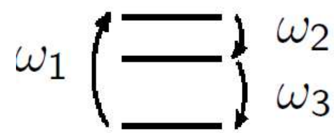
- Sub problem: *D*-level photon occupation
  - Classical: computing next state by matrix multiplication, *O*(D^2) operations
  - Quantum: computing next state by applying cubic gates, *O(1)* operations
    - Need 1-parameter family of cubic gates
    - Initial states simple, readout only needed at final step

Cubic gates have overhead, but once precompiled, operations cheap for eac

Solving cubic problem: mapping in action space

- Naive mapping in **energy space**
  - Direct mapping from resonant levels in energy space to hardware space restricted by
    (1) Tunability of level spacings and coupling
    (2) Unwanted terms in native Hamiltonian
    (3) Inefficient representation: 0 or 1 per qubit

$$H = igA_1^\dagger A_2 A_3 - ig$$



$$|n_j\rangle = \frac{(A_j^\dagger)^{n_j}}{\sqrt{n_j!}}$$

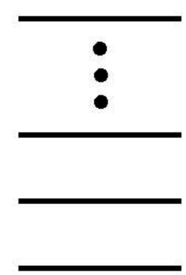
- More versatile mapping in **action space**
  - Action operators commute with Hamiltonian

$$S_2 = n_1 + n_3, \quad S_3 = n_1 + n_2$$

$$[H, S_2] = [H, S_3] = 0$$

  - Simultaneous eigen states of H, S_2 and S_3

$$|\psi\rangle = \sum_{j=0}^{\min(s_2, s_3)} c_j |s_2 - j, s_3 - s_2 + j, j\rangle$$



$$|\psi_j\rangle = |s_2 - j, s_3 - s_2 + j, j\rangle$$

$$|n_1, n_2, n_3\rangle = |n_1\rangle \otimes |n_2\rangle \otimes |$$

Temporal three-wave problem = Hamiltonian simula

- Occupation amplitudes satisfy Schrödinger equations

$$i\partial_t c_j = igh_{j+\frac{1}{2}}c_{j+1} - ig^*h_{j-\frac{1}{2}}c_{j-1}$$

$$h_{j-\frac{1}{2}} = \sqrt{j(s_2+1-j)(s_3-s_2+j)}$$

$$h_{-\frac{1}{2}} = h_{D+\frac{1}{2}} = 0$$

- Observables can be post processed from occupation probabilities

$$\langle n_1\rangle = \sum_{j=0}^{s_2}(s_2 - j)|c_j|^2$$

$$\langle n_2\rangle = \sum_{j=0}^{s_2}(s_3 - s_2 + j)|c_j|^2$$

$$\langle n_3\rangle = \sum_{j=0}^{s_2} j|c_j|^2$$

- Quantum number operators s Heisenberg equations

$$\partial_t^2 n_1 = -\partial_t^2 n_2 = -\partial_t^2 n_3$$
$$= 2|g|^2\left[s_2 s_3 - (2s_2 + 2s_3 + \right.$$

- Classical expectation values sa slightly different equations

$$\partial_t^2\langle n_1\rangle = -\partial_t^2\langle n_2\rangle = -\partial_t^2\langle n_3\rangle$$
$$= 2|g|^2\left[s_2 s_3 - (2s_2 + 2s_3)\right.$$

- Quantum system behaves like when wave packet is localized spontaneous emission is subd

Simplest nontrivial case requires *D* = 3 = (1+1/2) qub

- Readily realizable on hardware for s_2=2 and s_3=s. Hamiltonian matrix tridiago

$$h(\theta, s) = \begin{pmatrix} 0 & e^{i\theta}\sqrt{2(s-1)} & 0 \\ e^{-i\theta}\sqrt{2(s-1)} & 0 & e^{i\theta}\sqrt{2s} \\ 0 & e^{-i\theta}\sqrt{2s} & 0 \end{pmatrix}$$

