

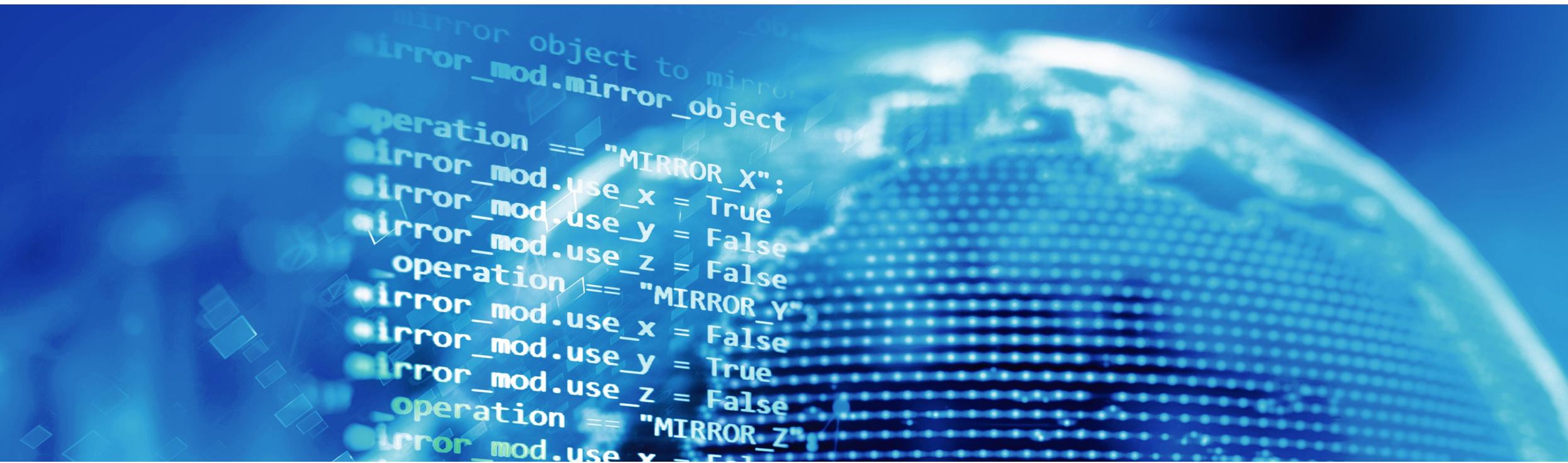
New Perspectives for the *ab-initio* Simulation and Diagnostics of Warm-dense Matter

T. Dornheim^{1,2}, J. Vorberger², Zh. Moldabekov^{1,2}, M. Böhme^{1,2,3},
K. Ramakrishna^{1,2,3}, M. Bonitz⁴, D. Kraus^{5,2}, T. Döppner⁶, T. Preston⁷, P. Tolias⁸



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¹CASUS | ²HZDR | ³TU Dresden | ⁴Kiel University | ⁵Rostock University | ⁶LLNL | ⁷European XFEL | ⁸KTH Stockholm



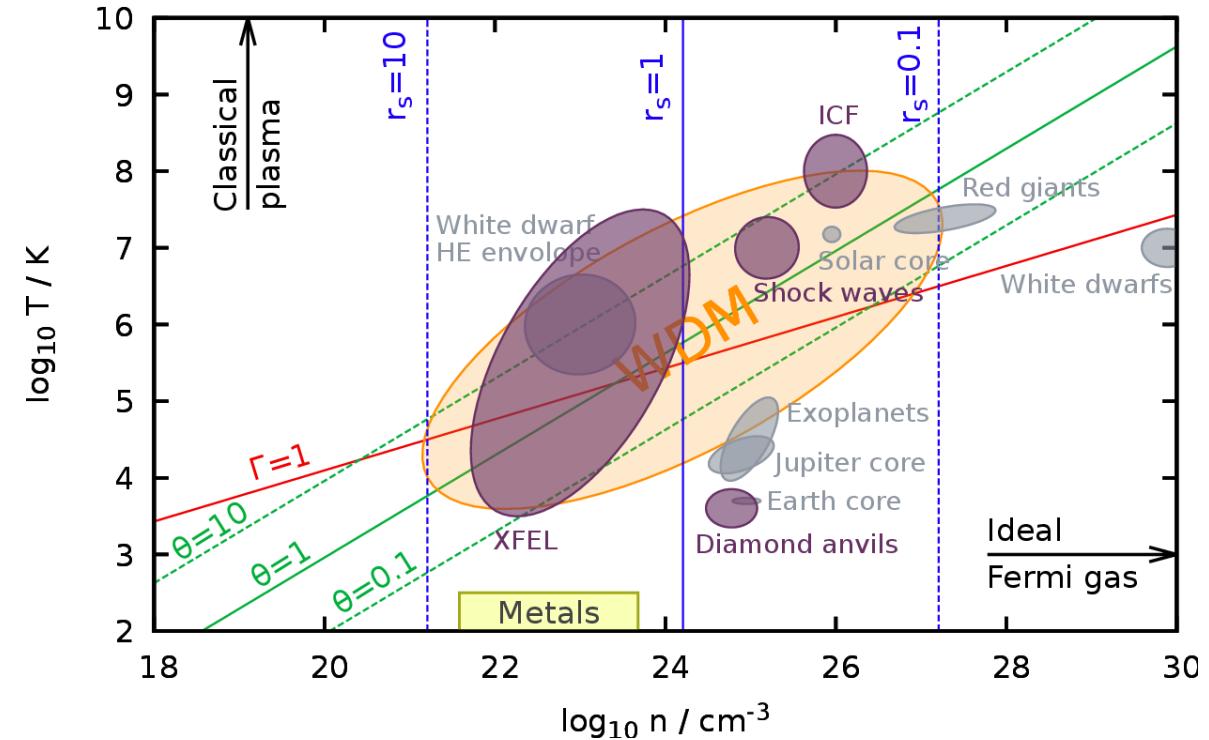
Introduction

Warm Dense Matter (WDM)

- Matter under extreme density / temperature ubiquitous throughout our universe

$$r_s \sim \theta \sim \Gamma \sim 1$$

$\rightarrow r_s = d/a_B$, density parameter, $\theta = k_B T/E_F$, $\Gamma = W/E_{\text{kin}}$



Taken from: **T. Dornheim**, S. Groth, and M. Bonitz, *Phys. Rep.* **744**, 1-86 (2018)

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- Examples: giant planet interiors, brown dwarfs

**Figure hidden
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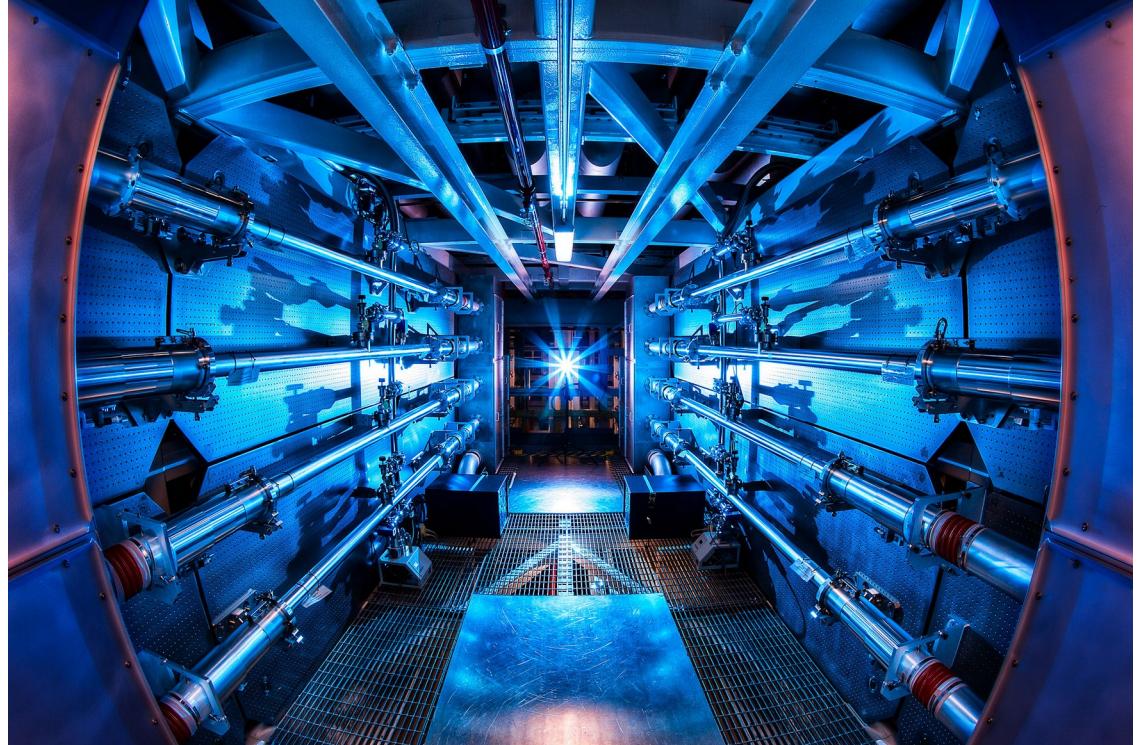
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- Examples: giant planet interiors, brown dwarfs
- WDM highly important for technological applications:
- Inertial confinement fusion, etc.

National Ignition Facility (NIF)



Taken from: Lawrence Livermore National Laboratory

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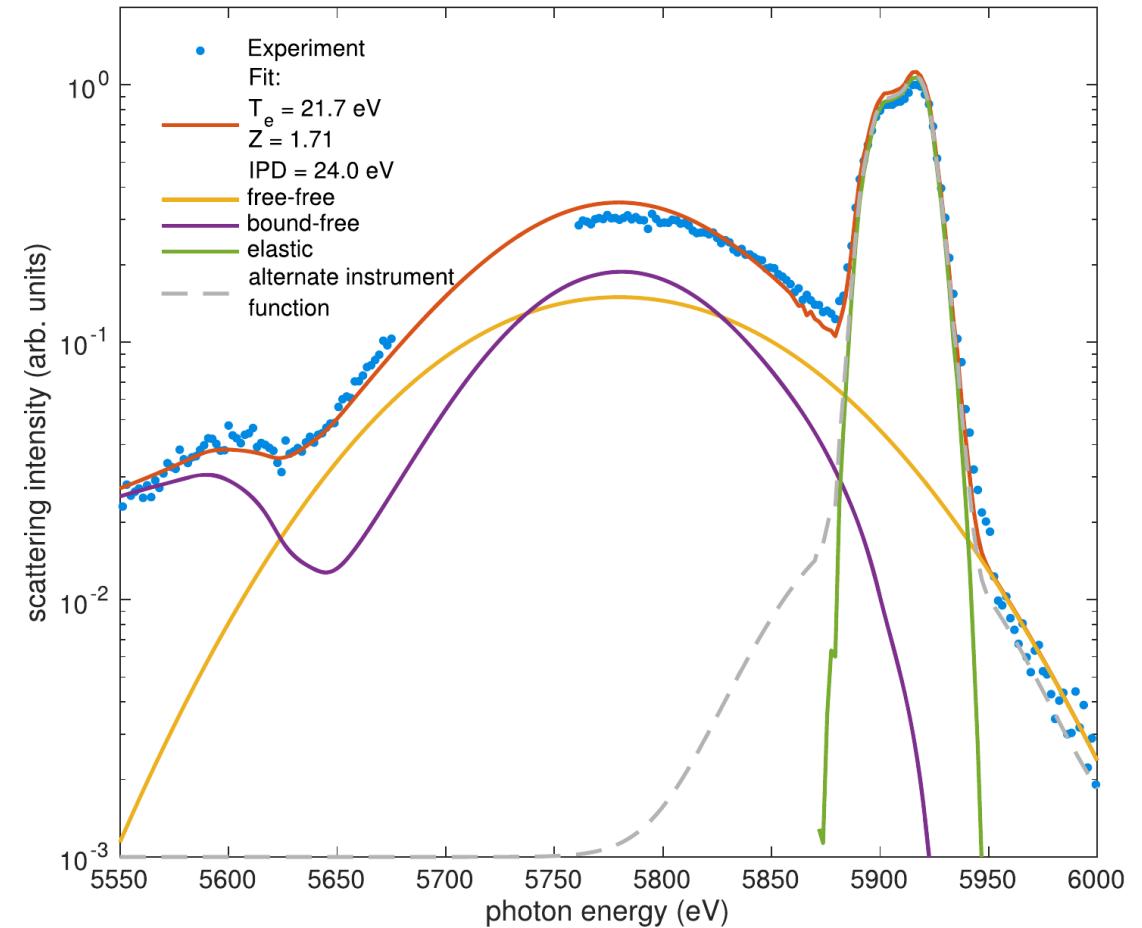
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Introduction

But: Rigorous WDM theory indispensable

- Diagnostics: parameters like T , n , Z , etc. cannot be measured and have to be inferred from theory
→ X-ray Thomson scattering (XRTS)

Isochorically heated graphite at LCLS (Stanford)



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- **WDM theory notoriously challenging**

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→ intricate interplay of:

- 1) Coulomb coupling

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Figure 1.1
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How to achieve a real *ab initio* description of WDM?

