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- Dr. Graziani F, Dr. Grabowski P, Dr. Remington BA, Ms. Karlton J
- Lawrence Livermore National Lab, Physical Sciences Division

Interface dynamics and flow fields structure in ideal and realistic fluids

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Thanks to colleagues: Ilyin DV, Goddard WA, Anisimov SI, Kadanoff LP

2022 Springer Nature Applied Sciences, 2022 Physics Letters A,
2021 Physics of Plasmas, 2021 Physica Scripta, 2020 Physics of Fluids,
2019 Proc Natl Acad Sci USA, 2019 Europhysics Letters, 2018 Physics of Plasmas,
2015 Physica Scripta.

Who am I?

- University of Western Australia (since end of 2016)
    - Prof. Abarzhi SI; Dr. Hill DL, Dr. Matthews MT, Dr. Pfefferle D;
    - Williams K, Wright CE, Wright J, Naveh A, Li JT, Pandian A.
  - Caltech (USA) – Mr. Ilyin DV [Prof Goddard WA];
  - Stanford (USA) – Dr. Jain SS, Dr Hwang HC, Dr Chan WCHR [Prof. Moin P].

- SIA:
    - Carnegie Mellon, U Chicago, Stanford, SUNY Stony Brook – USA;
    - Osaka U – Japan; U Bayreuth – Germany;
    - Landau Inst Theor Phys, Academy of Sciences – Russia.
  - Key discoveries are:
    - new class of fluids instabilities, inertial mechanism of interface stabilization, resolution of Landau 1944 paradox;
    - special self-similarity class; order in Rayleigh-Taylor mixing; group theory approach for fundamentals of fluids instabilities and interfacial mixing.
  - Key contributions to the community:
    - founding program ‘Turbulent Mixing and beyond’;
    - organizing high profile conference and symposia [at the KITP in Oct 2023];
    - editorial work [20+ edited books with lead journals and publishers]
  - Some recognitions
    - Awards – Natl Acad Sci, Natl Sci Foundation, Japan Soc Prom Sci, A v Humboldt
    - APS Fellow [‘for deep and abiding work on RT instabilities & community leadership’]
    - member of APS Committee on Scientific Publications.

Mathematics, Science, Engineering

- Mathematics
  - studies things that do not exist in nature;
  - gets a precise knowledge about objects existing in our mind only.

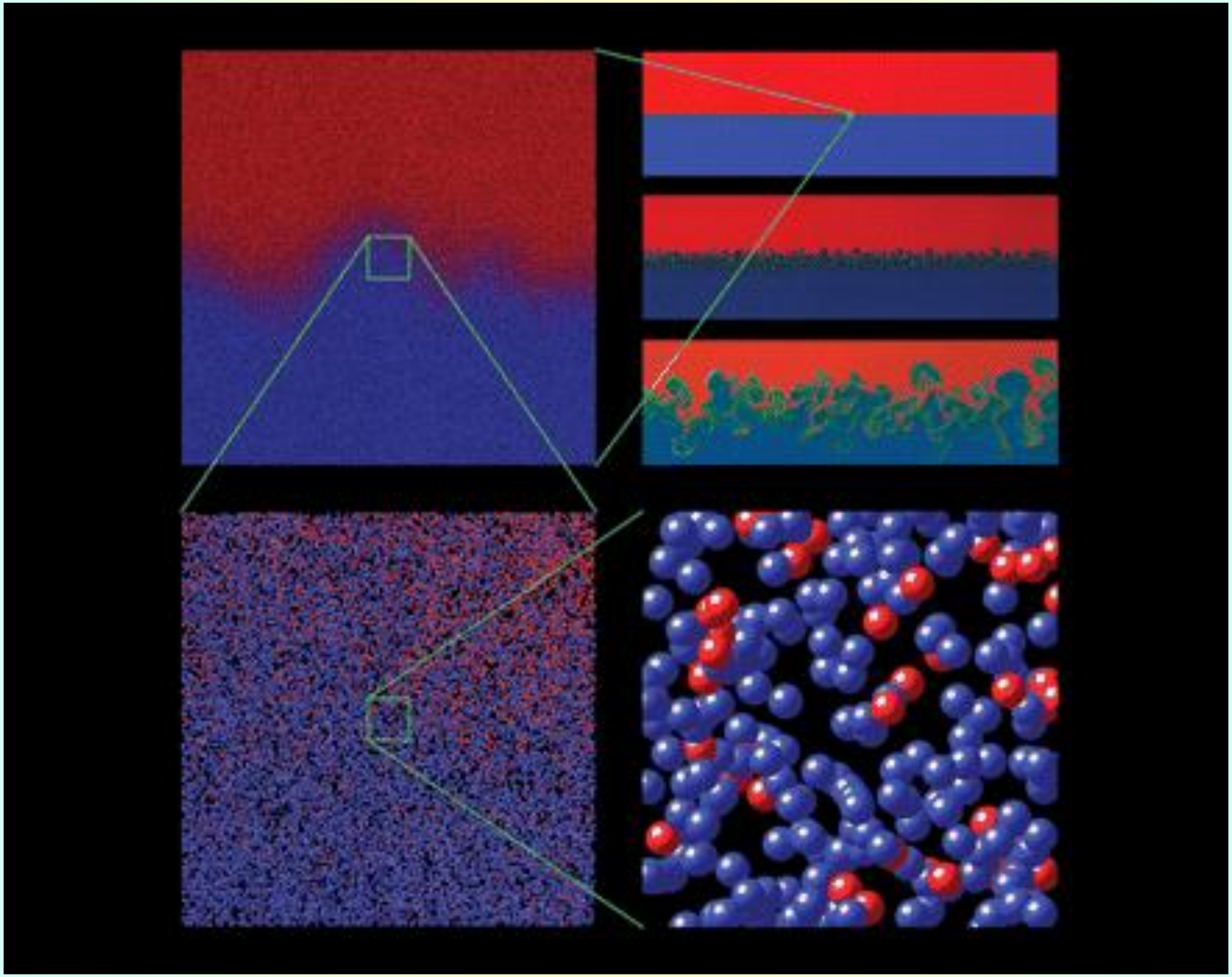
- Science
  - studies things that do exist in nature;
  - gets an approximate knowledge about objects independent of our mind.

- Engineering
  - control things that are man-made;
  - gets information with account for many minds and many natural processes.

What do they have in common?

Opinion Independent Results

Interfacial mixing from micro- to macro-scales



$g \sim 10^{10} g_{Earth}$

$\mathrm{Re} \sim 10^5$

Molecular dynamics simulations
of interfacial mixing

[Kadau et al. 2010]

Motivation and Outline

- Interfaces and interfacial mixing
  - exhibit non-equilibrium dynamics coupling micro to macro scales;
  - govern a broad range of processes
    - in plasmas, fluids, materials;
    - from celestial to atomic scales;
    - in high and in low energy density regimes.

- Understanding interfaces and interfacial mixing has crucial importance for
  - science, mathematics and engineering;
  - technology, energy and environment.

- We focus on stability of a phase boundary broadly defined.
  - develop rigorous theoretical framework;
  - capture physics of non-equilibrium dynamics of interfaces and interfacial mixing.
  - chart perspectives for future research.

What interfaces are?

- What interfaces are?
  - Interfaces appear obvious at a first glance.
  - Interface are a challenge to rigorously define.
  - The common wisdom is:
    - Interface is ‘thick’ and has fluxes (mass, heat, …) across it.
    - Front is ‘thin’ with zero fluxes.
    - Dynamics is stabilized by microscopic mechanisms.

- Interface is a place where balances are achieved by linking micro to macro scales.
  - Interface is stabilized by macroscopic inertial mechanism.
  - Interface can be destabilized by the acceleration.
  - Structure of macro fields in the bulk is set by micro transport at the interface.
  - Thermal heat flux and microscopic thermodynamics create vortical fields in the bulk.
  - New fluid instabilities are discovered for ideal fluids [conservative dynamics] and in realistic fluids [in three regimes – advection, diffusion, low Mach].

  - The classical ‘Landau 1944’ solution for Landau-Darrieus instability is a perfect mathematical match.

Interfaces are real.
They are globally stable.
They can be forced to destabilize.

Interface dynamics

is important to study:

Unstable interfaces and interfacial mixing control

- ✓ interstellar medium, core-collapse supernovae, thermonuclear flashes;
- ✓ inertial confinement fusion, magnetic fusion, Z-pinches in plasmas;
- ✓ light-material interaction, material transformation under impact, nano-fabrication;
- ✓ multi-phase geophysical flows, flows in ocean and atmosphere;
- ✓ energy and environment.

Interstellar media: clouds of molecular hydrogen

Birth of a star

Stalactites?
Stalagmites?

Eagle Nebula.

The fingers protrude from the wall
of a vast cloud of molecular hydrogen.

The gaseous tower are light-years long.

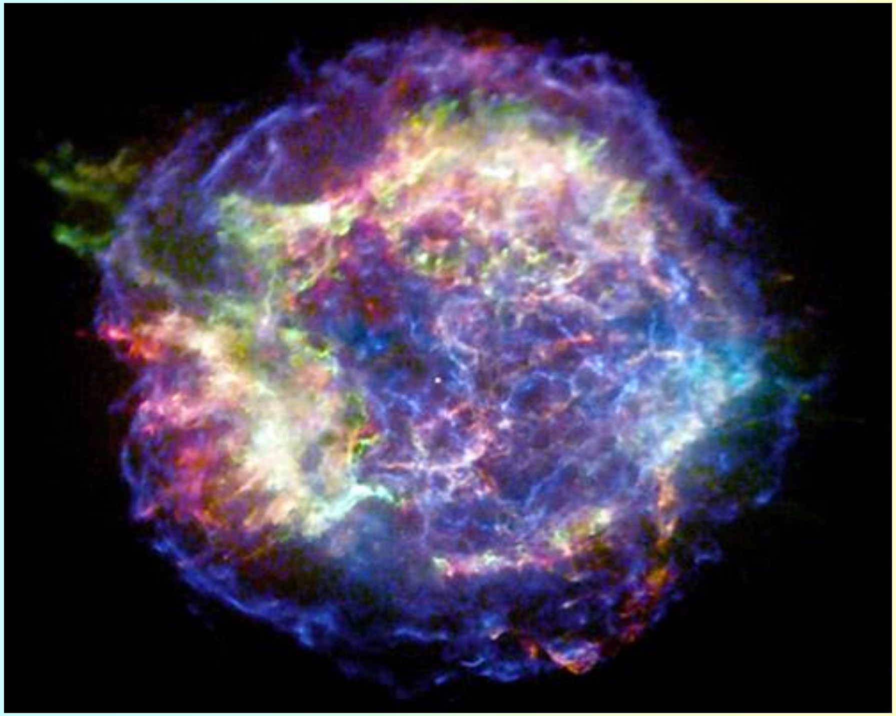
Hester and Cowen, Hubble pictures, 1995

The cloud stiffness can be caused by:
- the magnetic pressure by a large-scale primordial magnetic field.
- the ablation pressure by ionizing radiation of nearby stars.

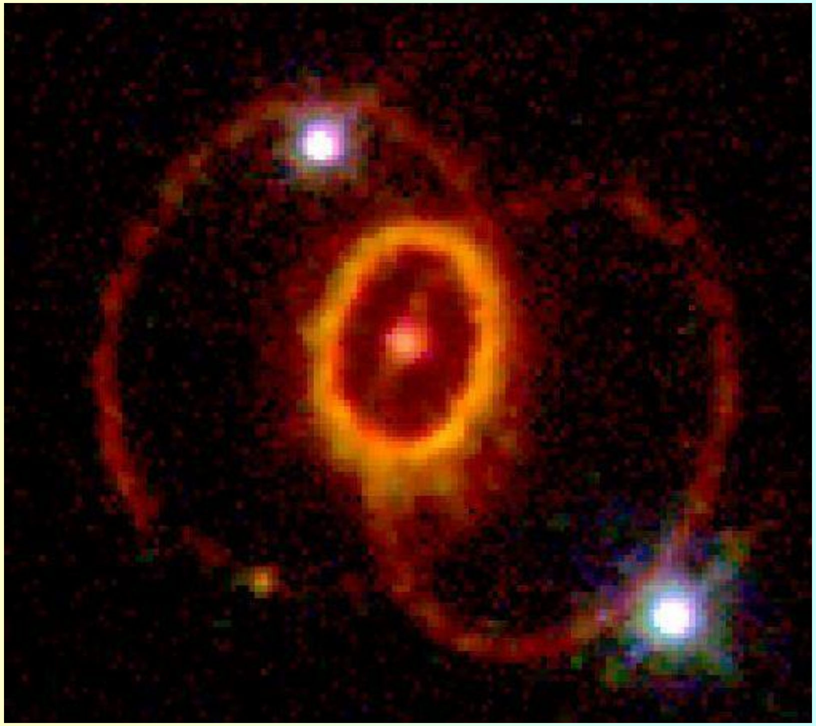
Ryutov et al. 2004;
Spitzer, 1978

Supernovae and nucleosynthesis

Death of a star



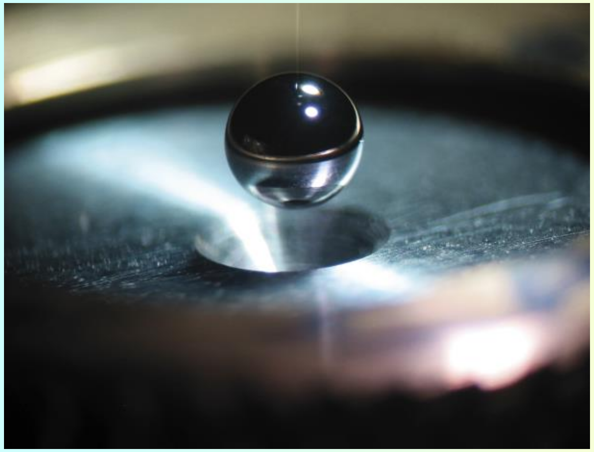
Kepler's supernova [discovered in 1604]



1987 supernova [Burrows, NASA,1994]

- Interfaces – type Ia
  - Interfacial mixing dominates propagation of thermonuclear flame front and provides conditions for synthesis of iron peak elements
- Fronts – type II
  - Interfacial mixing of layers of the progenitor star provides conditions for synthesis of heavy mass elements.