$$h = H/|g|$$

$$\exp(i\theta) = ig/|g|$$

$|2, s-2$

$|1, s-1$

$|0, s,$

- Solution to 3-level problem known analytically. Unitary to be implemented or

$$U = \begin{pmatrix} \frac{(s-1)\cos\lambda\tau + s}{2s-1} & -ie^{i\theta}\sqrt{\frac{s-1}{2s-1}}\sin\lambda\tau & e^{2i\theta}\frac{\sqrt{s(s-1)}}{2s-1}(\cos\lambda\tau - 1) \\ -ie^{-i\theta}\sqrt{\frac{s-1}{2s-1}}\sin\lambda\tau & \cos\lambda\tau & -ie^{i\theta}\sqrt{\frac{s}{2s-1}}\sin\lambda\tau \\ e^{-2i\theta}\frac{\sqrt{s(s-1)}}{2s-1}(\cos\lambda\tau - 1) & -ie^{-i\theta}\sqrt{\frac{s}{2s-1}}\sin\lambda\tau & \frac{s\cos\lambda\tau + s - 1}{2s-1} \end{pmatrix}$$

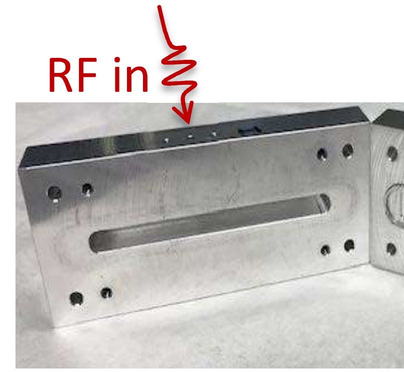
$U = \mathrm{ex}$

$\lambda = \sqrt{}$

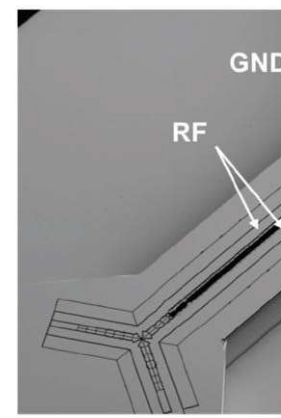
$U_2 = ($

Solving test problems: What quantum devices are a

- Leading architectures under development
  - **Superconducting qubits**: Nonlinear oscillators as registers. Control/readout by microwave pulses. Fabrication techniques already in use in semiconductor industry
  - **Trapped-ion qubits**: Hyperfine levels as registers. Control by electrodes, lasers, or microwaves. Readout by lasers. Potentially more versatile connection topology
  - **Photonic qubits**: Photon states as registers. Control by network of beam splitters and interferometers. Readout by single-photon detectors. Potentially miniaturizable and programable at room temperature
  - **Nuclear spins**: Angular momentum states of selected isotopes in B field as registers. Control/readout by electromagnetic pulses. Potentially operable at room temperature
- Noisy intermediate-scale quantum (NISQ) era: many qubits available, but not fault tolerant …



Superconductin



Ion trap at Sandia Nat

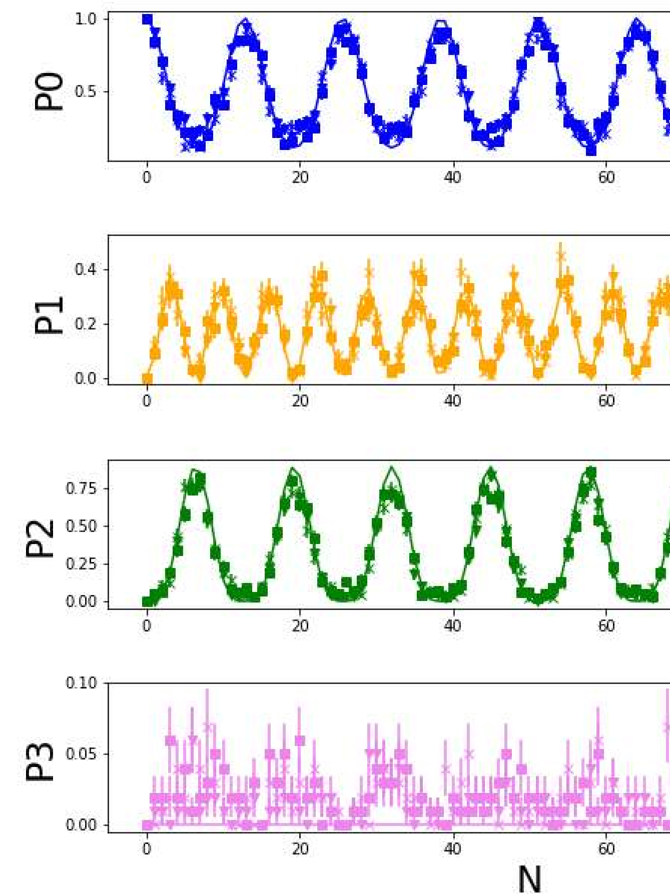
Realize cubic gates using standard gates