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Figure 1.17
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How to achieve a real *ab initio* description of WDM?

Can we do XRTS diagnostics of WDM without models / approximations?

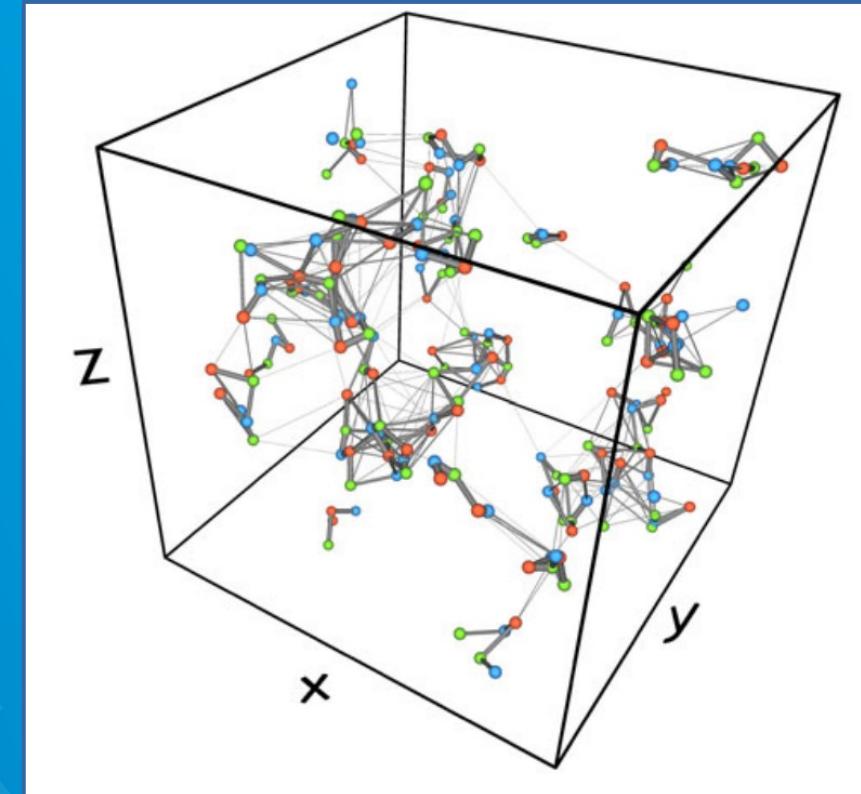
Ab-initio Quantum Monte Carlo (QMC) simulations

Problem:

- **Density functional theory (DFT)** etc. require external input about XC-effects
→ finite T : XC-free energy f_{xc}

Solution:

- **Quantum Monte Carlo** methods in principle allow for exact solution of quantum many-body problems without any empirical input
- Finite T : Path Integral Monte Carlo (PIMC)



Taken from: **T. Dornheim**, S. Groth, and M. Bonitz, *Contrib. Plasma Phys.* **59**, e201800157 (2019)

Previous result: XC-free energy of UEG

S. Groth, T. Dornheim, T. Sjostrom, F.D. Malone, W.M.C. Foulkes, and M. Bonitz, PRL 119, 135001 (2017)

Impact on thermal DFT simulation of warm dense hydrogen

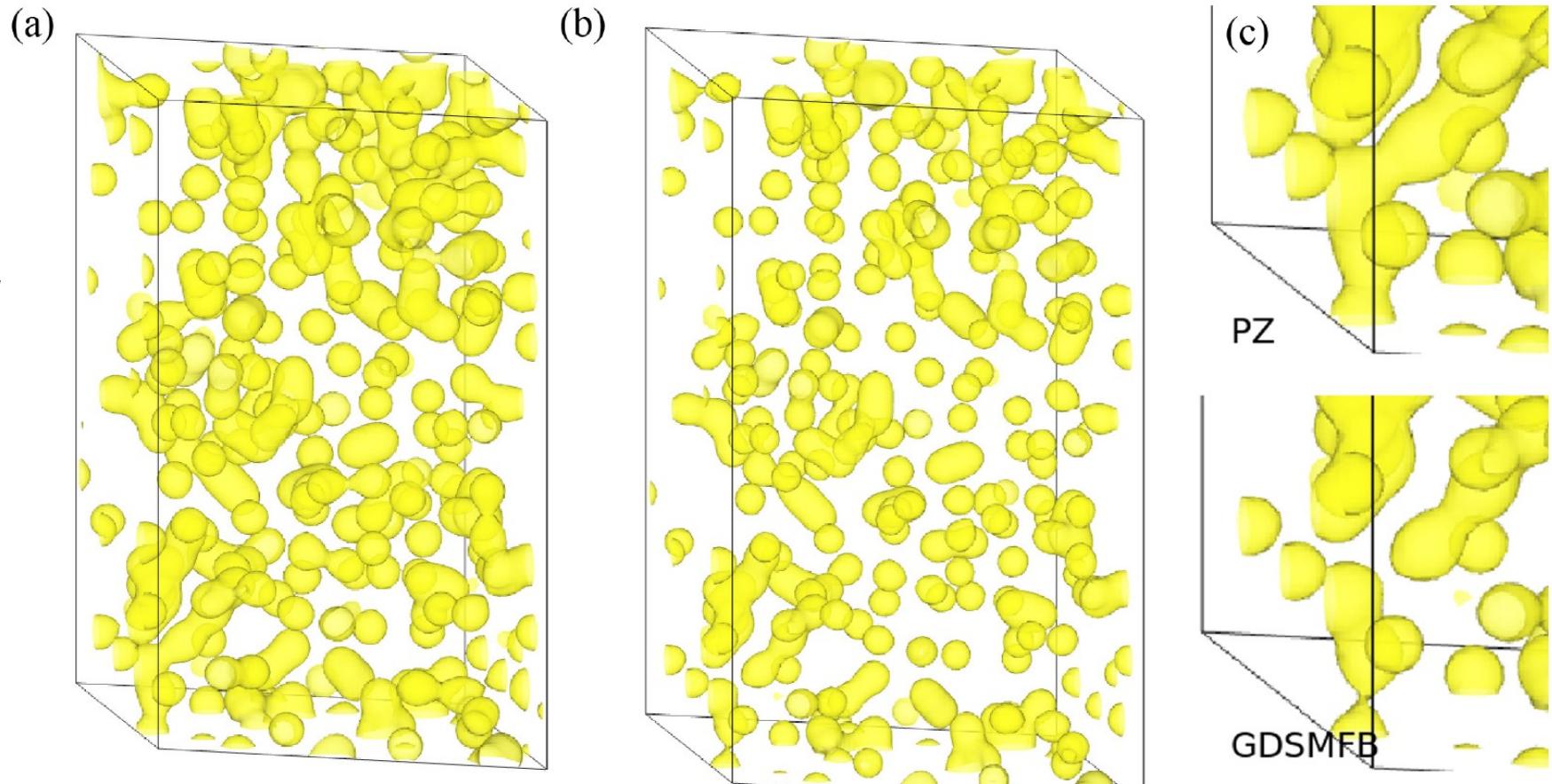
Example:

Hydrogen at T=65,000K

$$r_s = 2$$

(a) Ground-state LDA by Perdew
And Zunger, PRB (1980) [PZ]

(b) our thermal LDA
[GDSMFB]



Taken from: K. Ramakrishna, T.
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Phys. Rev. B **101**, 195129 (2020)



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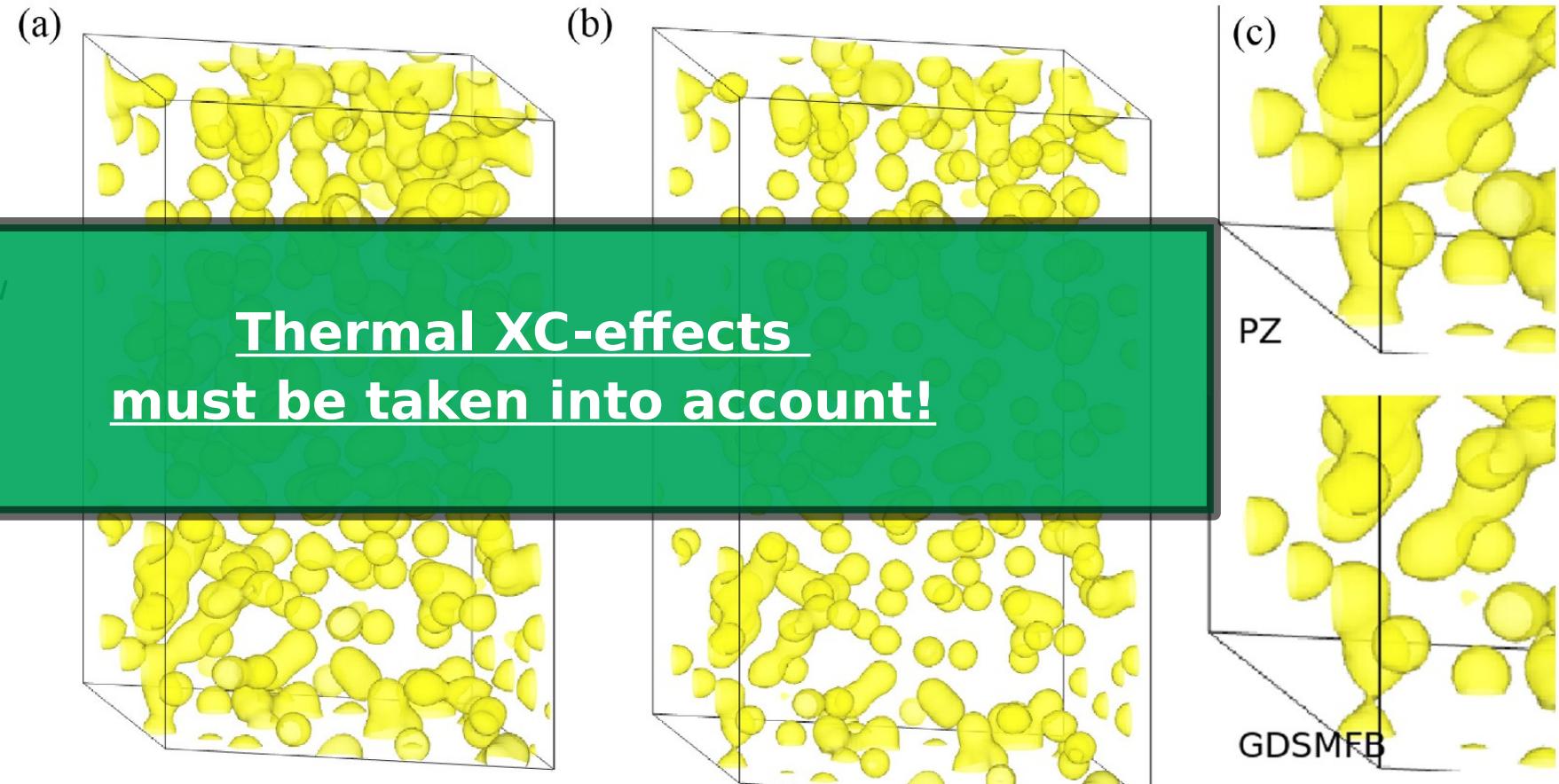
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Part II: Density response of warm dense electrons

Density response functions, local field correction

- Dynamic density response function

$$\chi(q, \omega) = \frac{\chi_0(q, \omega)}{1 - \frac{4\pi}{q^2} [1 - G(q, \omega)] \chi_0(q, \omega)}$$

→ $\chi_0(q, \omega)$ ideal density response function

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Important input for many applications:

- Advanced nonlocal XC-functionals for DFT
- Stopping power, electronic friction, ...
- Effective potentials
- Electrical/thermal conductivity
- Interpretation of XRTS experiments
- ...