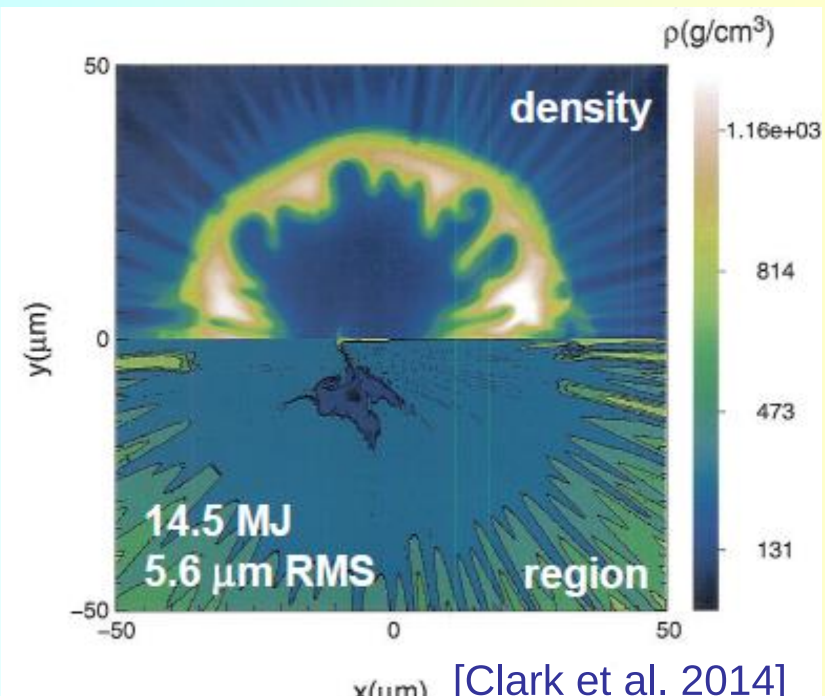
Fusion



NIF target is ~2mm

- Inertial confinement fusion
  - Indirect drive – National Ignition Facility –
    - The laser beam first irradiates a material.
    - The material then irradiates the target.
  - Direct drive – Nike, Gekko –
    - The laser beam irradiates the target.
- ICF record: 1.3 MJ energy was delivered to the NIF target toward the ignition threshold (2021).

Interfaces: Indirect drive



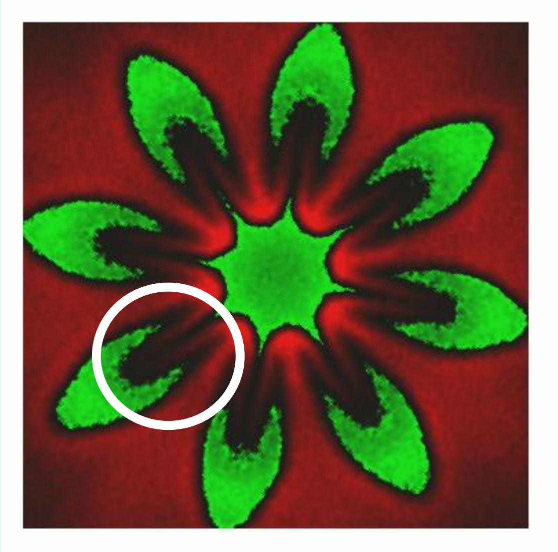
[Clark et al. 2014]

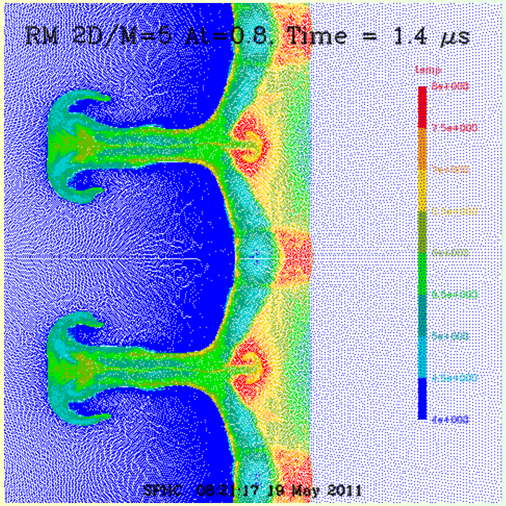
Fronts: Direct drive

[Nishihara et al. 1994]

Material melting and evaporation

MD simulations of material melting ~4 10^6 LJ atoms (bottom), ~50 nm, 0.2 um, ps



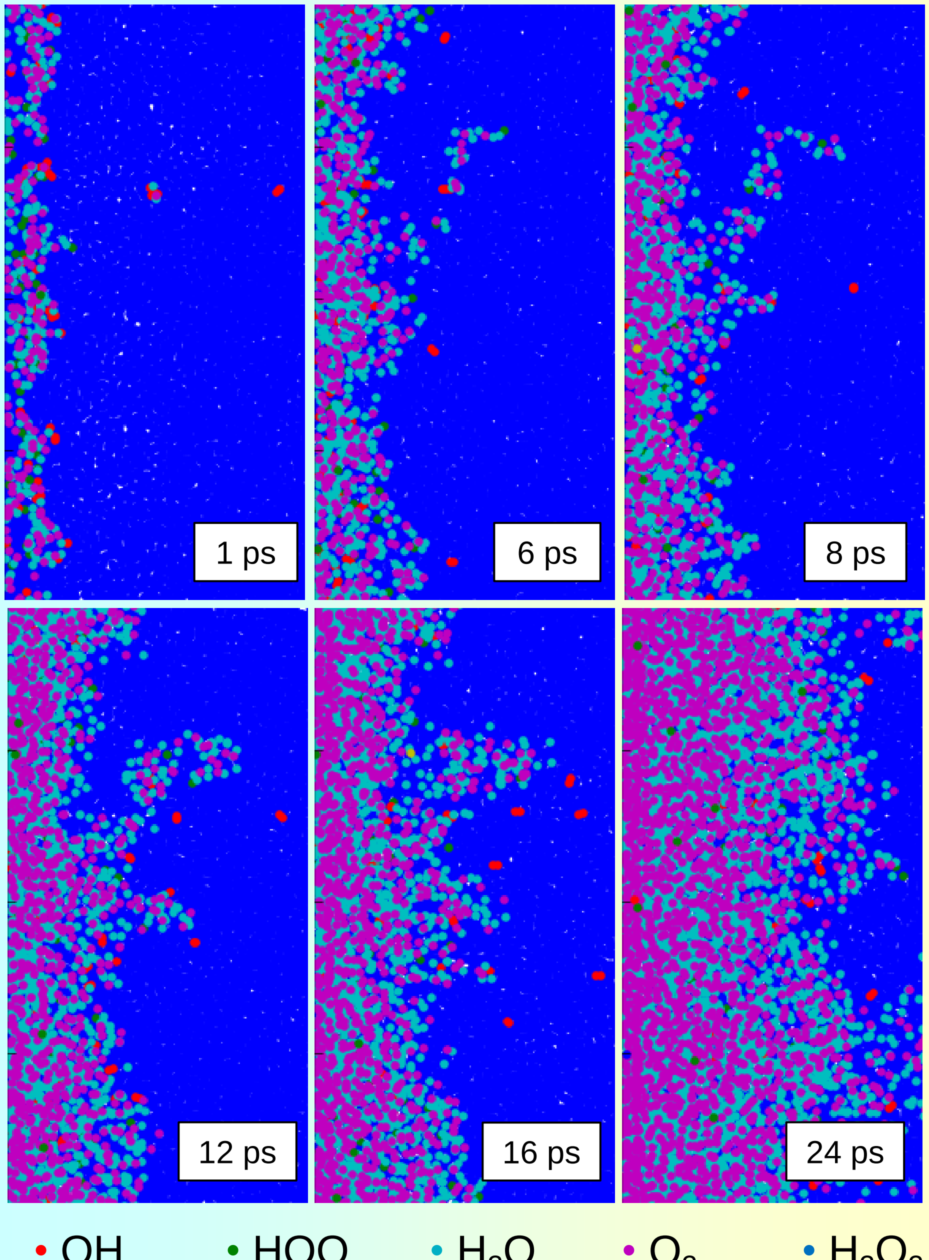


Fronts

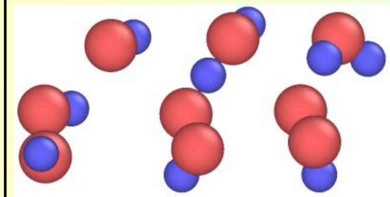
MD simulations ~ 2 x 10^8 LJ atoms (left) and SPHC simulations ~ 10^5 - 10^6 particles of the Richtmyer-Meshkov instability

Zhakhovsky et al. 2018, Dell et al. 2017

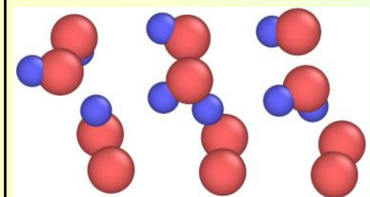
Chemically Reactive Flows



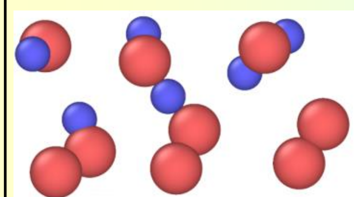
1. $OH + HOOH \Rightarrow HOO + H_2O$



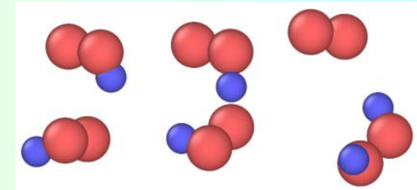
2. $HOO + HOOH \Rightarrow OH + H_2O + O_2$



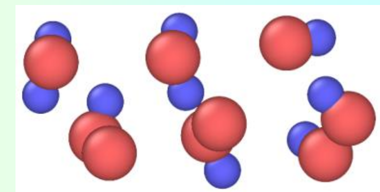
3. $HOO + OH \Rightarrow H_2O + O_2$



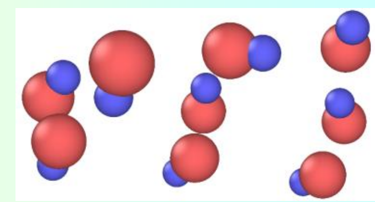
4. $HOO + HOO \Rightarrow HOOH + O_2$



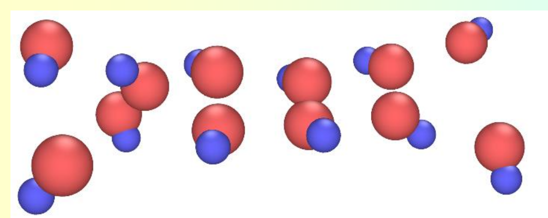
5. $HOO + H_2O \Rightarrow HOOH + OH$



6. $HOOH + M \Rightarrow OH + OH$



7. $OH + OH \Rightarrow HOOH \Rightarrow OH + OH$



Goddard et al 2019; Ilyin et al 2020, 2019

- H_2O_2 decomposition model by ReaxFF:
  - 5 species, 7 reactions
  - Energy scatters rather than diffuses.

Multiphase Geophysical Flows

Interfaces with mass and heat fluxes
tend to be stable at global scales.

Two rivers meet and do not mix.
The Green and Colorado Rivers Confluence
in Canyon Lands National Park.

Waters in the Pacific and Atlantic
oceans meet and do not mix.

Energy & Environment

In liquefied natural gas (LNG), dynamics of interfaces define:

- the gas mixture stability;
- the transportation security.