Quantum Cloud Services rigetti

Hardware: superconducting transmon qubits on 2D lattice, utilized Aspen-4-2Q-A

- Imbed 3 levels in 2 qubits: Utilize 00, 01, and 10 states
- Compile into standard gates[1], approximate single-step unitary matrix by

```
RZ(pi) 0
RX(1.570796326794897) 0
RZ(-0.9553166181245063) 0
RX(1.5707963267948948) 1
CZ 0 1
RX(pi/2) 0
RZ(0.48989794855663593) 0
RX(-pi/2) 0
RZ(1.42746659829489) 1
RX(pi/2) 1
RZ(0.39864643091397856) 1
RX(-pi/2) 1
CZ 0 1
RZ(-2.186276035465287) 0
RX(pi/2) 0
RZ(-1.7141260552949023) 1
RX(-1.5707963267948928) 1
```



Tracks exact dynamics if allo

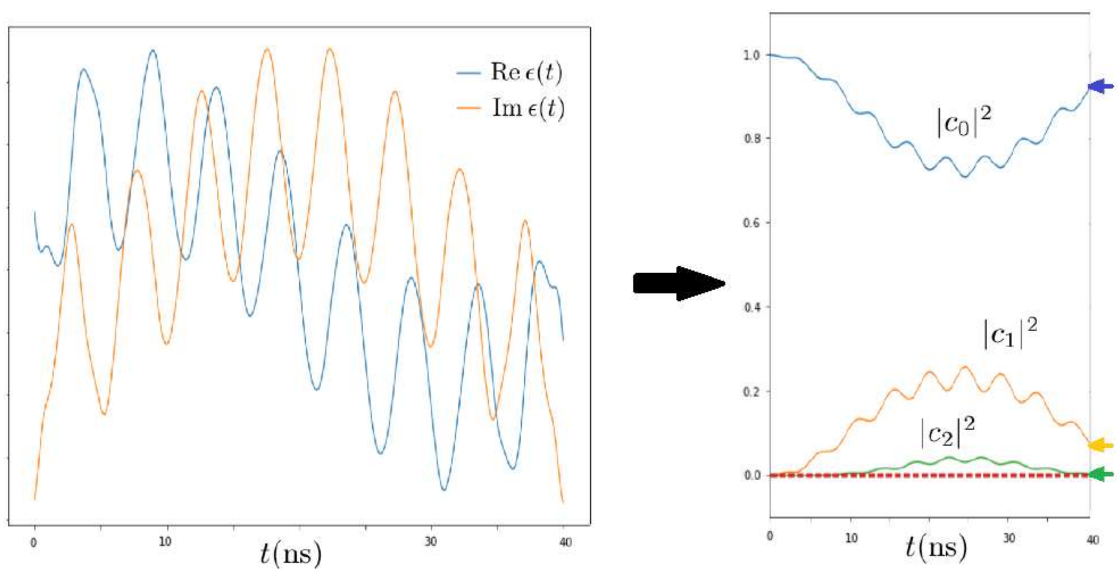
simplification $[U(\Delta\tau)]^N =$

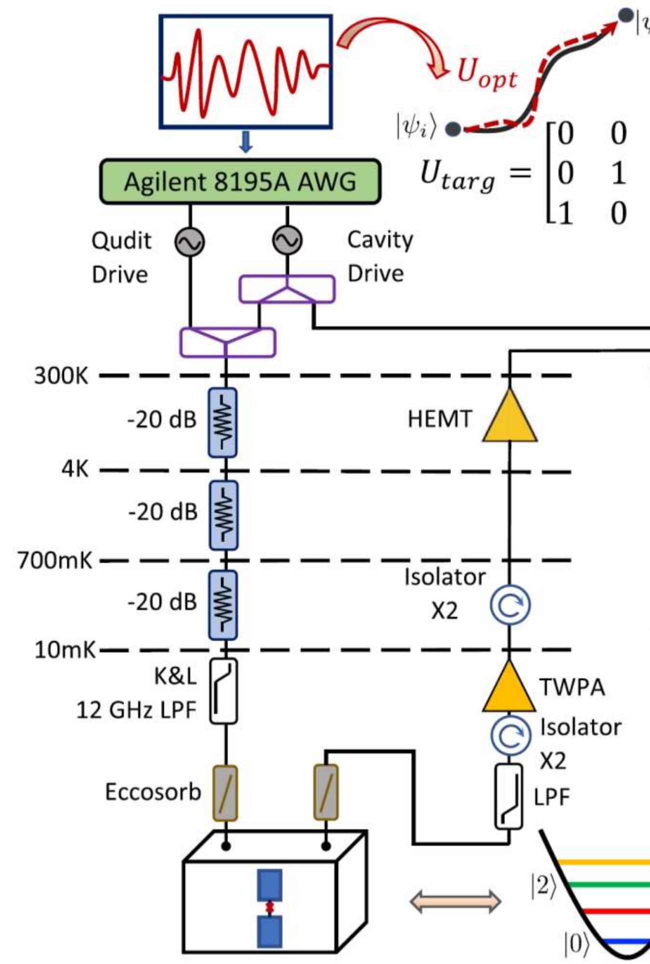
1. arXiv:1608.03355, (2017)

Realize cubic gates using optimal control

LLNL QuDIT[1]

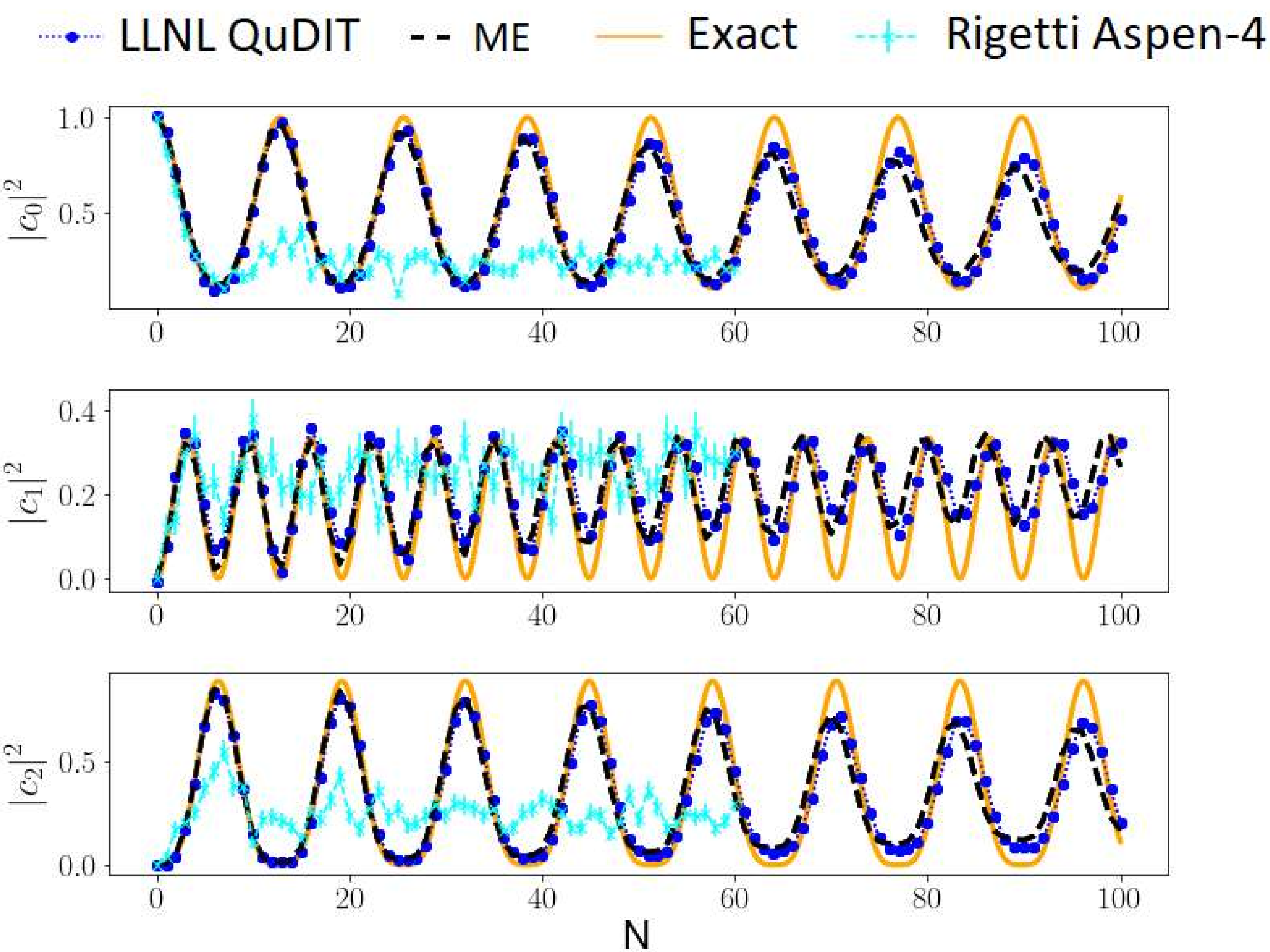
- Hardware: transmon inside a 3D microwave cavity, utilize three levels of a single qudit
- Control by customized waveform, single-step pulse optimized using GRAPE algorithm[2]