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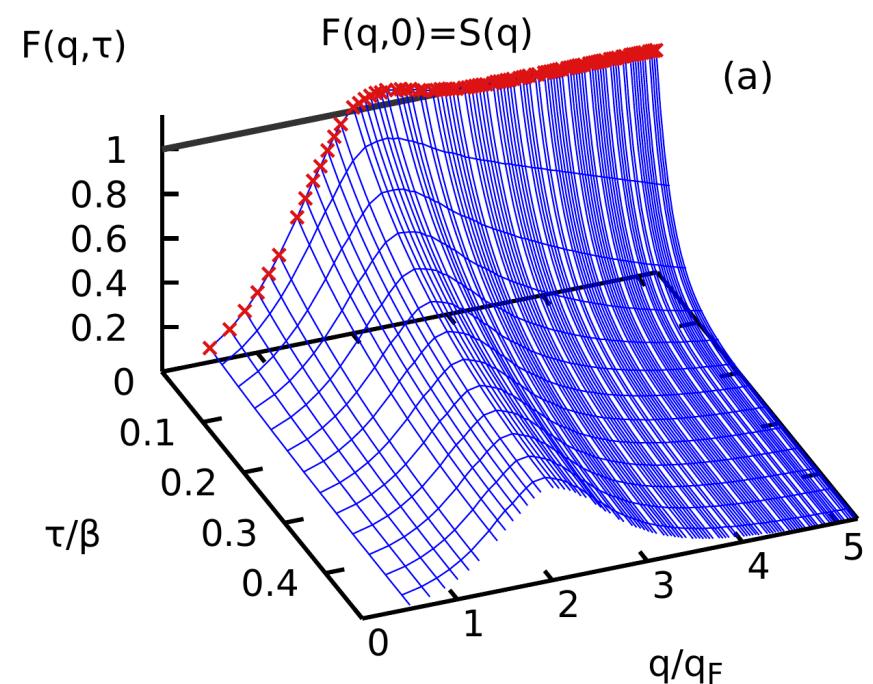
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Imaginary time $\tau \in [0, \beta]$

$$F(q, \tau) = \frac{1}{N} \langle \rho(q, \tau) \rho(-q, 0) \rangle$$



Taken from: **T. Dornheim**, T. Sjostrom, S. Tanaka, and J. Vorberger, Phys. Rev. B **101**, 045129 (2020)

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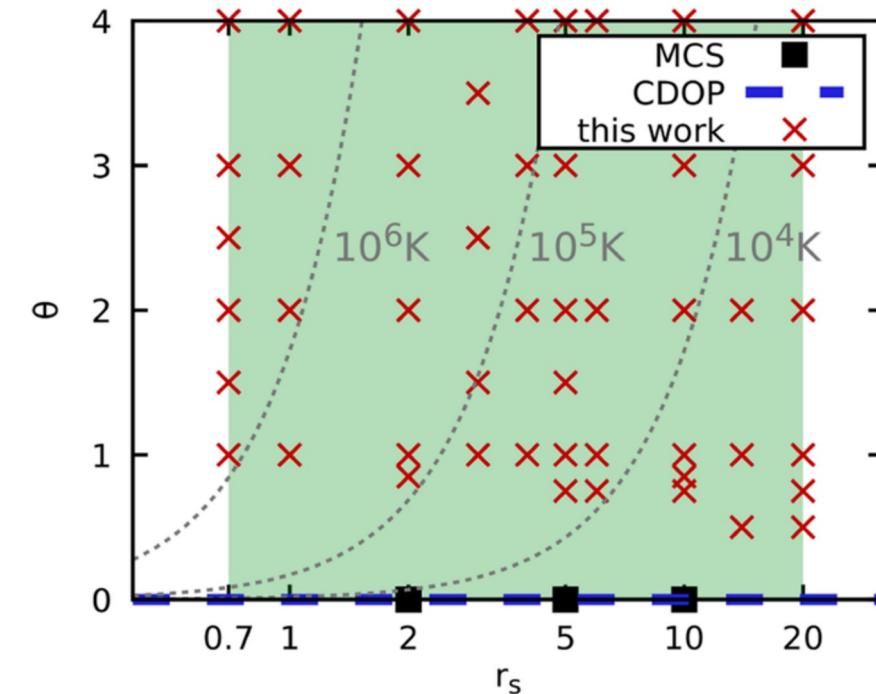
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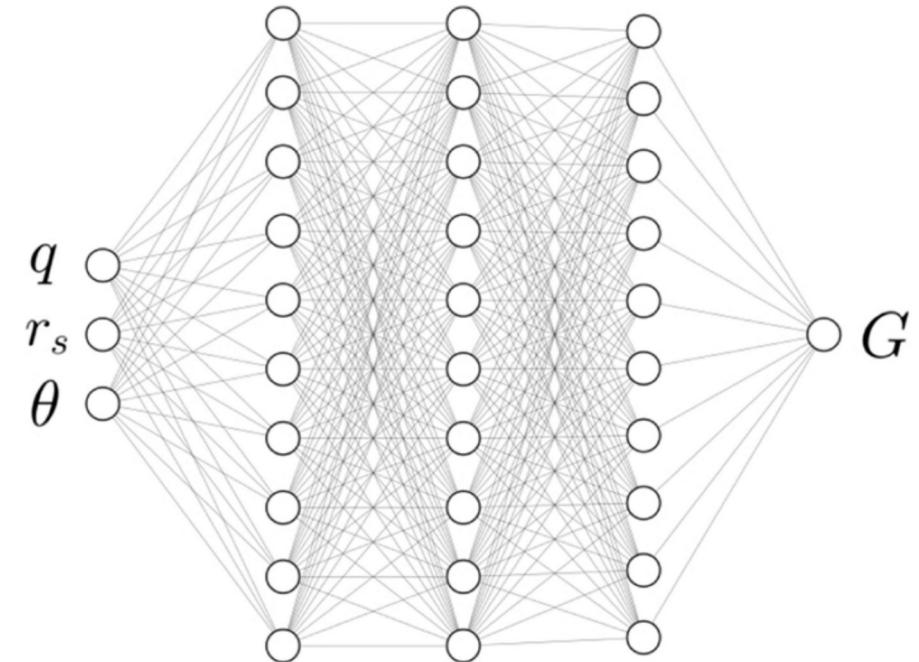
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Neural net representation covering full WDM regime.



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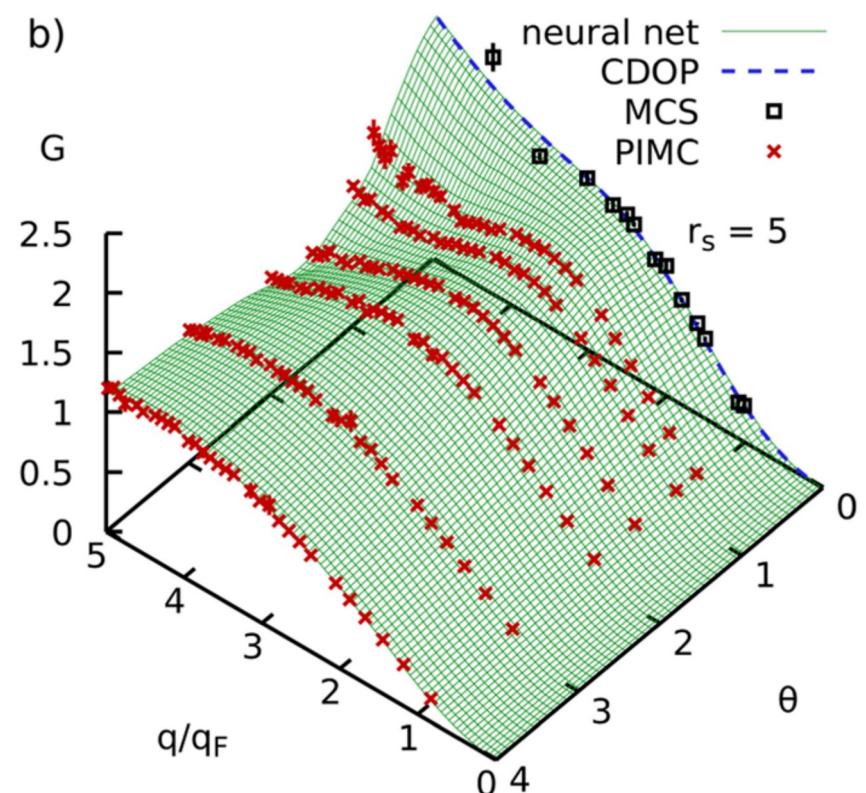
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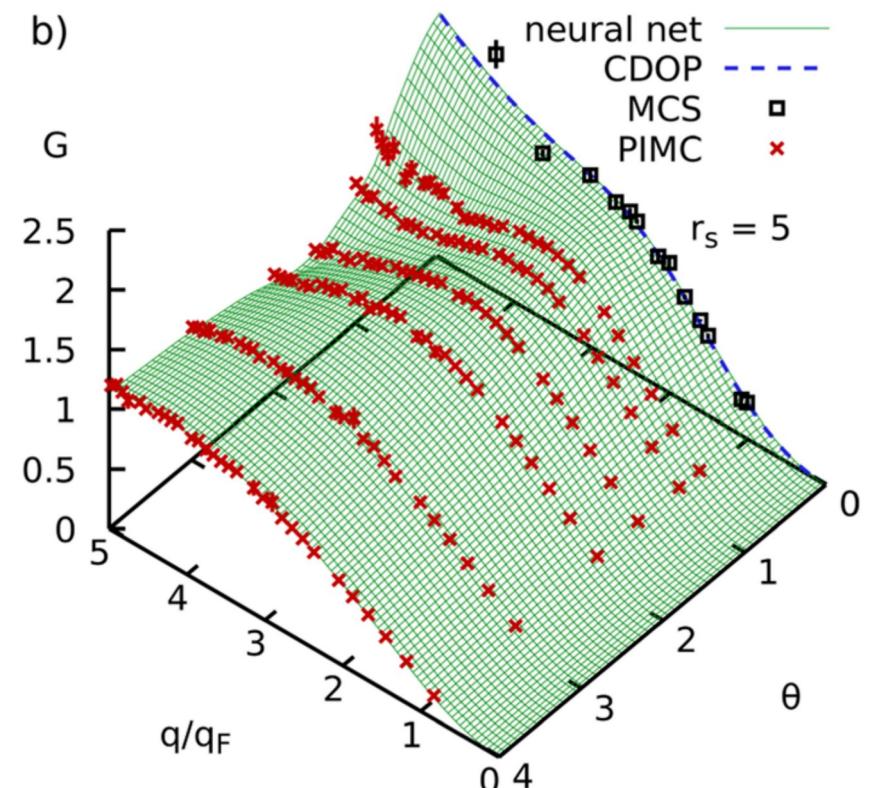
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First results for XC-kernel of hydrogen:

M. Böhme, Zh. Moldabekov, J. Vorberger, and T. Dornheim, *Phys. Rev. Lett.* **129**, 066402 (2022)



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Part II: Static density response of WDM

Density response of warm dense hydrogen

- **Exact exchange-correlation kernel of hydrogen (PIMC)**

- Benchmark Adiabatic LDA (ALDA) etc
- Influence of partial localization around ions?
- ...

- **UEG models break down at low density**

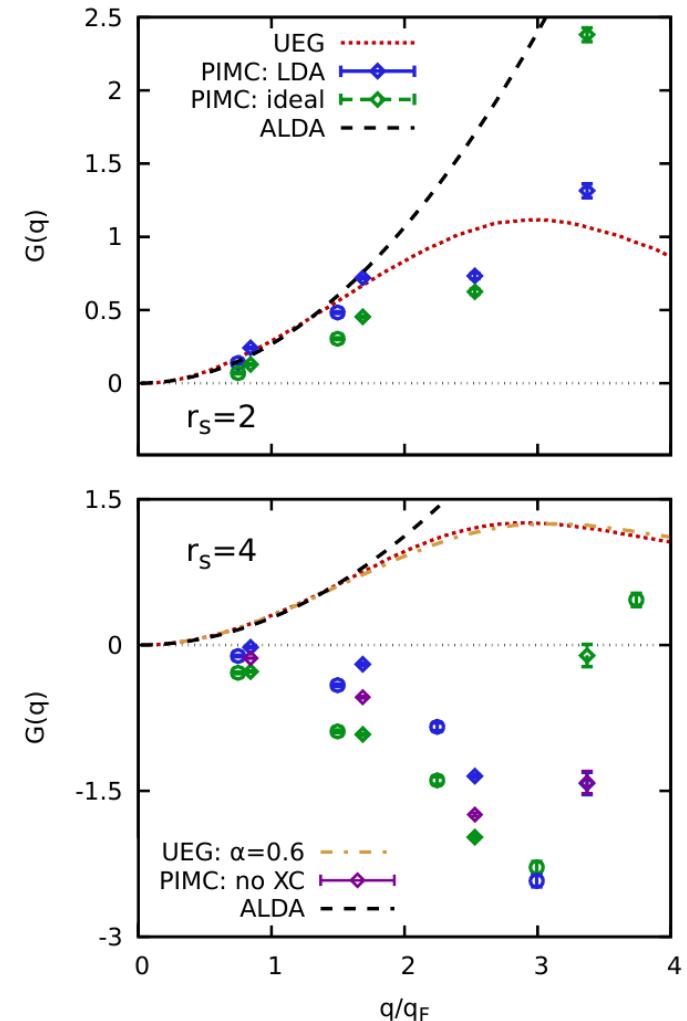
- **Future works:**

- input for time-dependent DFT, etc
- development of new XC-functionals
- predict NIF experiments
- ...