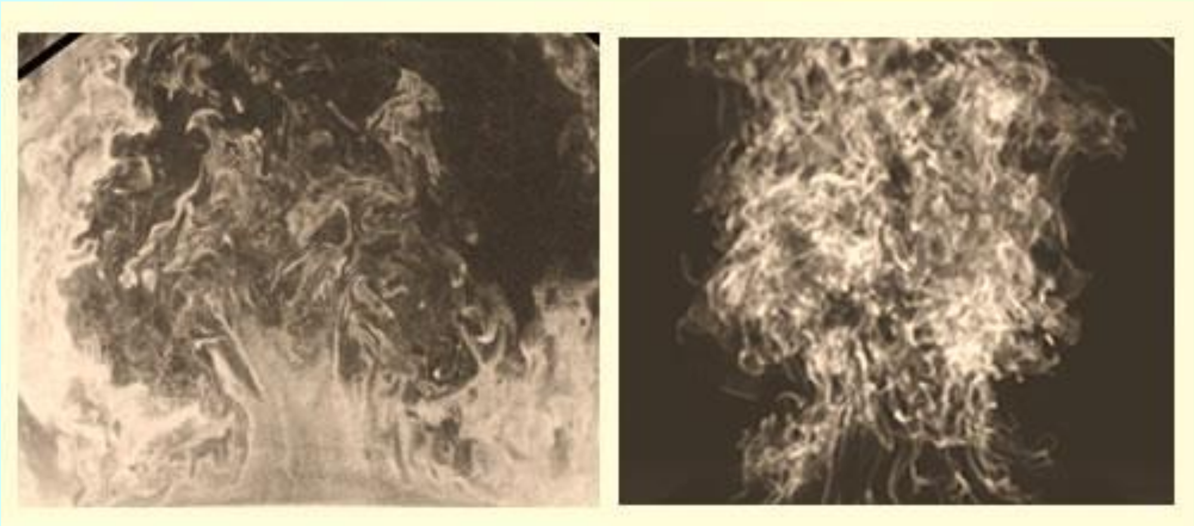
Oil Industry Insight 2016

Intense interfacial mixing is needed in industrial processes, e.g., water purification:

The acid mine drainage from the California Gulch mixes with the Arkansas River.

US Toxic Substances Hydrology Program 2018

Premixed and non-premixed combustion



hydrogen and methane

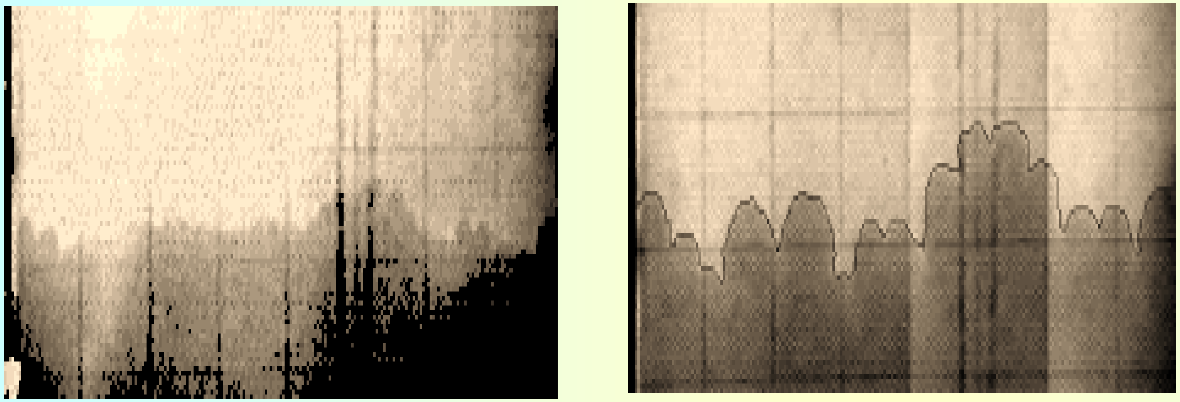
Fires

- non-premixed combustion

- Rayleigh-Taylor mixing

1 m x 1m

Tieszen et al, 2014



linear nonlinear

Flames

- premixed combustion

- Hele-Shaw (cells) mixing

1.0 mm x 200 mm

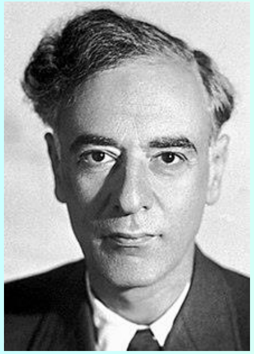
Ronney, 2016

Interface dynamics

is a source of paradigm shifts in science, mathematics, engineering

This problem is related to the 3rd prospect of Landau (1962 Nobel Laureate). Resolution of the 1st and the 2nd prospects of Landau were awarded with the Nobel prizes in 2003 (superconductors) and 1982 (phase transitions).

Magic and mystery of science and mathematics



Lev D. Landau

M. James Lighthill

Moscow 1965. USA-USSR Workshop.

Sitting: Andreev, Pines.

Standing: Khalatnikov, Rusinov, Pitaevskii, Eliashberg, Abrikosov, Martin, Kadanoff, Bardeen

Governing equations

- Conservation of mass, momentum and energy, heat flux equation and equation of state are:

inertial frame of reference $\mathbf{V}_0$

$$\partial\rho/\partial t + \partial\rho v_i/\partial x_i = 0 \qquad \partial\rho v_i/\partial t + \partial\rho v_i v_j/\partial x_j + \partial P/\partial x_i = 0$$

$$\partial E/\partial t + \partial(E+P)v_i/\partial x_i = 0 \qquad Q_i + \partial(\chi e)/\partial x_i = 0 \qquad P = s\rho e$$

$$(x_i, t) = (x, y, z, t) \qquad (\rho, \mathbf{v}, P, E, \mathbf{Q}) \qquad E = \rho\left(e + \mathbf{v}^2/2\right) \qquad W = e + P/\rho$$

- Multi-phase dynamics $\quad (\rho, \mathbf{v}, P, E, e, \mathbf{Q}, \chi, s)_{h(l)}$
- Free boundary (i.e., the interface) $\quad \theta(x_i, t) = 0 \quad \mathbf{n} = \nabla\theta/|\nabla\theta| \quad \mathbf{n}\cdot\boldsymbol{\tau} = 0$
- Boundary conditions at the free boundary $\quad \theta = -z + z^*(x, y, t)$
- Boundary conditions at the outside boundaries $\quad z \to \pm\infty$
- Boundary conditions for the thermal heat flux
- Initial conditions – initial perturbations of the interface and flow fields
- The problem is more challenging than the Millennium Navier-Stokes problem.

Boundary value problem

- There are two types of free boundaries - interfaces and fronts

$$\tilde{\mathbf{j}} = \rho\left(\mathbf{n}\,\dot{\theta}/|\nabla\theta| + \mathbf{v}\right)$$

- Conditions at the free boundary

- **Interface** has fluxes across it:

$$\left[\tilde{\mathbf{j}}\cdot\mathbf{n}\right] = 0$$

$$\left[\left(P + \left(\tilde{\mathbf{j}}\cdot\mathbf{n}\right)^2/\rho\right)\mathbf{n}\right] = 0$$

$$\left[\left(\tilde{\mathbf{j}}\cdot\mathbf{n}\right)\left(\tilde{\mathbf{j}}\cdot\boldsymbol{\tau}/\rho\right)\boldsymbol{\tau}\right] = 0$$

$$\left[\left(\tilde{\mathbf{j}}\cdot\mathbf{n}\right)\left(W + \tilde{\mathbf{j}}^2/2\rho^2\right) + \mathbf{Q}\cdot\mathbf{n}\right] = 0$$

Rankine-Hugoniot conditions

- **Front** has zero fluxes across it

$$\left[\tilde{\mathbf{j}}\cdot\mathbf{n}\right] = 0, \quad \tilde{\mathbf{j}}\cdot\mathbf{n}\Big|_{\theta=0^\pm} = 0$$

$$\left[\mathbf{v}\cdot\mathbf{n}\right] = 0$$

$$\left[P\right] = 0$$

$$\left[\mathbf{v}\cdot\boldsymbol{\tau}\right] = any$$

$$\left[W\right] = any$$

heat flux condition (the 1st time)

$$\left(\mathbf{Q}\cdot\boldsymbol{\tau}\right)\Big|_{\theta=0^+} = 0, \quad \left(\mathbf{Q}\cdot\boldsymbol{\tau}\right)\Big|_{\theta=0^-} = 0 \qquad \left(\mathbf{Q}\cdot\mathbf{n}\right)\Big|_{\theta=0^+} = 0, \quad \left(\mathbf{Q}\cdot\mathbf{n}\right)\Big|_{\theta=0^-} = 0$$

- Boundary conditions at the outside boundaries

$$\mathbf{v}\Big|_{z\to-\infty(+\infty)} = \mathbf{V}_{h(l)} = \left(0,0,V_{h(l)}\right) \qquad \mathbf{v}\Big|_{z\to-\infty(+\infty)} = 0$$

Solving interfacial boundary value problem

- Fluid phases are broadly defined.
  - These can be distinct kinds of matters.
  - These can be the same matter with distinct properties.
  - The matter can experience phase transitions and change chemical composition.
  - The matter can be out of thermodynamic equilibrium.
  - The matter can have a non-diffusive interfacial mass transport.

- Boundary value problem should be solved at nonlinear freely evolving discontinuity.

- Boundary value problems are a challenge to solve.
  - Initial value (Cauchy) problems can usually be solved.
  - Boundary value problem may / may not be solved and its solution may not be unique.

- This approach has important advantages:
  - On the side of fundamentals
    - The problem is treated rigorously.
    - Powerful theoretical methods are applied.
    - Physics of the process is explored.
  - On the side of applications
    - Reliable benchmarks are provided for diagnostics.
    - Theory is free from adjustable parameters.
    - Theoretical results have high predictive capability in a broad regime.

Historically, in science we solve boundary value problems.

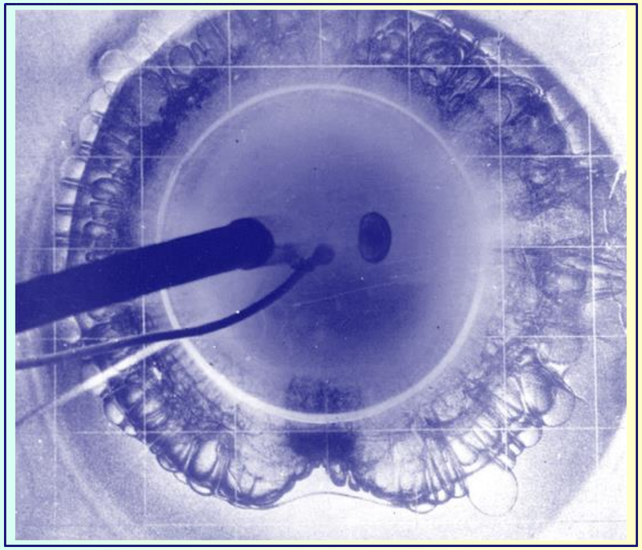
Interfacial dynamics

Theoretical approaches

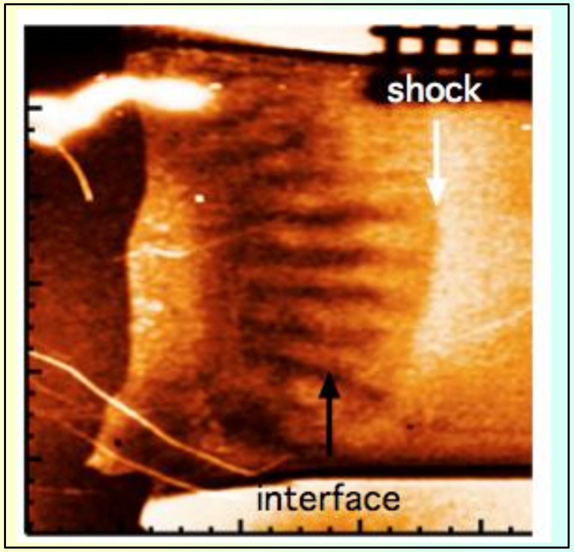
Dynamics of fronts

- Fronts
  - Separate (‘immiscible’) fluids of different densities and are neutrally stable.
  - Can be destabilized by accelerations and shocks.
  - Can lead to Rayleigh-Taylor (RT) and Richtmyer-Meshkov (RM) instabilities.
- Group theory approach captures complexity of RT/RM dynamics
  - Scale-dependent RT dynamics is multi-scale and interfacial.
  - Self-similar RT mixing with constant acceleration may keep order.
  - There is special self-similar class for RT/RM mixing with variable acceleration:
    - Self-similar dynamics can vary significantly, from super-ballistics to sub-diffusion.
    - Self-similar dynamics is sensitive to deterministic conditions for any acceleration.

Abarzhi et al. 1998, 2003, 2008, 2010, 2013, 2019, 2021, 2022



RT mixing is interfacial process
Re ~3.2 x 10^6 [Meshkov 1990, 2006,2019].



RT mixing may keep order
Re >10^5 [Robey et al. 2003].

Dynamics of interfaces

- Stability of interface separating distinct (‘miscible’) fluids is a long-standing problem.