1. Phys. Rev. Lett. 125, 170502 (2
2. Comput. Phys. Commun. 184, 1

Use precompiled gates to simulate real-time dynam



- Long-time evolution customized gate. Res Master Equation (MI and are close to Exac
- Short-time dynamics shallow gates, seque standard gates perfo
- Decay and dephasing fidelity after ~100 ga for both standard/cu

1. Phys. Rev. A 103, 062608 (20

Interpolated control pulses may also achieve high fi

- Numerical optimizations expensive, shortcut by interpolation. Works well for 3-level parametric gates

Target Hamiltonian

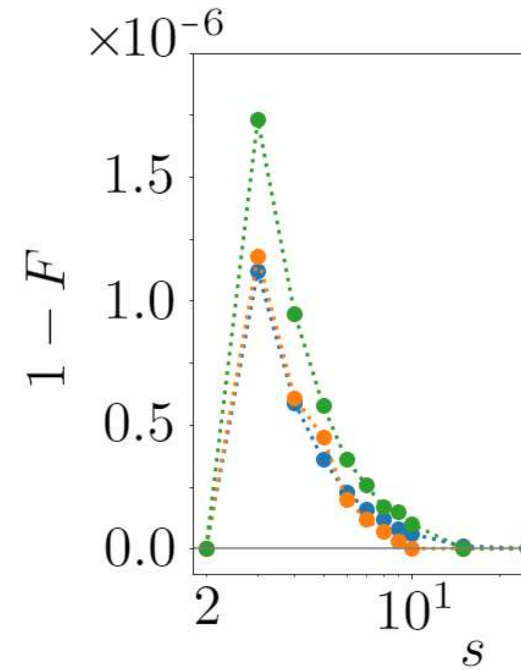
$$h(s) = \frac{\sqrt{2s}}{(1-2^{-1/2})}\left[(1-\xi)K(2) + (\xi - 1/\sqrt{2})K(\infty)\right]$$

$$\xi(s) = \sqrt{1-1/s}$$

$$K(2) = \begin{pmatrix} 0 & 1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad K(\infty) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Control pulse

$$\epsilon_I(s) = [(1-\xi)\epsilon_O(2) + (\xi - 1/\sqrt{2})\epsilon_O(\infty)]/(1-1/\sqrt{2})$$



Fidelity:

$$F(\rho, \sigma) = \mathrm{tr}\sqrt{}$$

density matrices a

σ : optimized puls

ρ : interpolated pu

Cubic gates as building blocks for future application

- Cubic gates can be programmed, no need for native cubic couplings
  Action space mapping reduces nonlinear problem to Hamiltonian si
- ➢ **Quantum computers useful for simulating nonlinear/nonnative in**

- Simulations realized on Rigetti Aspen-4 using standard gates, on LLN
  using customized gates, and both limited by decoherence up to ~10
- ➢ **Customized gates enable larger simulation depth on NISQ hardwar**

- Future directions:
  - Generalize mapping to other nonlinear interactions
  - Implement gates on hardware with more qubits/higher fidelity
  - Utilize N-wave gates as building blocks for realistic applications

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