Exchange—correlation kernel of warm dense hydrogen

Taken from: M. Böhme, Zh. Moldabekov,
J. Vorberger, and **T. Dornheim**, *Phys.
Rev. Lett.* **129**, 066402 (2022)



Part II: Static density response of WDM

Density response of real materials

- Compute exchange–correlation kernel from DFT simulations

→ Problem: DFT limited to single-electron density

→ Solution: Perturb system, compute density response

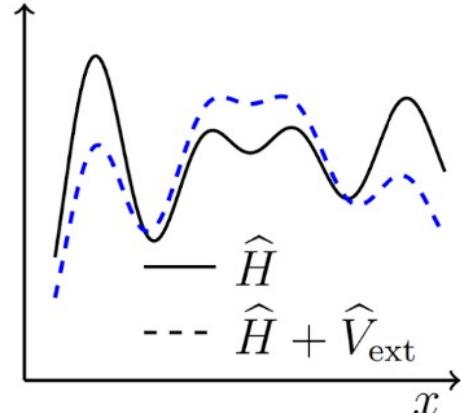


- **DFT gives access to exchange–correlation kernel**

Taken from: Zh. Moldabekov,
 M. Böhme, J. Vorberger, D. Blaschke,
 and T. Dornheim, arXiv:2209.00928

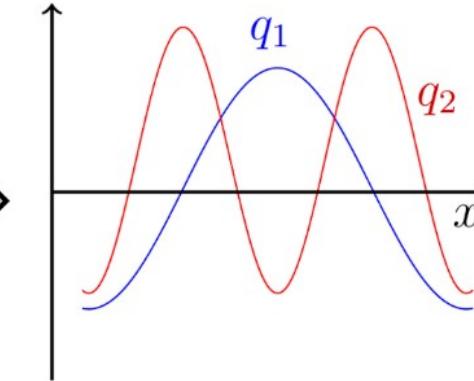
DFT : Single particle

$n(x)$ density



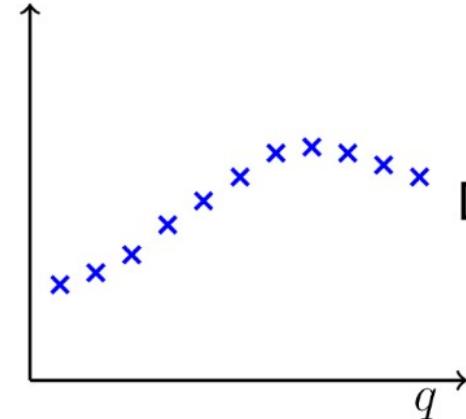
Harmonic

$\Delta n(x)$ perturbations



Exact e – e correlations

$K_{xc}(q)$



Applications :

$S(q, \omega), \epsilon(q, \omega),$
 $\sigma_{el}(q, \omega), \sigma_{th}(q, \omega),$
 $\frac{\delta E}{\delta l}, S(q), g(r),$
 $\phi_{eff}(r) \dots$

Part II: Static density response of WDM

Density response of real materials

- **Compute exchange–correlation kernel from DFT simulations**

→ Problem: DFT limited to single-electron density

→ Solution: Perturb system, compute density response

- **DFT gives access to exchange–correlation kernel**

- **DFT capable to generate electronic XC-effects**

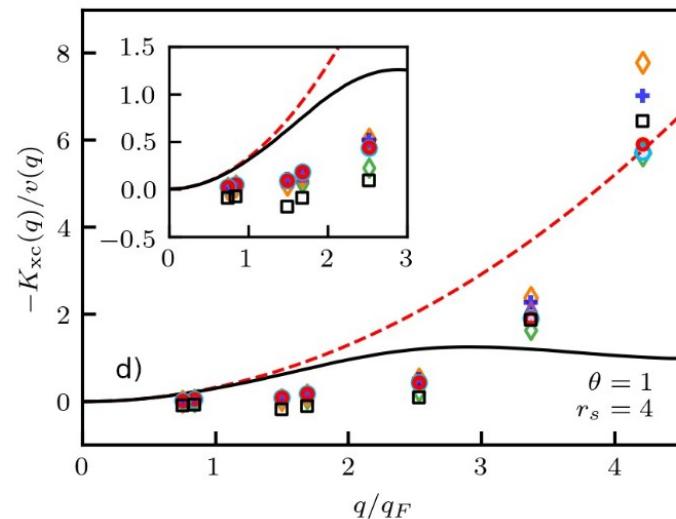
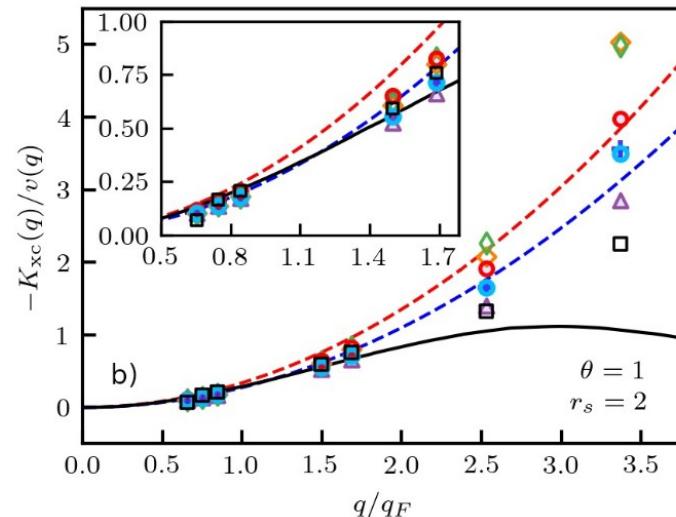
→ insights into performance of XC-functionals

→ XC-effects of real materials

→ ...

Exchange–correlation kernel of warm dense hydrogen

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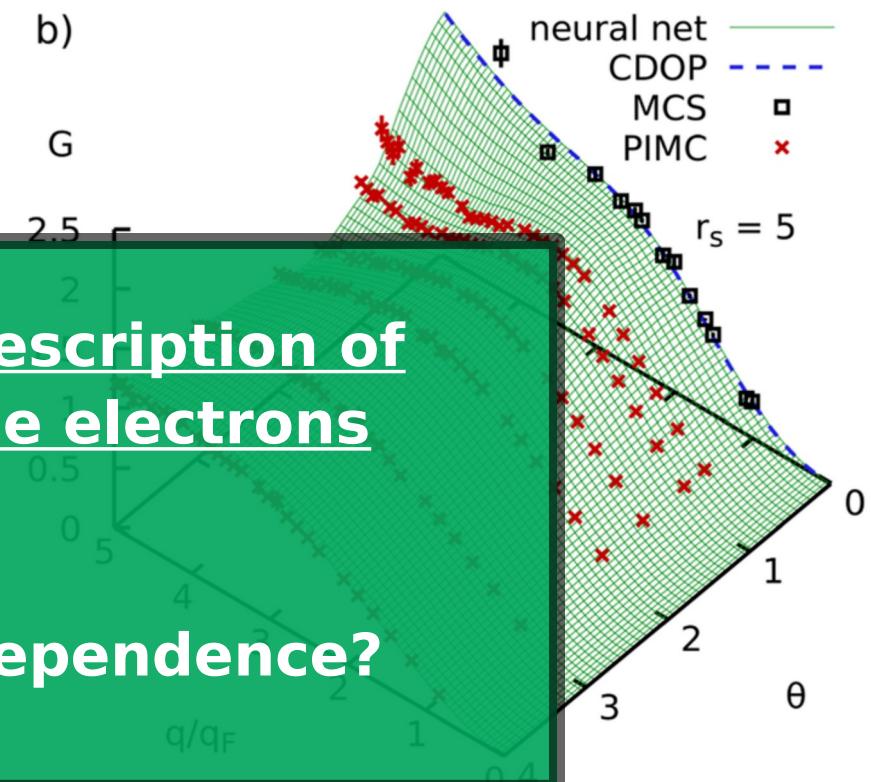
$\rightarrow G(q, \omega)$ dynamic local field correction, containing all electronic XC-effects

Wave-number resolved description of XC-effects of warm dense electrons

What about frequency-dependence?

- Static limit: Exact QMC results for $\chi(q) := \chi(q, 0)$, $G(q)$

- Extensive PIMC data for LFC $G(q)$ for $\sim 50 r_s$ - θ combinations



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Need for dynamic properties of WDM

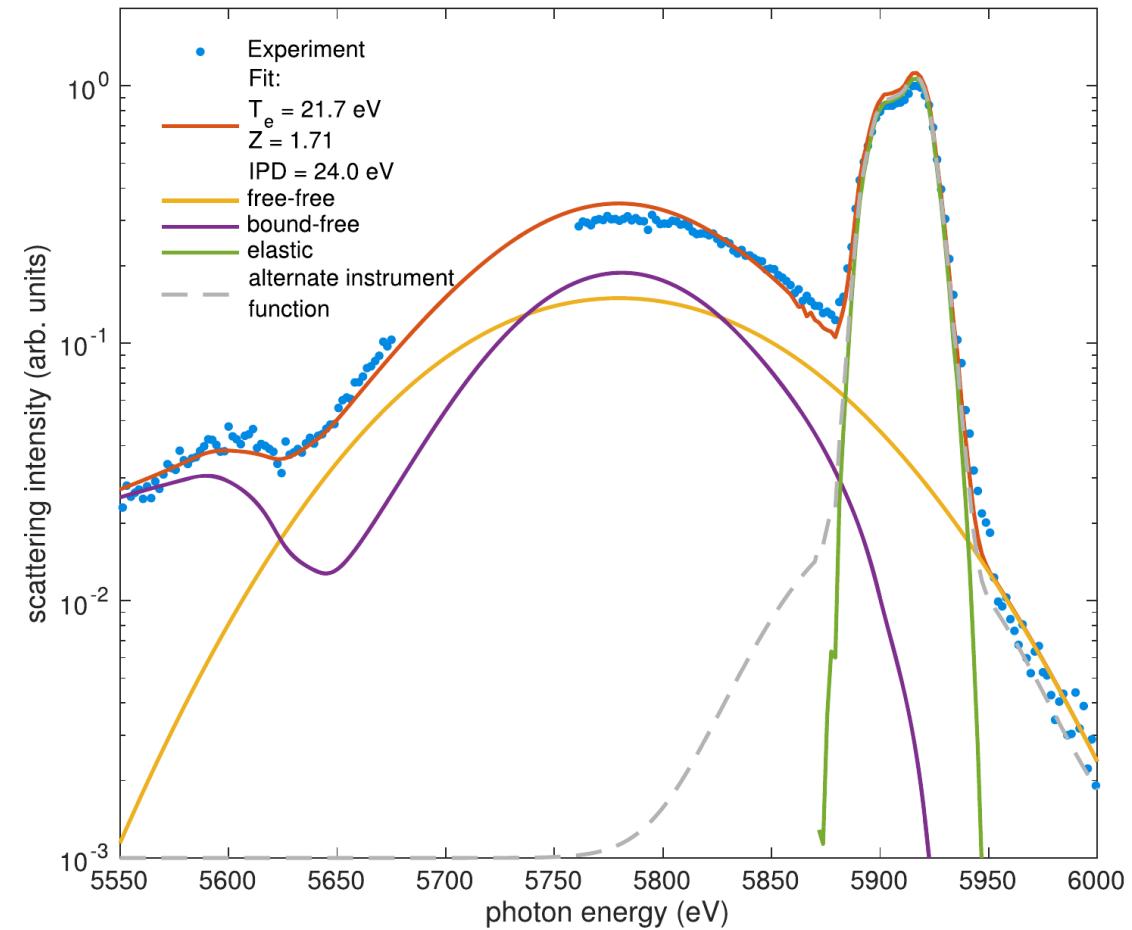
- **WDM Diagnostics:** obtain plasma parameters from XRTS experiments

→ Dynamic structure factor

$$F(q, t) = \frac{1}{N} \langle \rho(q, t) \rho(-q, 0) \rangle$$

$$\Rightarrow S(q, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt F(q, t) e^{i\omega t}$$

Isochorically heated graphite at LCLS (Stanford)