- Classical approach was developed by Landau:
  - Studies dynamics as a phase boundary (application - flames in premixed combustion)
  - Analyzes the dynamics of an interface for ideal incompressible fluids;
  - Balances the fluxes of mass and momentum at the interface;
  - Implements a special condition for the perturbed mass flux at the interface;
  - Postulates constancy of the interface velocity.

- Landau 1944 found:
  - The interface is unconditionally unstable.
  - The instability is the Landau-Darrieus (LD) instability (a.k.a. Darrieus-Landau instability).

- There are important challenges.
  - This result contradicts the experiments.
  - Stable flames exist and are observed in the laboratory.
  - The achieved consensus of extensive studies is:
    - Interface is stable at small scales, due to microscopic effects (e.g., dissipation).
    - Interface is unstable at global scales (which are challenging to set in laboratory).
    - Interface is globally unstable even for inertial dynamics with zero acceleration

- Hence it appears the 3rd prospect of Landau (1962 Nobel prize).
  - 2nd is theory of superconductors (resolution is awarded with 2003 Nobel prize).
  - 1st is theory of phase transitions (resolution is awarded with 1982 Nobel prize).

Some of seminal studies of the interface stability

since 1944…

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Foundations of nonlinear physics and applied mathematics

$$(\partial A/\partial T) = \Delta_0 A + \beta(\partial^2 A/\partial Y^2) - D|A|^2 A$$

- Complex systems: generic mathematical models $\beta = \beta_r + i\beta_i \quad D = D_r + iD_i$
  - Ginzburg-Landau equation (real and complex);
  - Nonlinear Schrödinger equation (exactly integrable systems);
  - Kuramoto-Sivashinsky equation, amplitude equation, reaction-diffusion equation.

$$F = aM^2 + bM^4 + c(\nabla M)^2$$

- Statistical physics in classical and quantum systems
  - Landau theory for phase transitions in solids, liquids, gases, glasses;
  - Condensed matter physics - super-conductivity, super-fluidity, Bose-Einstein condensation:
    - Bardeen-Cooper-Schrieffer (BCS) & Abrikosov-Ginzburg-Gorkov (AGG) models
    - Gross-Pitaevskii equation.

- Stochasticity and chaos as scale-dependent and scale-invariant complexity
  - Turbulence with self-similar dynamics and with fluctuations' spectra $\sim k^{\alpha}$
  - Chaos with scale-dependent dynamics and with fluctuations' spectra $\sim e^{\gamma k}$
  - Anomalous scaling in realistic turbulent processes with fluctuations spectra $\sim k^{\alpha} e^{\gamma k}$

Our theory of interface dynamics

- Develop general framework for a broad range of conditions;
- Find the mechanisms of interface stabilization and destabilization;
- Discover the novel class of fluid instabilities in ideal and realistic fluids.

Principles, key ingredients, main yields

- Theory has to be physically meaningful and mathematically rigorous to be able to
  - Analyze the interface dynamics in a broad range of conditions for ideal and realistic fluids.
  - Relate to Landau's framework and to other approaches.
  - Link with group theory approach for nonlinear and self-similar dynamics of the interface.

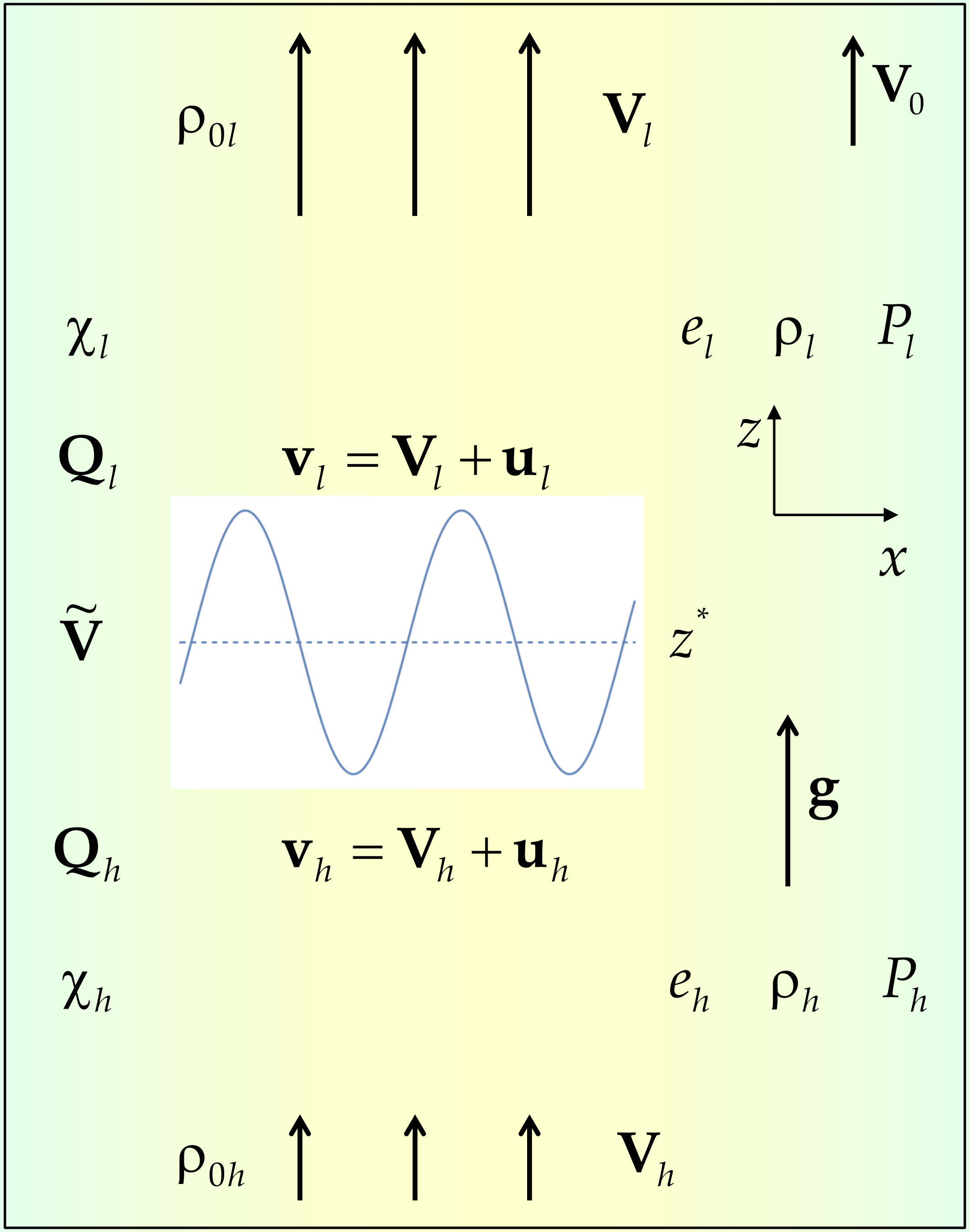
- We solve the boundary value problem at the (unstable and nonlinear) interface:
  - Flow quantities often experience sharp changes.
  - The dynamics is usually observed from a far field.
  - Small-scale interfacial processes are a challenge to accurately diagnose.

- Key ingredients are:
  - The interface velocity is free from the postulate of the constancy.
  - Flow fields are formally represented.
  - Thermal effects are accurately evaluated.
  - Structure of flow fields is rigorously defined.
  - Fundamental solutions are fully identified (eigenvalues & eigenvectors, degeneracy).

- Main yields are:
  - Interface is stabilized by macroscopic inertial mechanism balancing destabilizing acceleration.
  - Microscopic thermodynamics and thermal heat flux produce vortical structures in the bulk.
  - Macroscopic fields structure in the bulk is linked to microscopic transport at the interface.

  - The fluid instabilities are found that were never discussed before.
  - Landau 1944 solution is identified as a perfect mathematical match.

Continuous media from a far field



Methodology

- Generalized distribution functions are applied $\delta(\theta), H(\theta)$ in inertial frame $\mathbf{V}_0$.
- Interface velocity is: $\tilde{\mathbf{V}}$

  - steady planar interface

  $$\tilde{\mathbf{V}} = \tilde{\mathbf{V}}_0$$

  - in general case

  $$\tilde{\mathbf{V}} \neq \tilde{\mathbf{V}}_0 \qquad \tilde{\mathbf{V}}\mathbf{n} = -\mathbf{v}\mathbf{n}\big|_{\theta=0^{\pm}} = -\left(\tilde{\mathbf{j}}/\rho\right)\mathbf{n}\big|_{\theta=0^{\pm}}$$

- Flow fields are: $\mathbf{v} = \mathbf{V} + \mathbf{u} \qquad \mathbf{u} = \nabla\Phi + \nabla\times\mathbf{\Psi}$

$$\rho = \rho_0 + \bar{\rho},\ P = P_0 + p,\ e = e_0 + \bar{e},\ \mathbf{Q} = \mathbf{Q_0} + \mathbf{q},\ \tilde{\mathbf{j}} = \mathbf{J} + \hat{\mathbf{j}}; \quad (\mathbf{n},\boldsymbol{\tau}) = (\mathbf{n},\boldsymbol{\tau})_\mathbf{0} + (\mathbf{n},\boldsymbol{\tau})_\mathbf{1}$$

- Thermal heat flux is: $\left[\left(\mathbf{Q}_0\cdot\mathbf{n}_0\right)\right] = -\left[\left(\mathbf{J}\cdot\mathbf{n}_0\right)\left(W_0 + \left(\mathbf{J}^2/2\rho_0^2\right)\right)\right]$
- Initial conditions set 2D dynamics as: $(x,z): x \to x+\lambda, \quad k = 2\pi/\lambda, \quad 1/k, 1/kV_h$
- Fields in the bulk are: $\left(\Phi, p, \Psi, \bar{\rho}, \bar{e}\right) = \left(\hat{\Phi}, \hat{p}, \hat{\Psi}, \hat{\rho}, \hat{e}\right) exp\left(ikx - Kz + \Omega t\right)$

$$M_Z : 5\times5, 3\times3 \qquad M_Z\mathbf{Z} = 0 \qquad \mathbf{Z} = \left(\Phi, p, \Psi, \bar{\rho}, \bar{e}\right)^{\mathrm{T}} \qquad \mathbf{\Psi} = (0, \Psi, 0)$$

- Boundary value problem solutions are: $\mathbf{r} = \left(\Phi_h, \Phi_l, V_h z^*, \Psi_l, \bar{e}_h/kV_h, \bar{e}_l/kV_h\right)^{\mathrm{T}}$

$$\mathrm{M} : 6\times6, 4\times4 \qquad \mathrm{M}\mathbf{r} = 0 \qquad \mathbf{r} = C_i\mathbf{r}_i \qquad \mathbf{r}_i\left(\Omega_i, \mathbf{e}_i\right)$$

Inertial and accelerated dynamics in ideal fluids

- Discover characteristics of conservative dynamics.
- Identify mechanisms of stabilization and destabilization.
- Resolve Landau 1944 paradox.

Conservative dynamics: ideal incompressible fluids

$$\rho_0 V^2 / P_0 \to \infty$$

- Leading order: $\left(\rho, \mathbf{v}, P, W, \tilde{\mathbf{j}}\right)_{h(l)} = \left(\rho_0, \mathbf{V}, P_0, W_0, \mathbf{J}\right)_{h(l)}$

$$[J_n] = 0, \quad \left[\left(P_0 + \frac{J_n^2}{\rho_0}\right)\mathbf{n}_0\right] = 0, \quad [J_n W_0] = 0$$

$$\mathbf{J} = \rho_0 \mathbf{V}$$

$$J_n = \mathbf{J} \cdot \mathbf{n}_0$$

- First order: $\nabla \cdot \mathbf{u} = 0, \; \left(\partial/\partial t + \mathbf{V} \cdot \nabla\right)\mathbf{u} + \nabla p/\rho_0 = 0$

$$[j_n] = 0, \; \left[\left(p + \frac{2 J_n j_n}{\rho_0}\right)\mathbf{n}_0\right] = 0, \; \left[J_n\left(\mathbf{J} \cdot \boldsymbol{\tau}_1 + \mathbf{j} \cdot \boldsymbol{\tau}_0\right)\frac{\boldsymbol{\tau}_0}{\rho_0}\right] = 0, \; \left[J_n\left(w + \frac{\mathbf{J} \cdot \mathbf{j}}{\rho_0^2}\right)\right] = 0$$

$$\left(\mathbf{u}, p, \mathbf{j}, w\right)\Big|_{z \to \pm\infty} = 0$$

$$\mathbf{j} = \rho\left(\mathbf{u} + \mathbf{n}_0 \dot{\theta}\right), \; j_n = \mathbf{j} \cdot \mathbf{n}_0$$

- Interface velocity is: $\tilde{\mathbf{V}} = \tilde{\mathbf{V}}_0 + \tilde{\mathbf{v}}$ $\qquad \tilde{\mathbf{v}} \cdot \mathbf{n}_0 = -\left(\mathbf{u} \cdot \mathbf{n}_0 + \dot{\theta}\right)\Big|_{\theta=0}$