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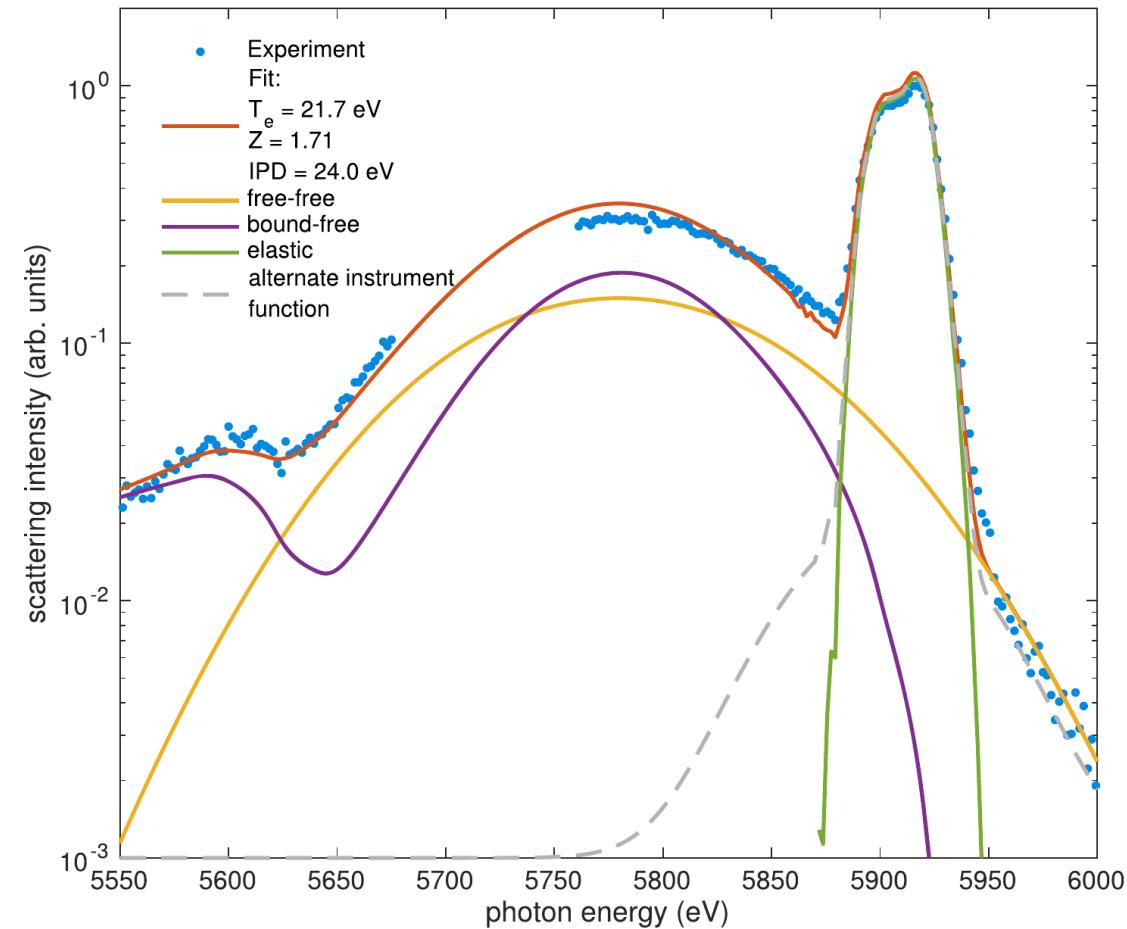
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- **Rigorous description of dynamic properties even more challenging than TD equilibrium**

- TD-DFT: adiabatic approximation, no XC-kernel
- Green functions: approximation in coupling
- PIMC: Imaginary time, **analytic continuation** possible for UEG!

Isochorically heated graphite at LCLS (Stanford)



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Part II: Dynamic density response of WDM

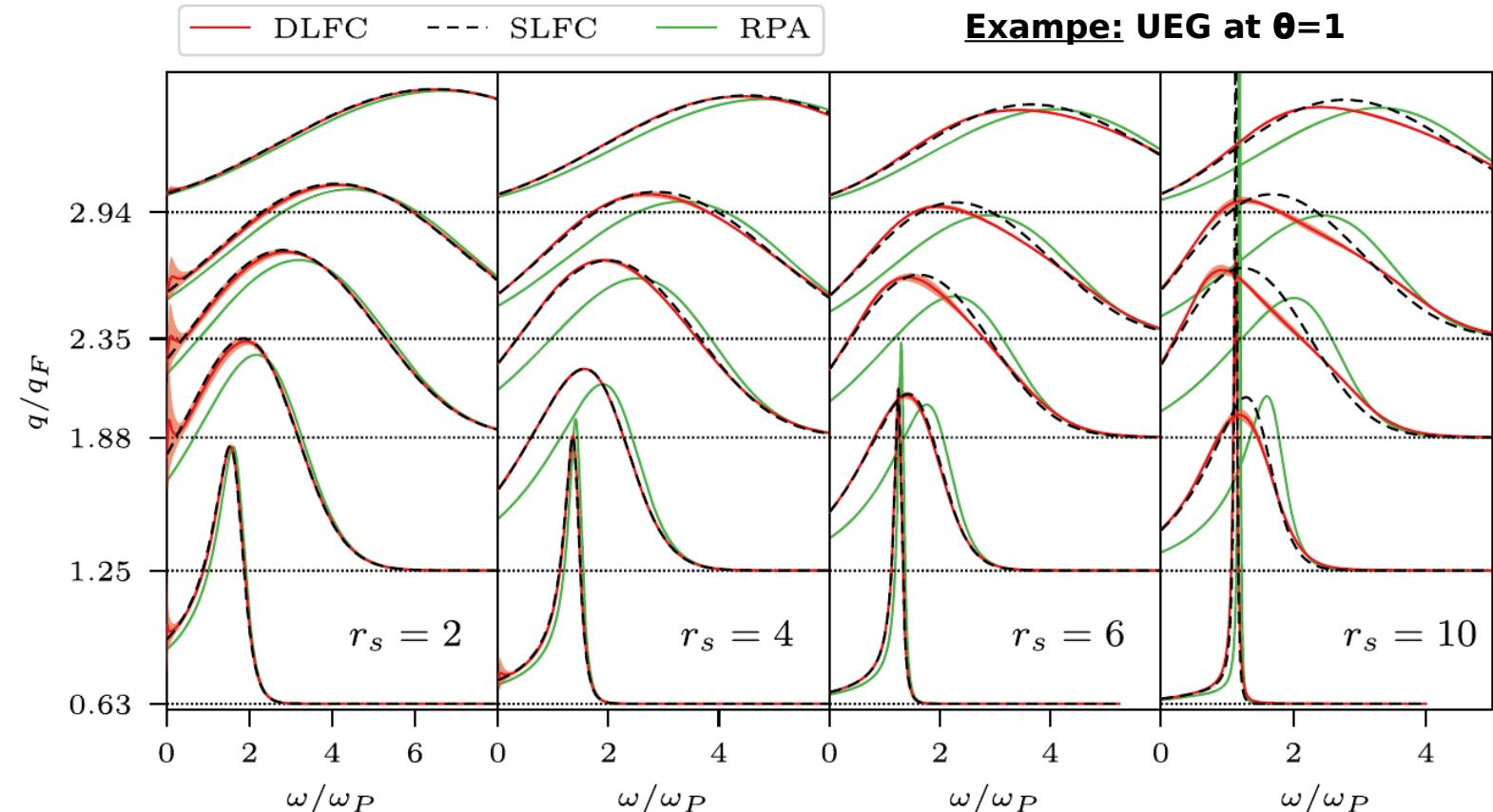
First *ab-initio* results for dynamic structure factor of UEG

DLFC: exact solutions

RPA: $G(\mathbf{q},\omega)=0$

SLFC: $G(\mathbf{q},\omega)=G(\mathbf{q},0)$

→ static approximation



Taken from: **T. Dornheim**, S. Groth, J. Vorberger, and M. Bonitz,
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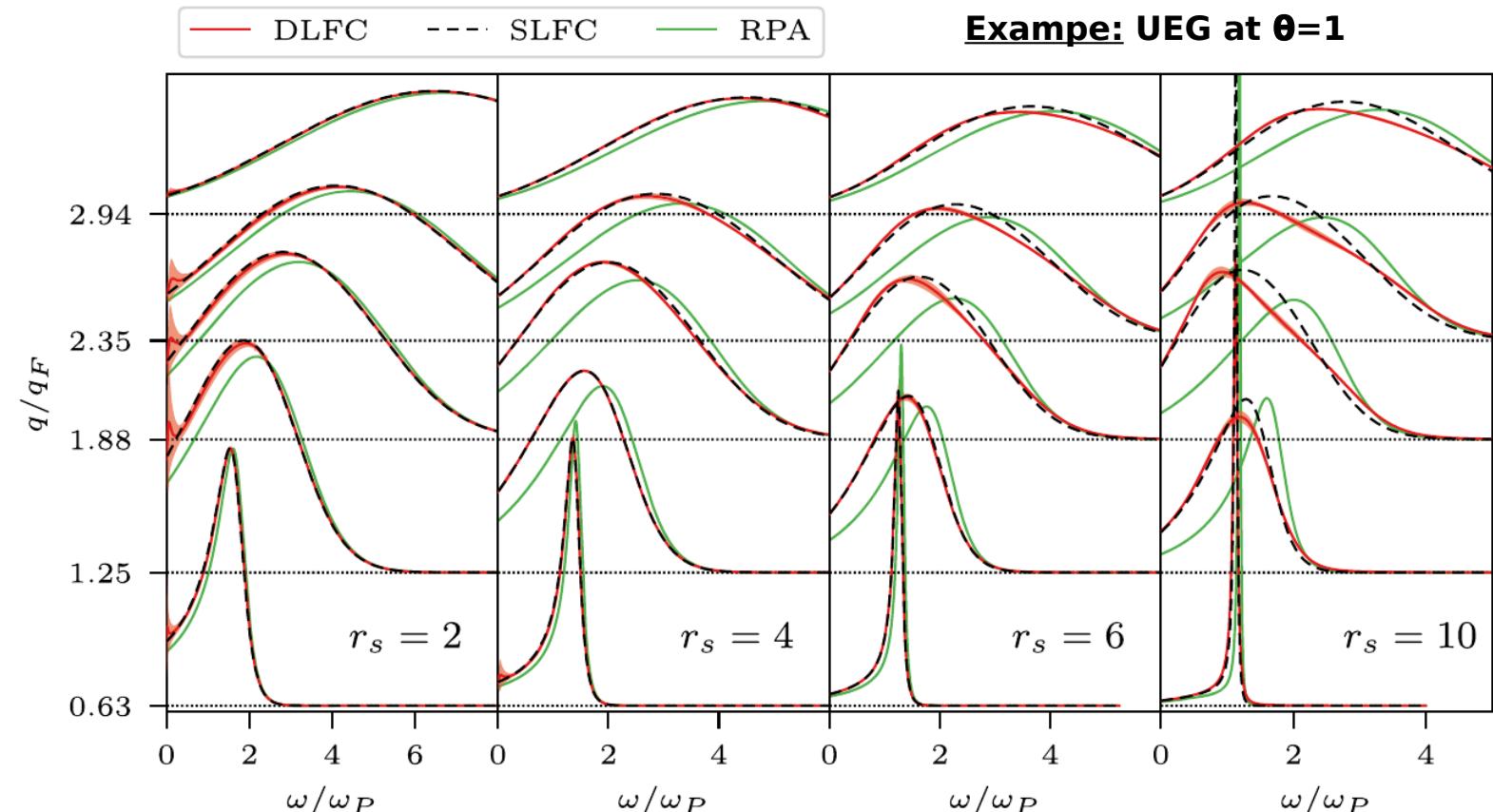
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- Metallic density ($r_s=2,4$):

→ red-shift compared to RPA

→ static approximation quasi-exact



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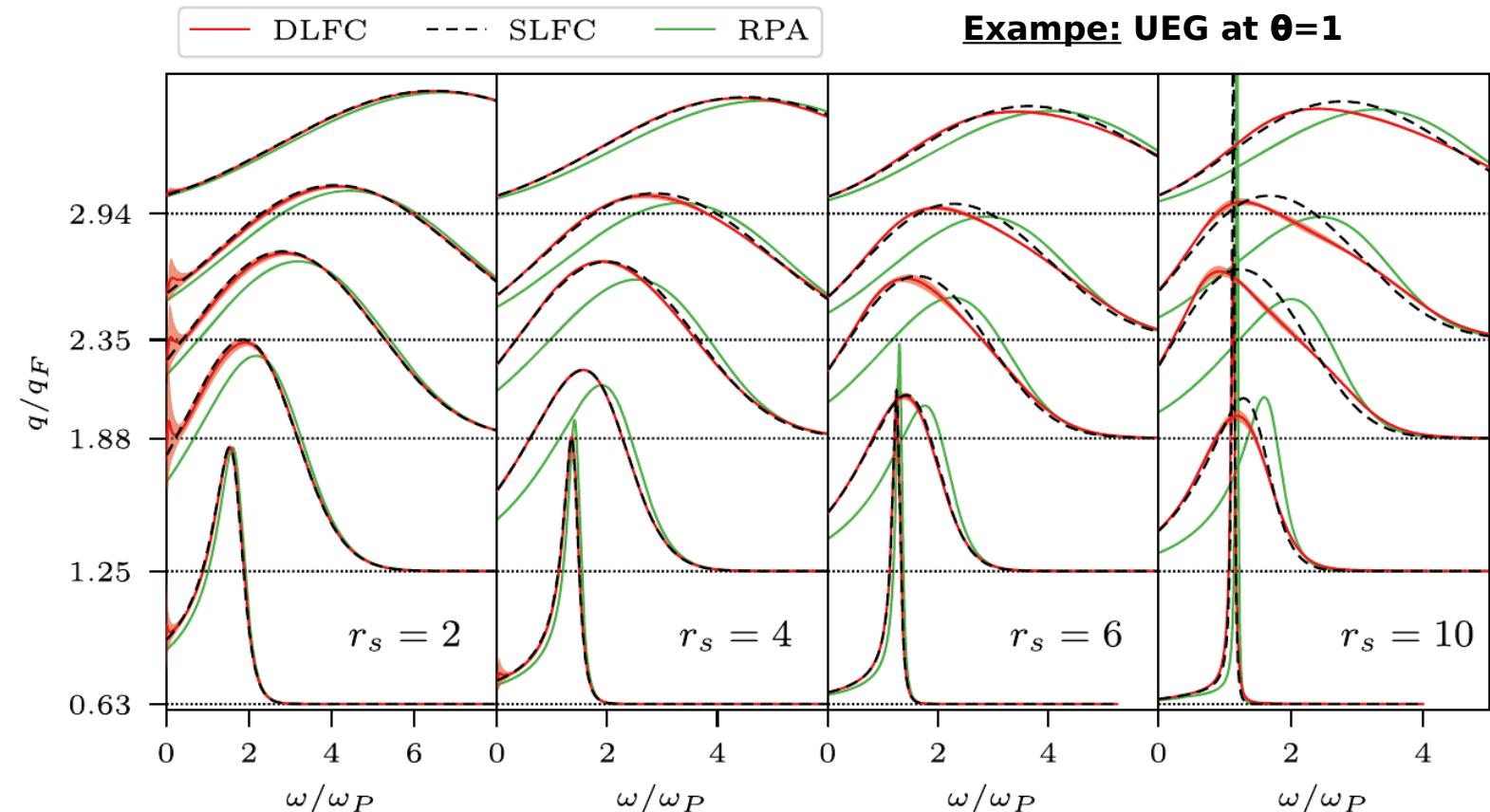
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- Electron liquid ($r_s=10$):