- Fields in the bulk are:

$$\left(\Phi, p\right)_h = \hat{\Phi}_h \left(1, -k\rho_0 V\left(1 - \Omega/kV\right)\right)_h e^{ikx + kz + \Omega t}$$

$$\left(\Phi, p\right)_l = \hat{\Phi}_l \left(1, k\rho_0 V\left(1 + \Omega/kV\right)\right)_l e^{ikx - kz + \Omega t}$$

$$\mathbf{r} = \left(\Phi_h, \Phi_l, V_h z^*, \Psi_l\right)^{\mathrm{T}}$$

$$z^* = Z^* e^{ikx + \Omega t}, \quad \Psi_l = \hat{\Psi} e^{ikx - (\Omega/V_l)z + \Omega t}$$

Inertial dynamics

Conservative inertial dynamics

- Boundary conditions

$$[j_n]=0, [(p+2J_n j_n/\rho)\mathbf{n}_0]=0, [J_n(\mathbf{J}\cdot\boldsymbol{\tau}_1+\mathbf{j}\cdot\boldsymbol{\tau}_0)/\rho]=0, [J_n(w+(\mathbf{J}\cdot\mathbf{j})^2/\rho^2)]=0$$

$$\mathbf{r}=(\Phi_h,\Phi_l,V_h z^*,\Psi_l)^{\mathrm{T}} \qquad \mathbf{r}=C_i\mathbf{r}_i \qquad \mathbf{r}_i(\Omega_i,\mathbf{e}_i) \qquad \hat{\mathbf{e}}=(\hat{\Phi}_h,\hat{\Phi}_l,Z^*,\hat{\Psi})^{\mathrm{T}}$$

- Solutions

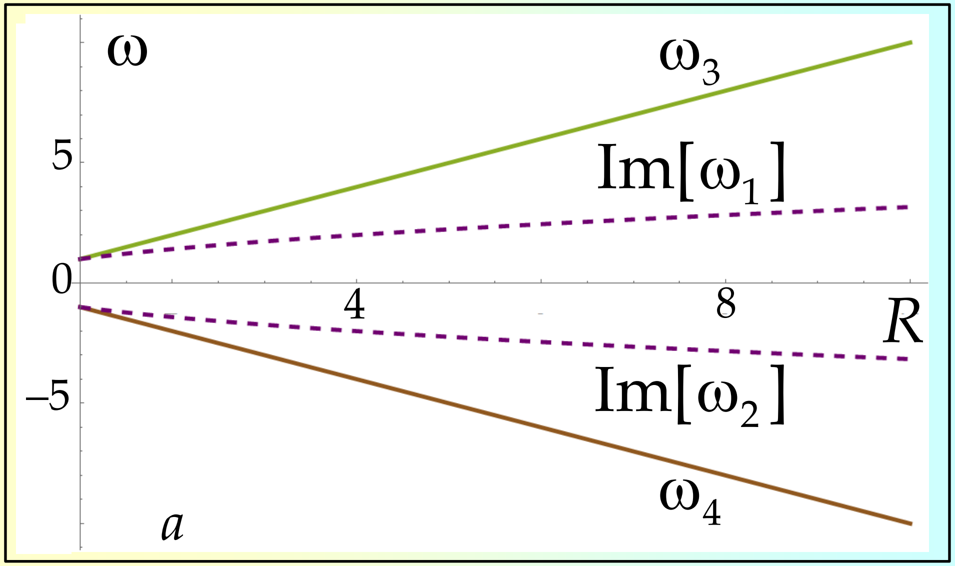
$$\omega_1=i\sqrt{R}, \hat{\mathbf{e}}_1=(*,*,1,0)^{\mathrm{T}}$$

$$\omega_2=-i\sqrt{R}, \hat{\mathbf{e}}_2=(*,*,1,0)^{\mathrm{T}}$$

$$\omega_3=R, \hat{\mathbf{e}}_3=(0,*,0,1)^{\mathrm{T}}$$

$$\omega_4=-R, \hat{\mathbf{e}}_4=(*,*,0,1)^{\mathrm{T}}$$

$$\omega=\Omega/kV_h \qquad R=\rho_{0h}/\rho_{0l} \qquad V/k$$



- Integration constants

$$C_1=C_2^*=C, C_3=any, C_4=0$$

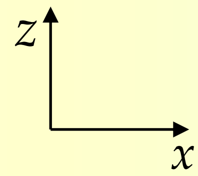
$$\mathbf{r}_{CD}=(\mathbf{r}_1+\mathbf{r}_2)/2$$

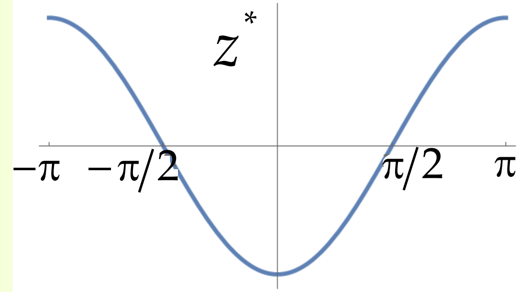
$$M=\begin{pmatrix} -R & -1 & -\omega+R\omega & i \\ 1 & -1 & 1-R & i\omega/R \\ R-R\omega & R+\omega & 0 & -2iR \\ \omega & -\omega & \omega-R\omega & iR \end{pmatrix} \qquad M\mathbf{r}=0 \qquad P_M\dot{\mathbf{r}}=S_M\mathbf{r}$$

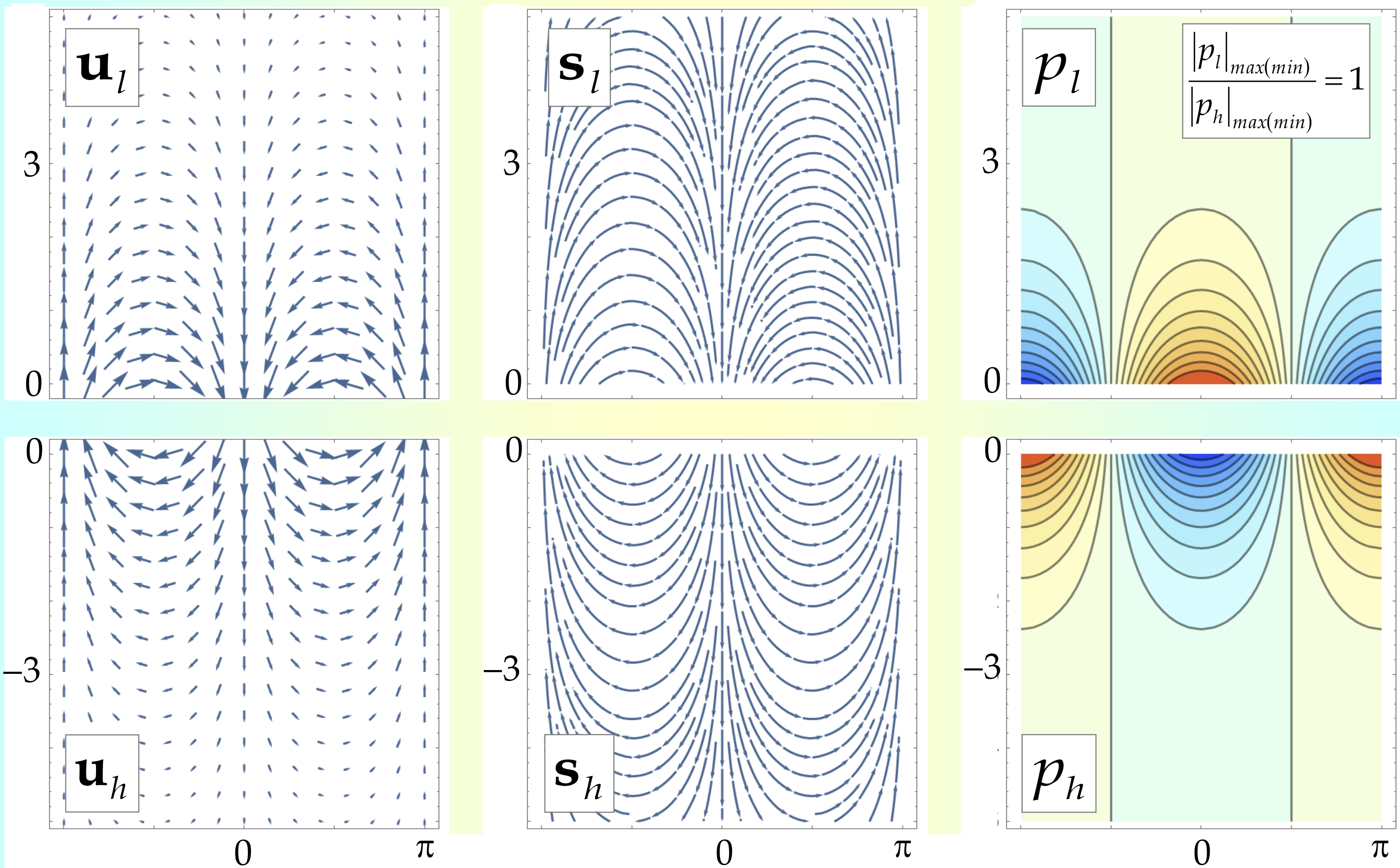
- Non-degenerate dynamics

$$\mathbf{r}_{CD}(\omega_{CD}, \mathbf{e}_{CD}) \qquad R = 5$$

$$\mathbf{r}_{CD} = (\mathbf{r}_1 + \mathbf{r}_2)/2 \qquad t = \pi/2$$







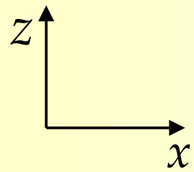
CD

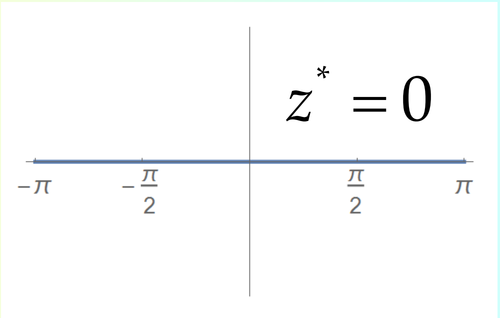
$\mathbf{r}_3(\omega_3, \mathbf{e}_3)$

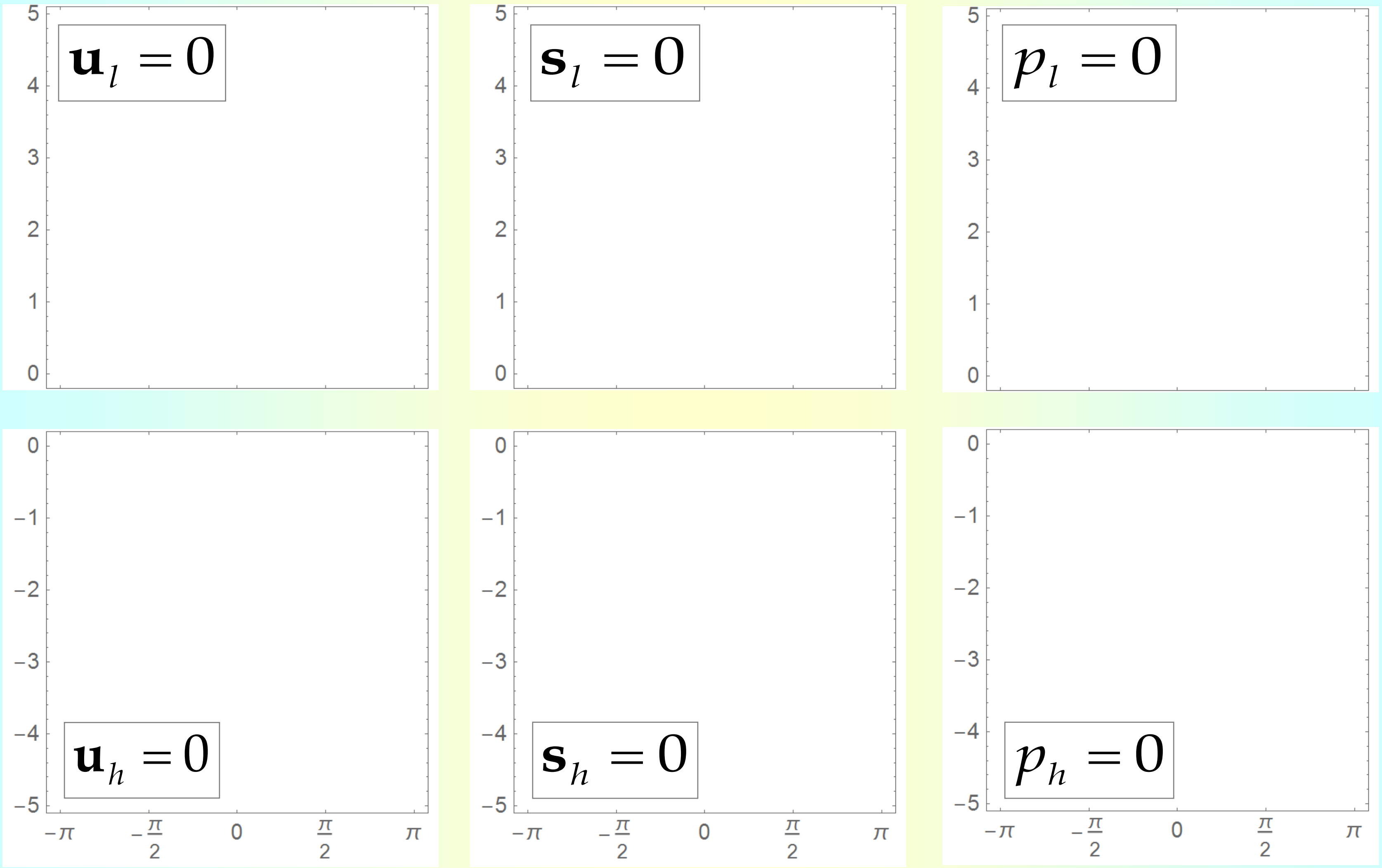
$R = 5$

$G = 0$

$t = \pi/2$







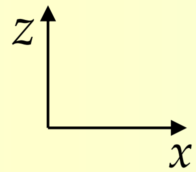
CD

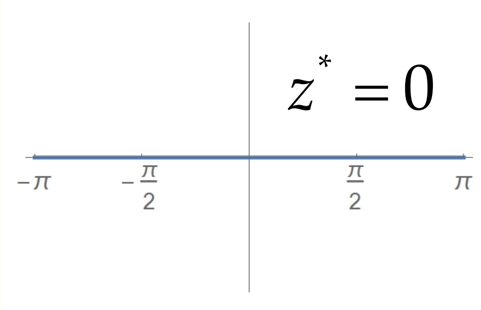
$\mathbf{r}_4(\omega_4, \mathbf{e}_4)$

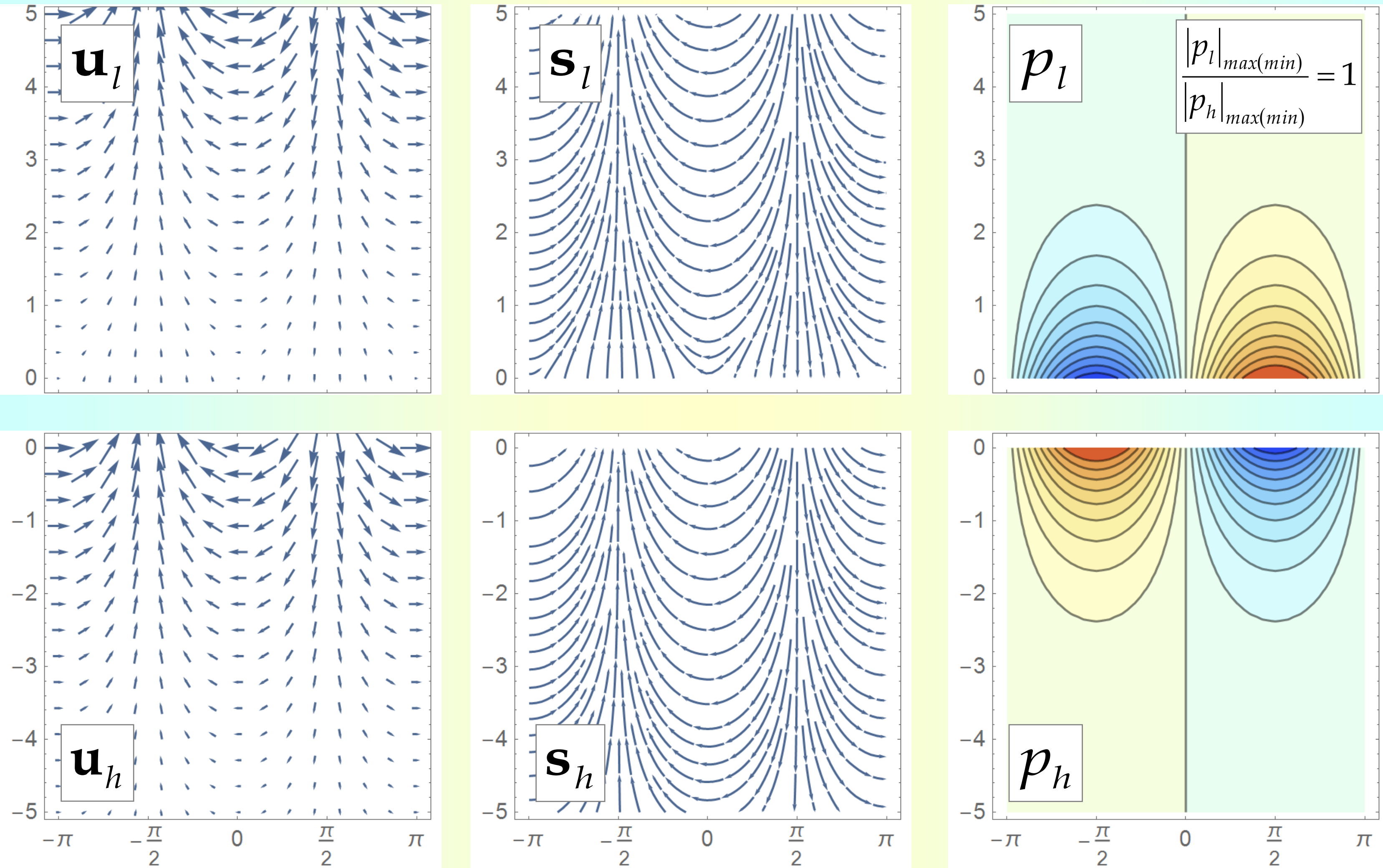
$R = 5$

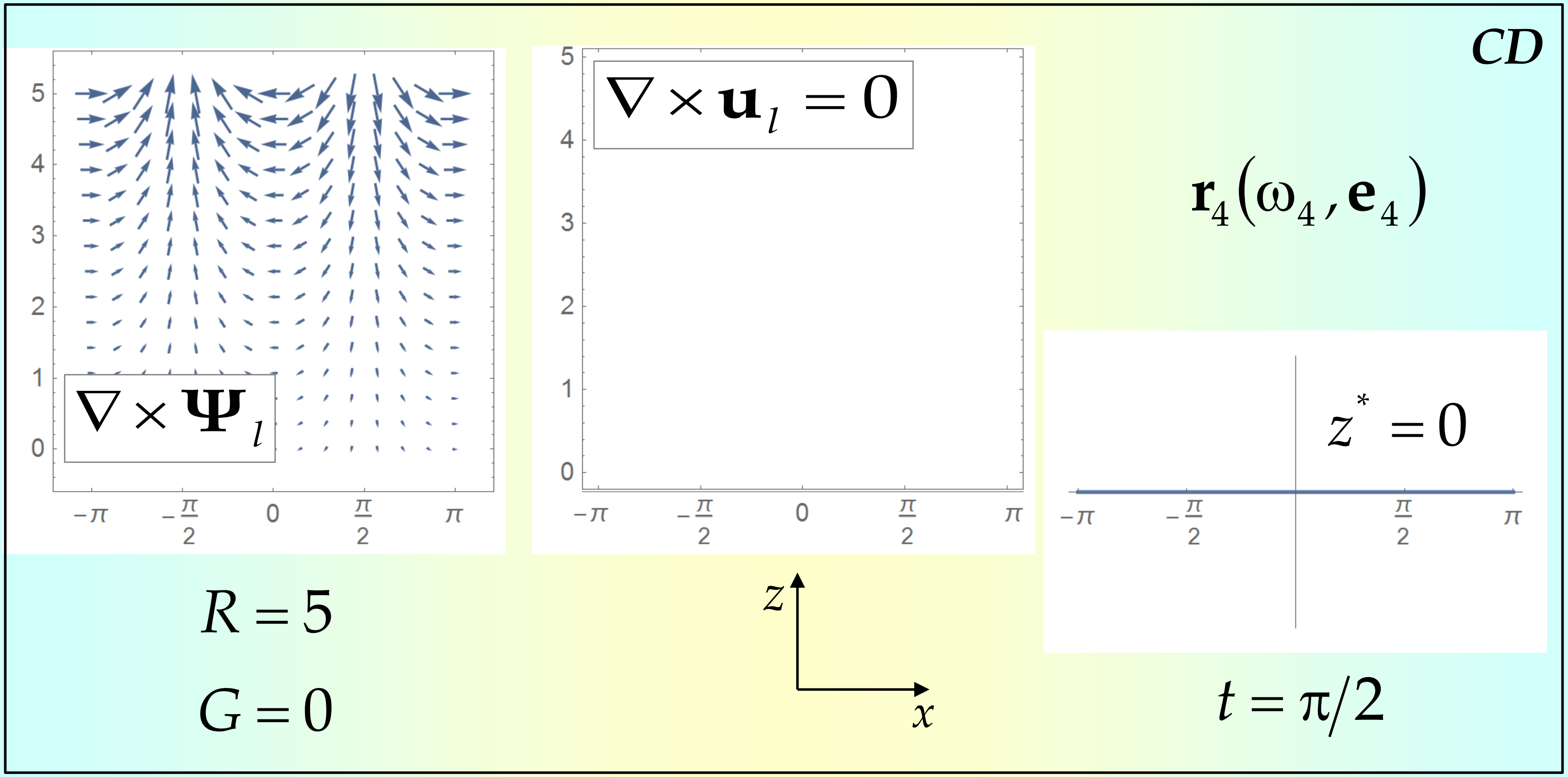
$G = 0$

$t = \pi/2$








CD
$\nabla \times \mathbf{\Psi}_l$
$\nabla \times \mathbf{u}_l = 0$
$\mathbf{r}_4(\omega_4, \mathbf{e}_4)$
$z^* = 0$
$R = 5$
$G = 0$
$t = \pi/2$
z
x

Classical Landau's dynamics

- Boundary conditions

$$[j_n]=0,\ [(p+2J_n j_n/\rho)\mathbf{n}_0]=0,\ [J_n(\mathbf{J}\cdot\boldsymbol{\tau}_1+\mathbf{j}\cdot\boldsymbol{\tau}_0)/\rho]=0,\ [\mathbf{u}\cdot\mathbf{n}_0]=0$$

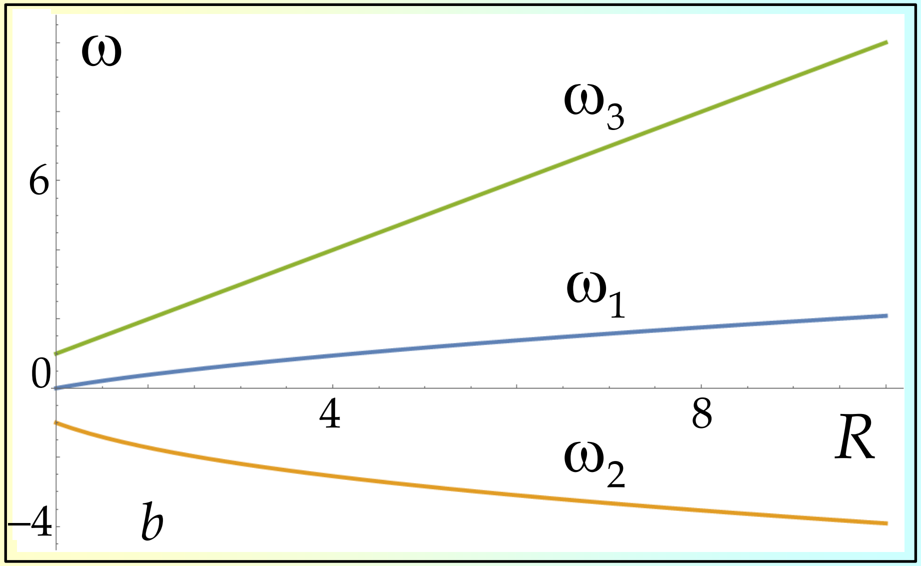
$$\mathbf{r}=(\Phi_h,\Phi_l,V_h z^*,\Psi_l)^{\mathrm{T}} \qquad \mathbf{r}=C_i\mathbf{r}_i \qquad \mathbf{r}_i(\Omega_i,\mathbf{e}_i) \qquad \hat{\mathbf{e}}=(\hat{\Phi}_h,\hat{\Phi}_l,Z^*,\hat{\Psi})^{\mathrm{T}}$$