→ non-trivial shape of $S(\mathbf{q},\omega)$

→ negative dispersion relation



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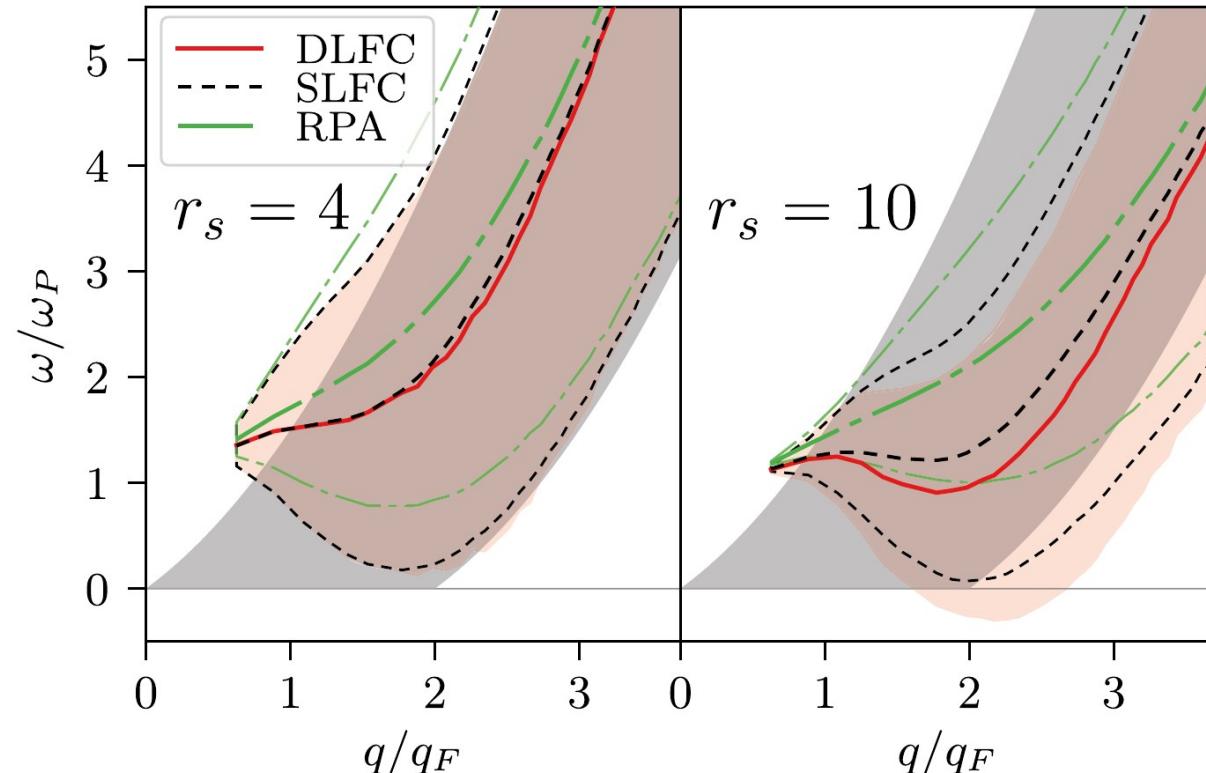
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Example: UEG at $\theta=1$



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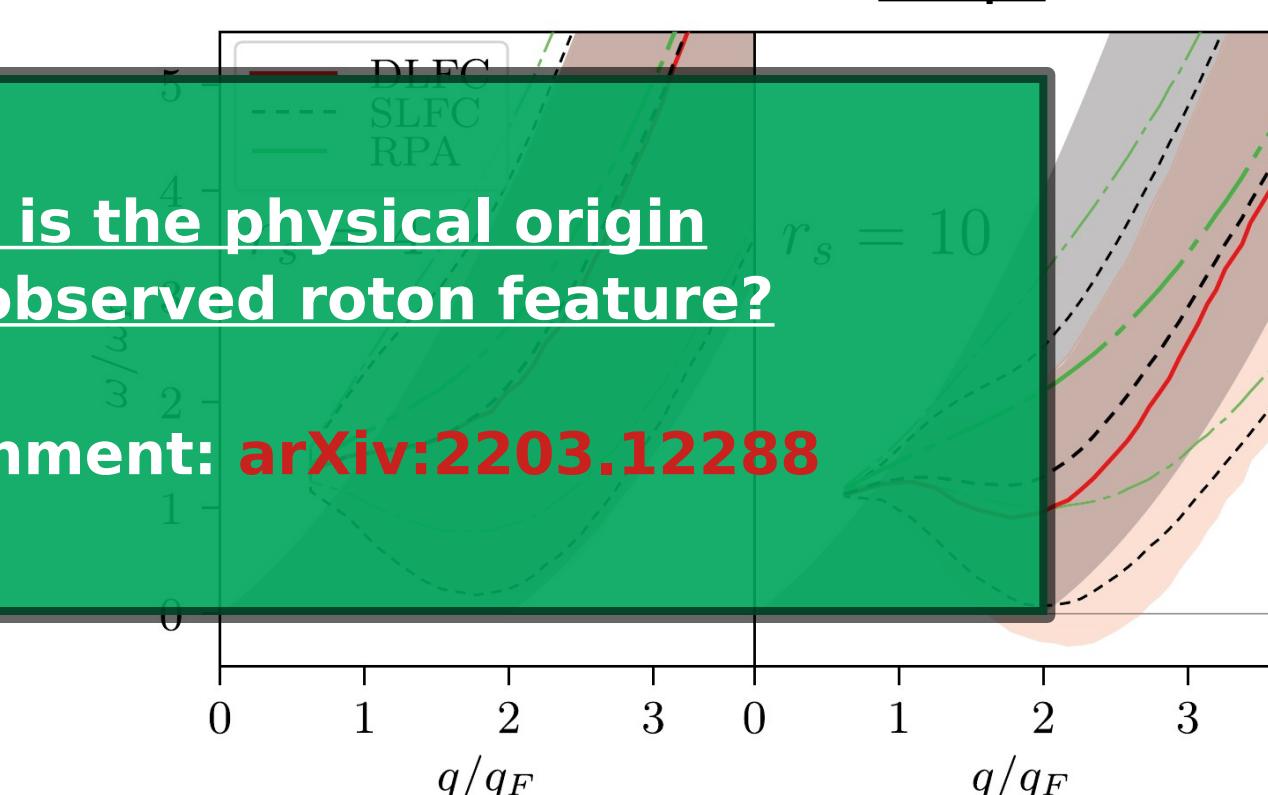
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**What is the physical origin
of the observed roton feature?**

Pair alignment: [arXiv:2203.12288](https://arxiv.org/abs/2203.12288)



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Taken from: **T. Dornheim**, S. Groth, J. Vorberger, and M. Bonitz,
Phys. Rev. Lett. **121**, 255001 (2018)

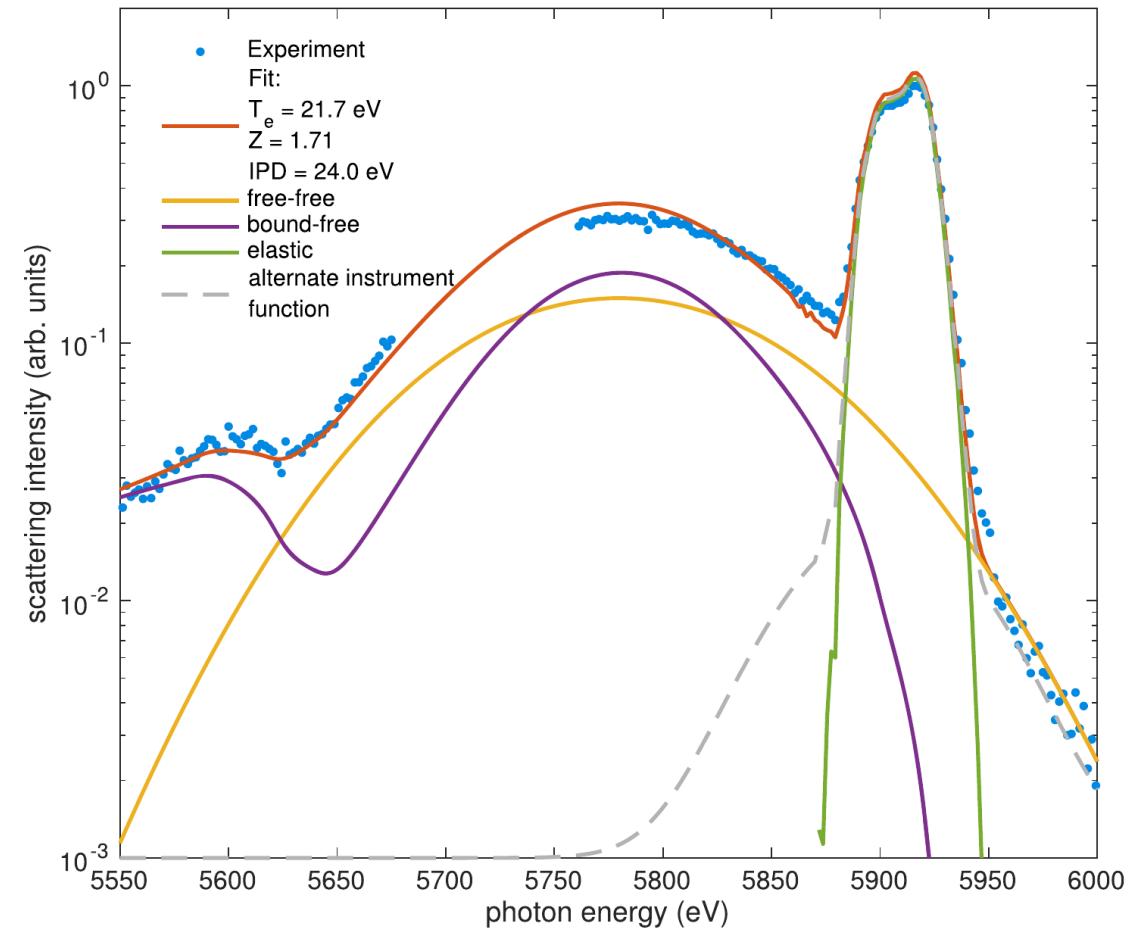
Part III: Dynamic properties in the imaginary time

X-ray Thomson scattering (XRTS)

- Standard way: construct a model for $S(q, \omega)$, convolve with instrument function $R(\omega)$, fit to XRTS signal $I(q, \omega)$

$$I(\mathbf{q}, \omega) = S(\mathbf{q}, \omega) \circledast R(\omega)$$

Isochorically heated graphite at LCLS (Stanford)



Taken from: D. Kraus *et al.*, *Plasma Phys. Control. Fusion* **61**, 014015 (2019)