- Solutions

$$\omega_{1(2)}=\left(-R\pm\sqrt{(R^3+R^2-R)}\right)/(1+R),$$

$$\hat{\mathbf{e}}_{1(2)}=(*,*,*,1)^{\mathrm{T}};$$

$$\omega_3=R,\ \hat{\mathbf{e}}_3=(0,*,0,1)^{\mathrm{T}}$$



- Integration constants

$$C_1=i.c.,C_2=0,C_3=any$$

$$\mathbf{r}_{LD}=\mathbf{r}_1$$

- Degenerate dynamics: needs energy excess at the interface

$$L=\begin{pmatrix}-R & -1 & -\omega+R\omega & i\\ 1 & -1 & 1-R & i\omega/R\\ R-R\omega & R+\omega & 0 & -2iR\\ -1 & -1 & 0 & i\end{pmatrix} \qquad S_L=\begin{pmatrix}-R & -1 & 0 & i\\ 1 & -1 & 1-R & 0\\ R & R & 0 & -2iR\\ -1 & -1 & 0 & i\end{pmatrix} \qquad P_L=\begin{pmatrix}0 & 0 & 1-R & 0\\ 0 & 0 & 0 & -i/R\\ R & -1 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$$L\mathbf{r}=0$$

$$P_L\dot{\mathbf{r}}=S_L\mathbf{r}$$

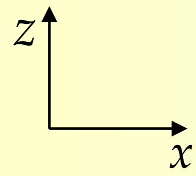
$$det\,P_L=0$$

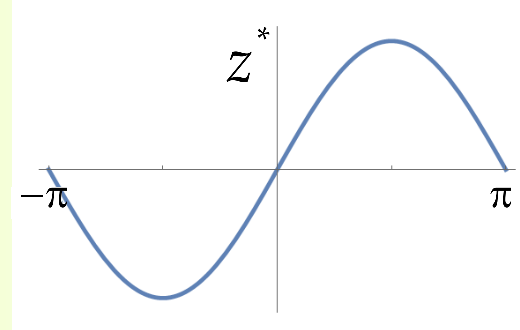
$\mathbf{r}_{LD}(\omega_{LD}, \mathbf{e}_{LD})$

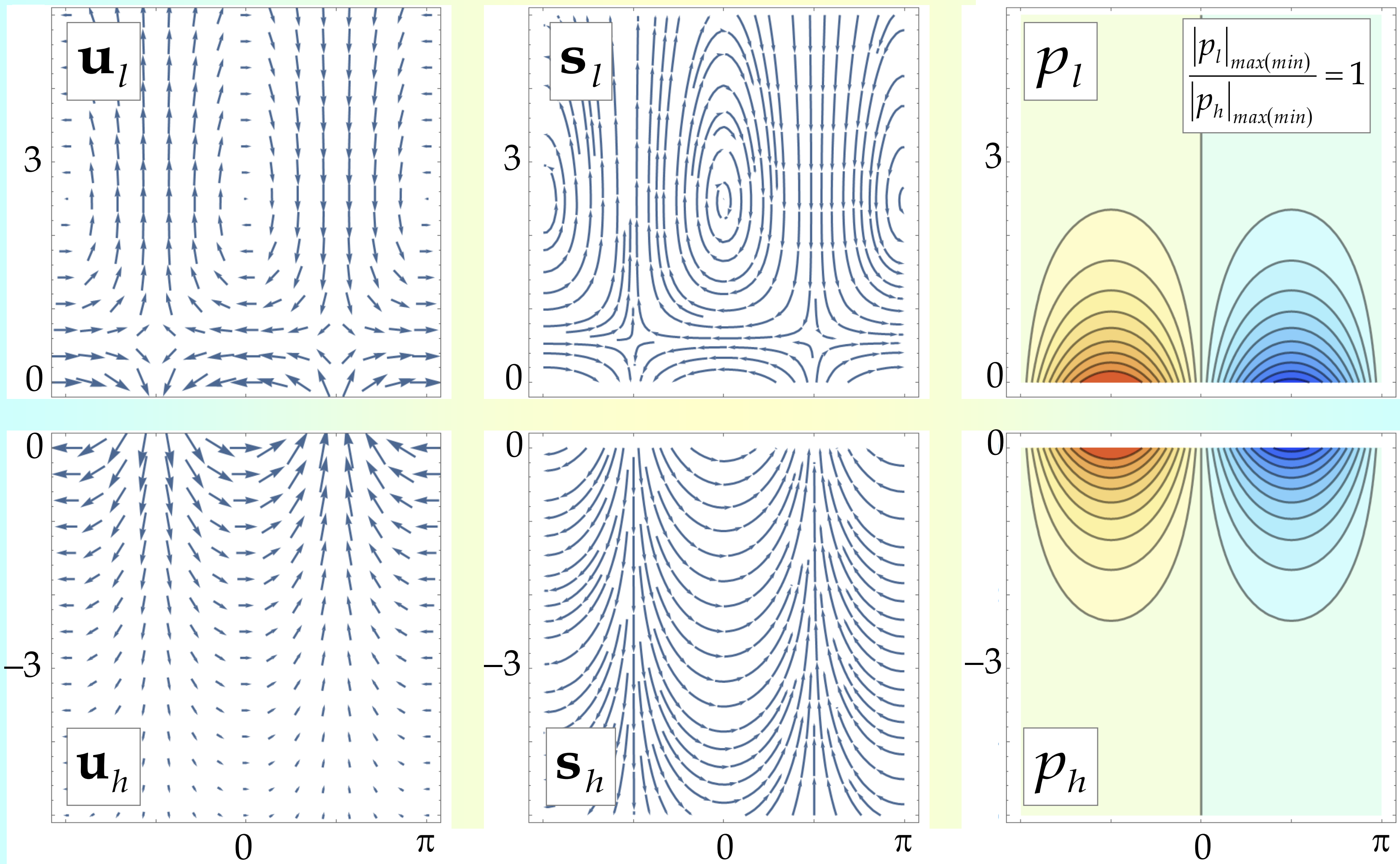
$\mathbf{r}_{LD} = \mathbf{r}_1$

$R = 5$

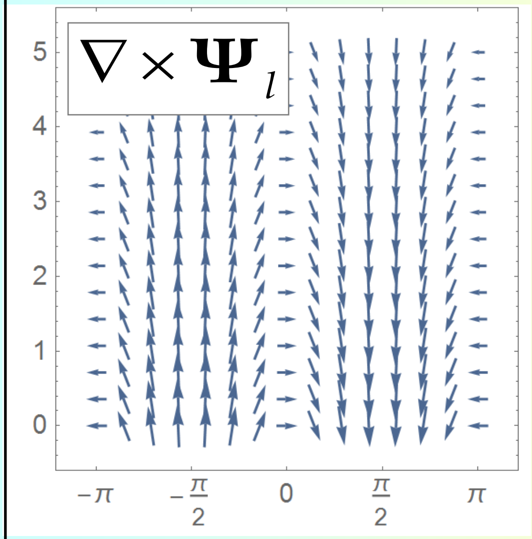
$t = \pi/2$

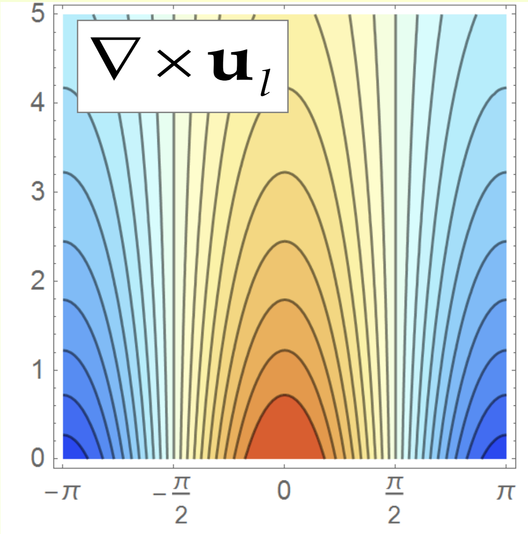




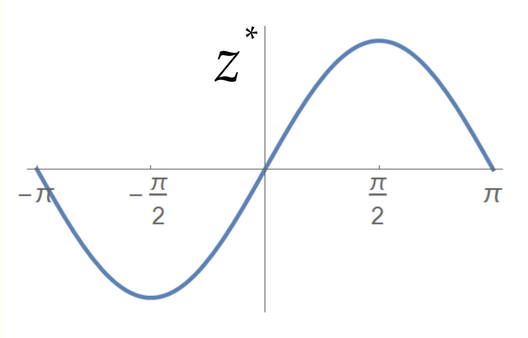


LD



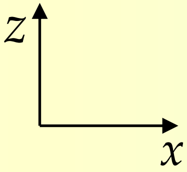


$$\mathbf{r}_{LD}\left(\omega_{LD}, \mathbf{e}_{LD}\right)$$



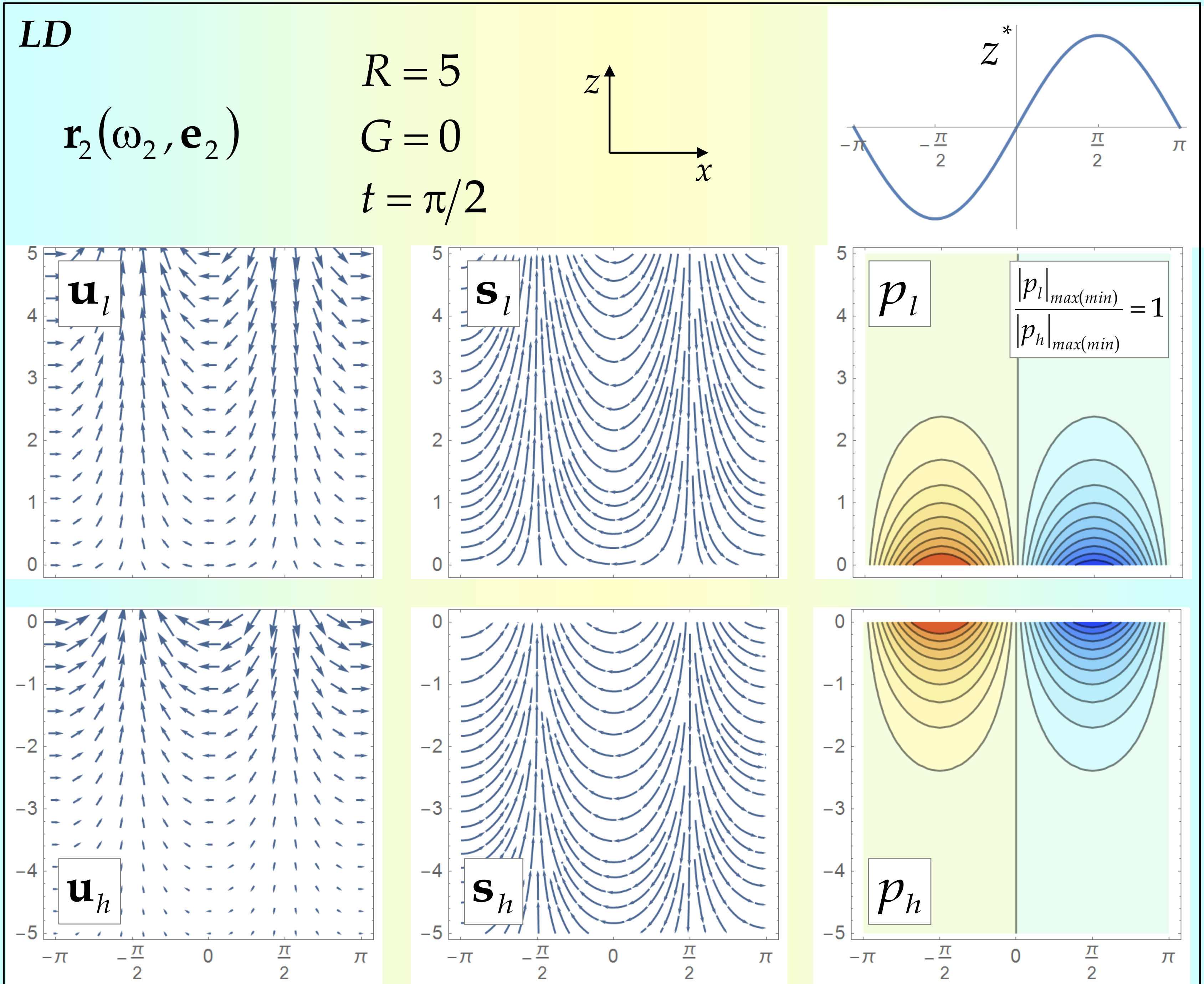
$$R = 5$$

$$t = \pi/2$$



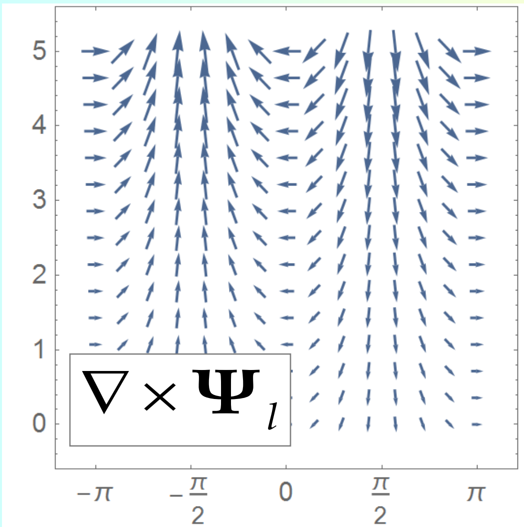
$$\tilde{k}/k = \left(-R + \sqrt{-R + R^2 + R^3}\right)/R(1+R)$$