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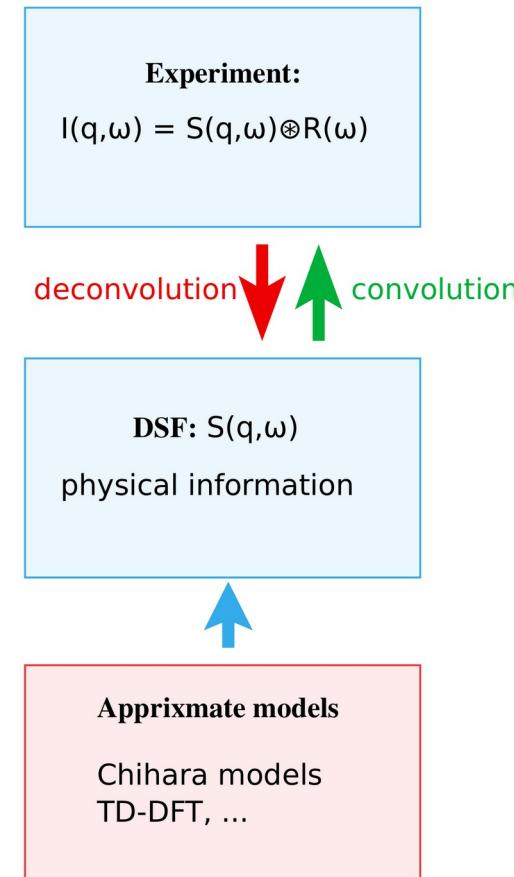
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Taken from: **T. Dornheim**, Zh. Moldabekov, P. Tolias, M. Böhme, and J. Vorberger, arXiv:2209.02254 (submitted)

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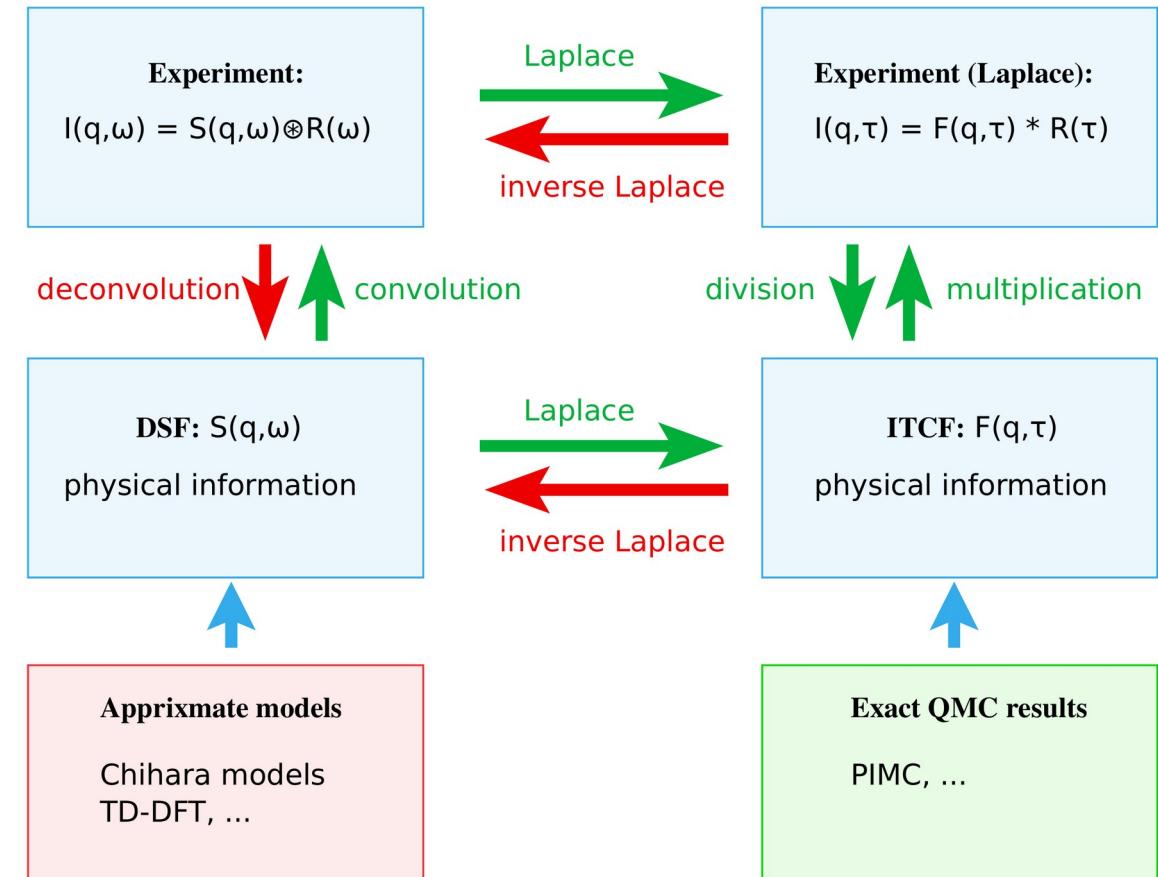
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- exact QMC simulations

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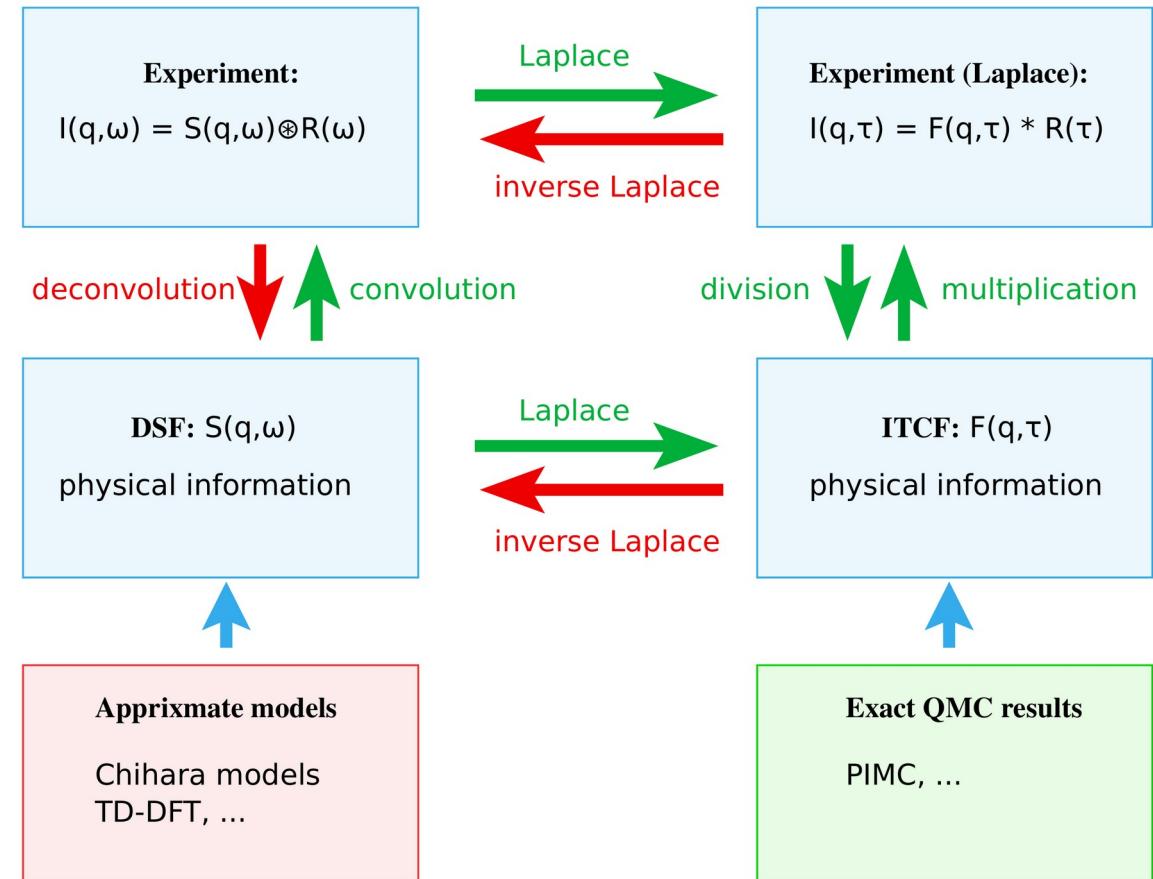
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Model-free temperature from XRTS experiments:

- **Detailed balance in the τ -domain:**

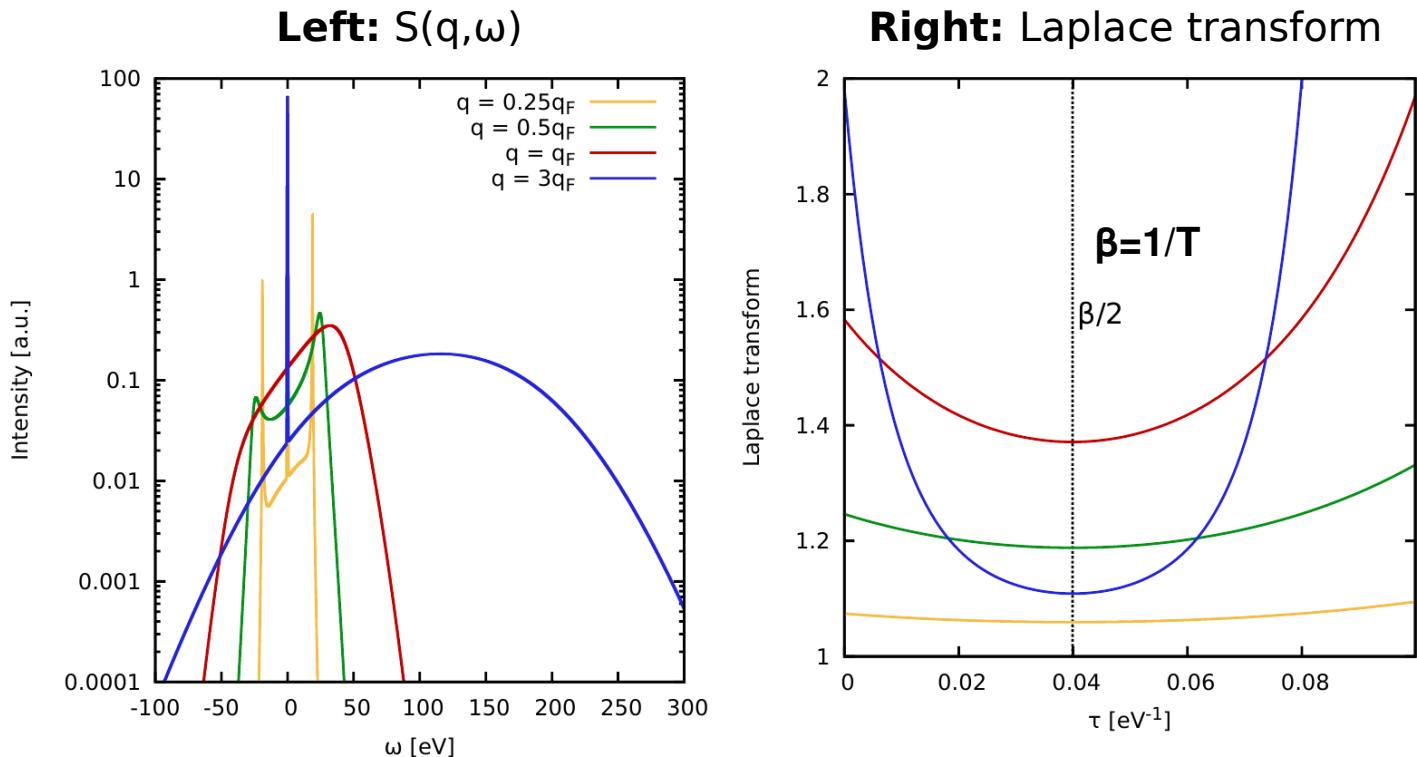
- works for all wave numbers
- no explicit resolution of plasmon required

Laplace transform:

$$\mathcal{L} [S(\mathbf{q}, \omega)] = \int_{-\infty}^{\infty} d\omega e^{-\tau\omega} S(\mathbf{q}, \omega)$$

→ symmetry around $\tau=(2T)^{-1}$

$$S(\mathbf{q}, -\omega) = S(\mathbf{q}, \omega) e^{-\beta\omega}$$



Taken from: T. Dornheim *et al.*, in preparation

Part III: Dynamic properties in the imaginary time

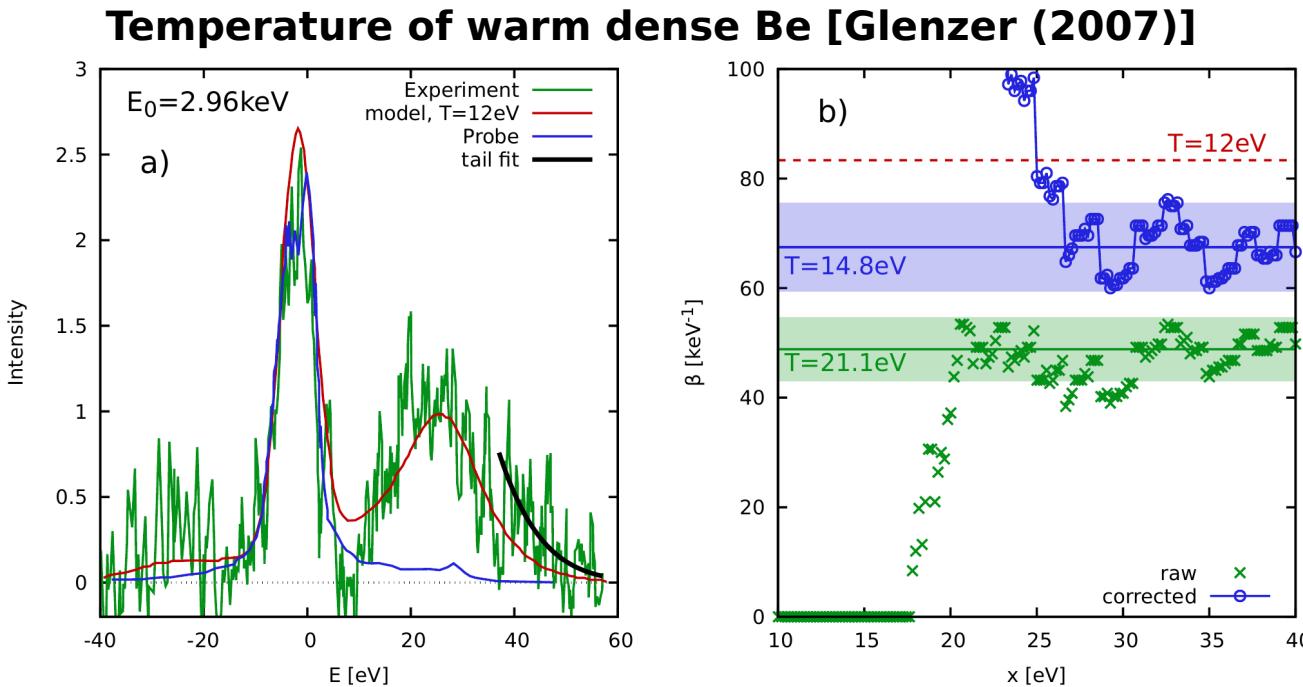
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$$\mathcal{L} [S(\mathbf{q}, \omega)] = \frac{\mathcal{L} [S(\mathbf{q}, \omega) \circledast R(\omega)]}{\mathcal{L} [R(\omega)]}$$

Taken from: **T. Dornheim, M. Böhme, D. Kraus, T. Döppner, T. Preston, Zh. Moldabekov, and J. Vorberger, arXiv:2206.12805**

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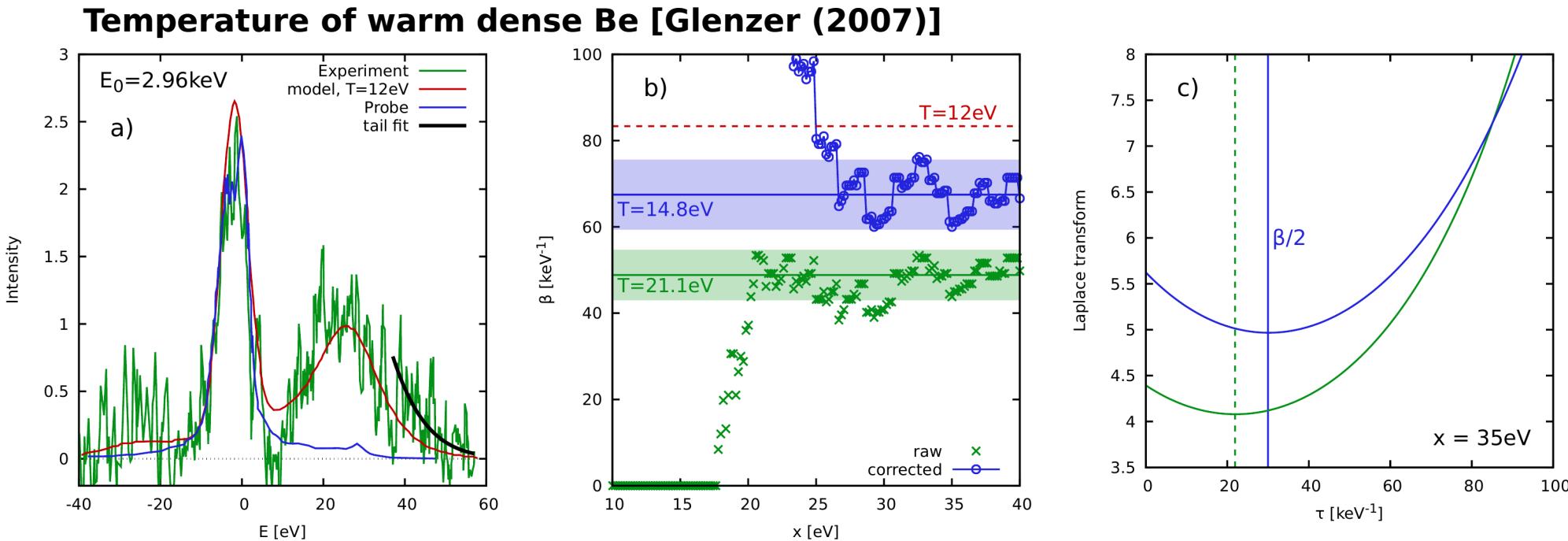
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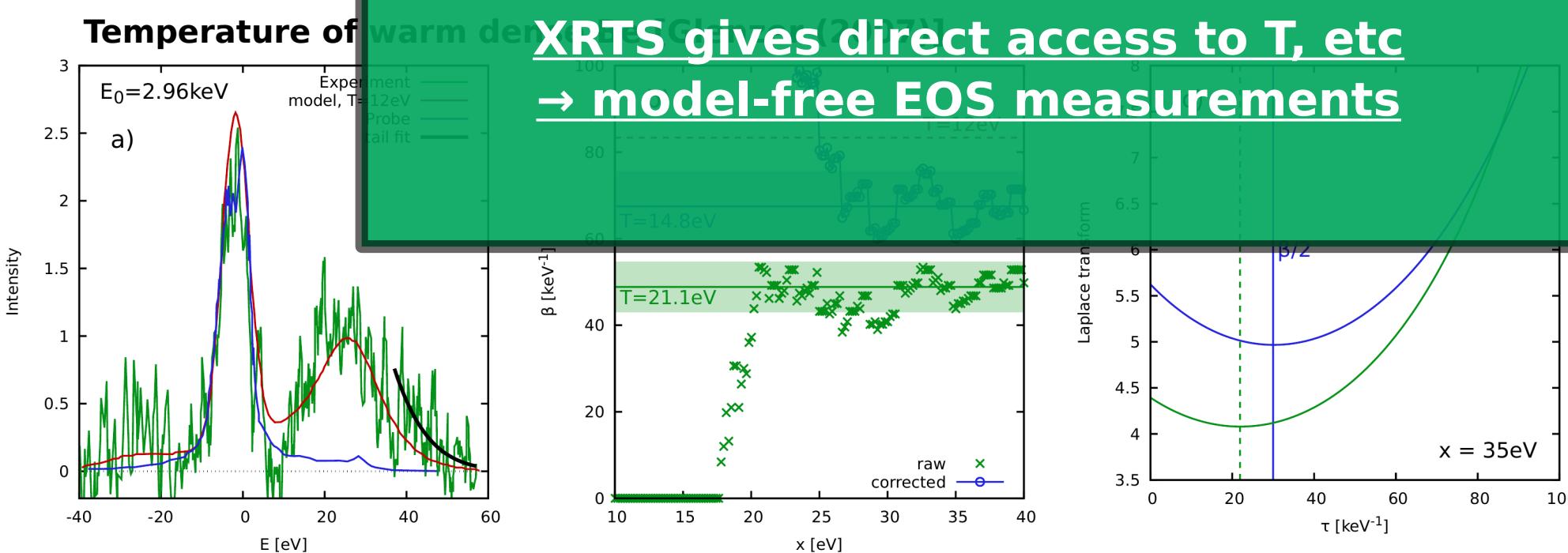
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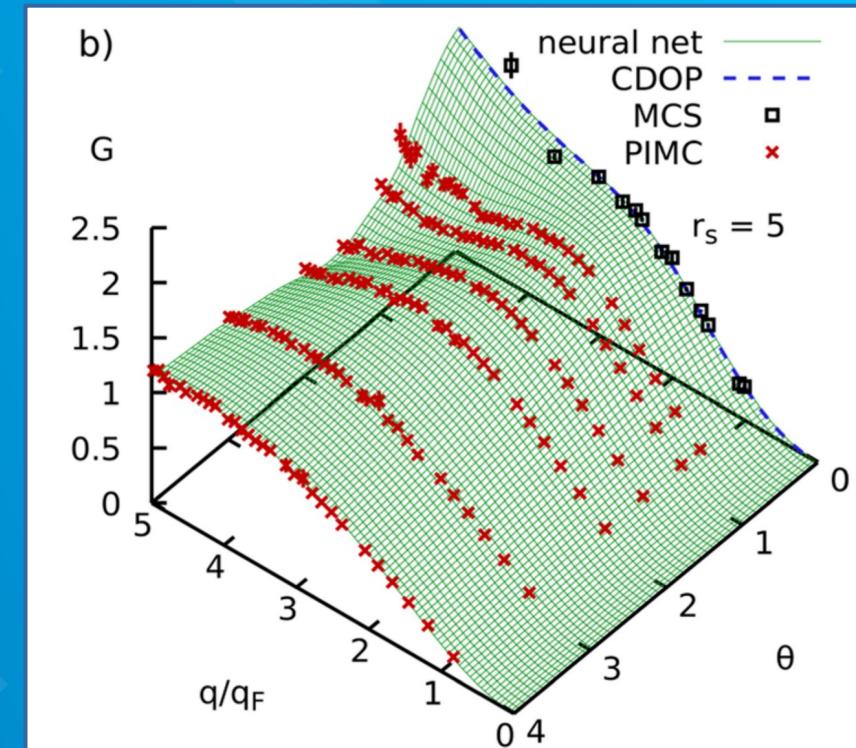
Summary and Outlook

Ab initio theory of WDM

- need for finite-T XC functionals based on PIMC results
- static density response: PIMC + neural net
- PIMC results for warm dense hydrogen
- DFT framework for the study of XC-effects
- dynamic density response: PIMC + analytic continuation

Key pre-print:

arXiv:2209.00928



Taken from: **T. Dornheim**, J. Vorberger, S. Groth, N. Hoffmann, Zh. Moldabekov, and M. Bonitz, JCP **151**, 194104 (2019)

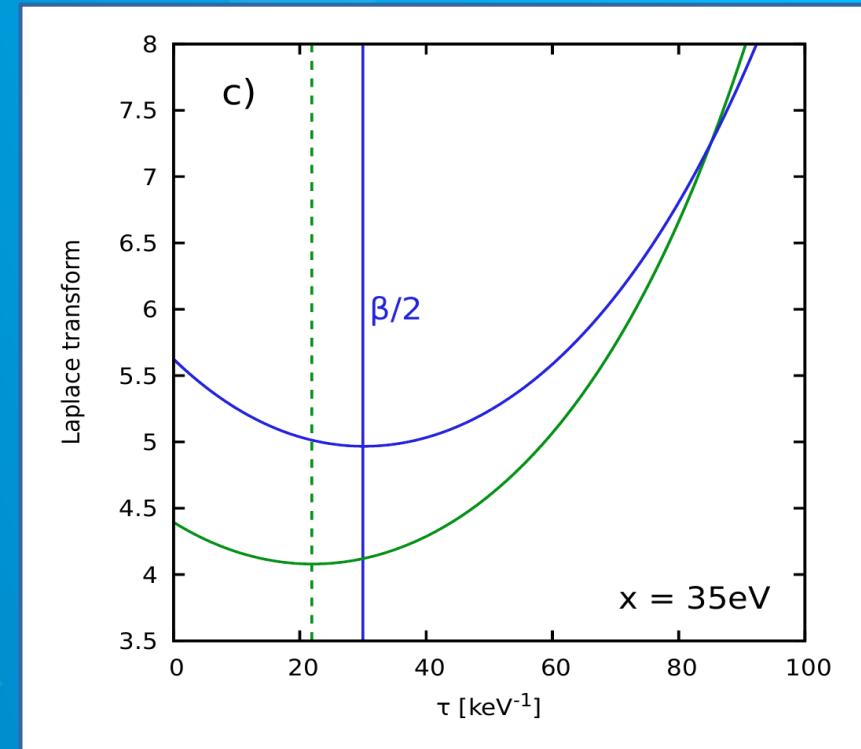
Physics in the imaginary time

- Usual ω -representation equivalent to τ -domain
- **Model-free T-diagnostics** etc.
- Future works: physical insights from the τ -domain

Key pre-prints:

arXiv:2206.12805

arXiv:2209.02254



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