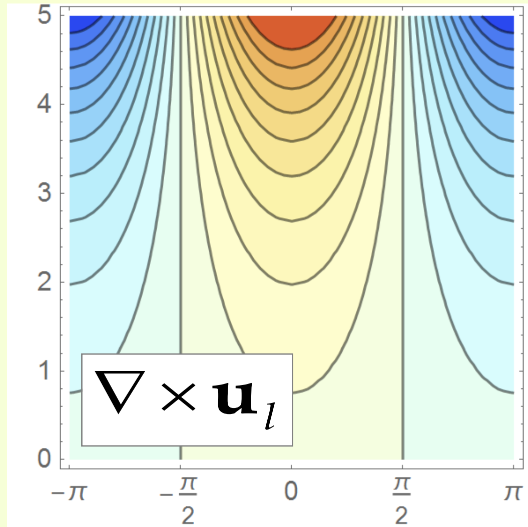
$$R_m = 2 + \sqrt{5} \approx 4.24$$



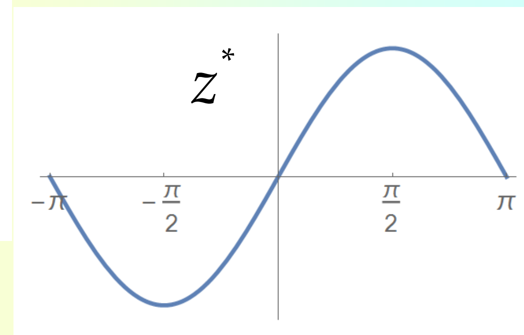
LD
$\mathbf{r}_2(\omega_2, \mathbf{e}_2)$
$R = 5$
$G = 0$
$t = \pi/2$
z
x
z^*
$\mathbf{u}_l$
$\mathbf{s}_l$
p_l
$\frac{|p_l|_{max(min)}}{|p_h|_{max(min)}} = 1$
$\mathbf{u}_h$
$\mathbf{s}_h$
p_h

LD



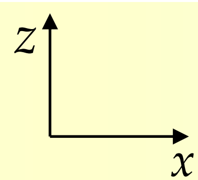


$\mathbf{r}_2(\omega_2, \mathbf{e}_2)$



$R = 5$

$G = 0$



$t = \pi/2$

Stabilization of inertial dynamics

- In the laboratory frame of reference the interface velocity is

$$\tilde{\mathbf{V}} = \tilde{\mathbf{V}}_0 + \tilde{\mathbf{v}} \qquad \tilde{\mathbf{v}} \cdot \mathbf{n}_0 = -\left(\mathbf{u} \cdot \mathbf{n}_0 + \dot{\theta}\right)\Big|_{\theta=0^+}$$

- For the conservative dynamics

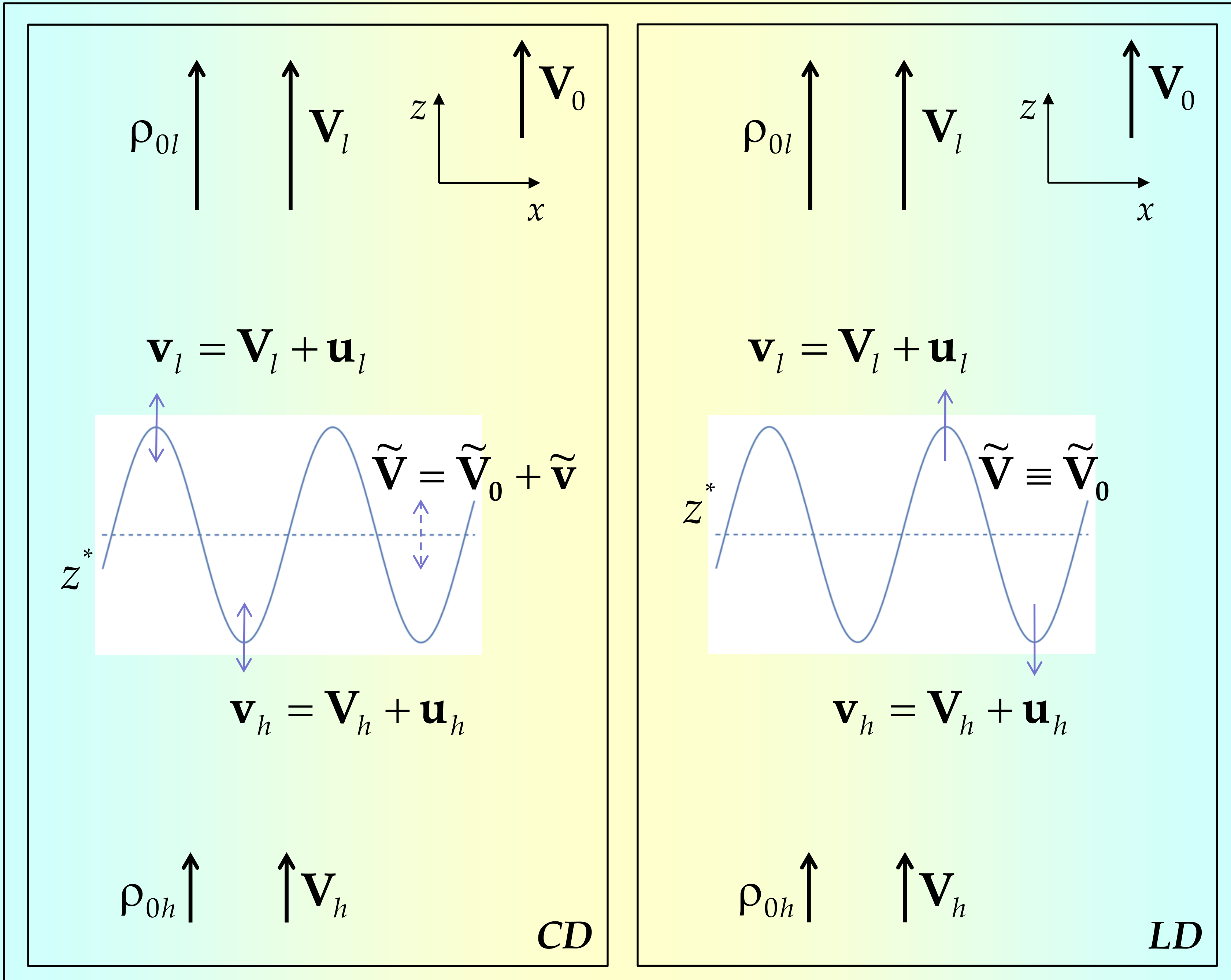
$$\mathbf{r}_{CD} \qquad \mathbf{u} \cdot \mathbf{n}_0\big|_{\theta=0} \sim e^{\pm i\sqrt{R}t},\ \dot{\theta}\big|_{\theta=0} \sim e^{\pm i\sqrt{R}t} \qquad \left(\tilde{\mathbf{V}} - \tilde{\mathbf{V}}_0\right)\cdot \mathbf{n}_0 \sim e^{\pm i\sqrt{R}\,t}$$

- Inertial stabilization mechanism of the conservative dynamics
    - The interface is slightly perturbed.
    - The parcels of the heavy and the light fluids follow the interface perturbation.
    - This causes the change of momentum of the fluid system.
    - The dynamics is inertial.
    - To conserve the momentum, the interface as a whole slightly changes its velocity.
    - The reactive force occurs and stabilizes the dynamics.

- For the classic Landau's dynamics $\left[\mathbf{u} \cdot \mathbf{n}_0\right] = 0 \qquad \mathbf{u} \cdot \mathbf{n}_0, \dot{\theta}\big|_{\theta=0} \sim e^{\omega_{LD}\,t}$

$$\mathbf{r}_{LD} \qquad \left(\mathbf{u} \cdot \mathbf{n}_0 + \dot{\theta}\right)\Big|_{\theta=0} = 0,\ \tilde{\mathbf{v}} = 0 \qquad \tilde{\mathbf{V}} \equiv \tilde{\mathbf{V}}_0$$

- The classic Landau's dynamics is the perfect match.
- Inertial stabilization mechanism is absent due to constancy of the interface velocity.

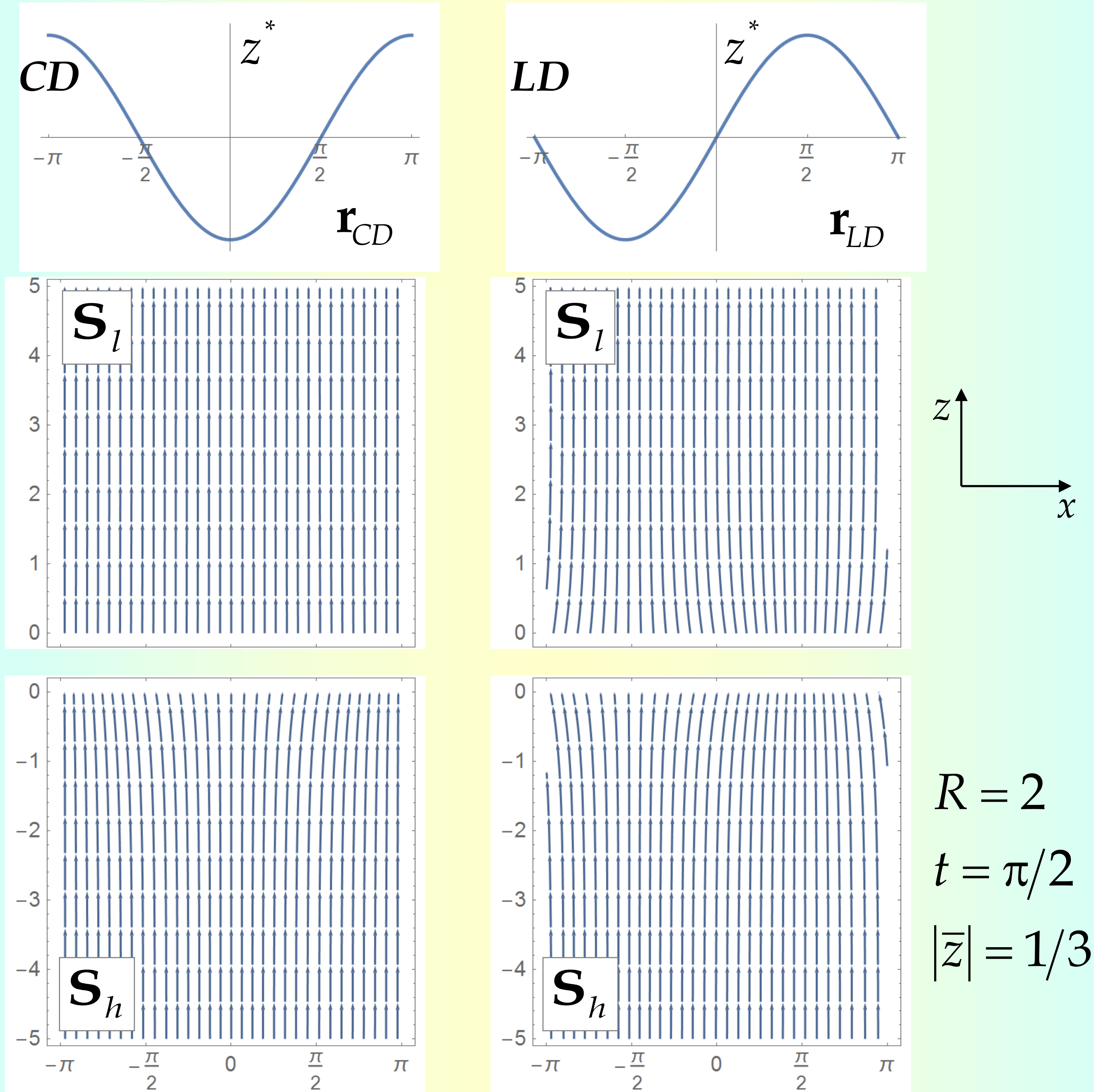


ρ_{0l}
$\mathbf{V}_l$
$\mathbf{V}_0$
z
x
$\mathbf{v}_l = \mathbf{V}_l + \mathbf{u}_l$
$\tilde{\mathbf{V}} = \tilde{\mathbf{V}}_0 + \tilde{\mathbf{v}}$
z^*
$\mathbf{v}_h = \mathbf{V}_h + \mathbf{u}_h$
ρ_{0h}
$\mathbf{V}_h$
CD
ρ_{0l}
$\mathbf{V}_l$
$\mathbf{V}_0$
z
x
$\mathbf{v}_l = \mathbf{V}_l + \mathbf{u}_l$
$\tilde{\mathbf{V}} \equiv \tilde{\mathbf{V}}_0$
z^*
$\mathbf{v}_h = \mathbf{V}_h + \mathbf{u}_h$
ρ_{0h}
$\mathbf{V}_h$
LD

Properties of inertial dynamics in ideal fluids

	CD	LD
Conservation properties	Conserves mass, momentum and energy at the interface	Conserves mass and momentum, has zero perturbed mass flux at the interface
Interface velocity	Slight stable oscillations near constant value	Constant value (postulate)
Flow field	Potential velocity fields	Vortical field is present in the light fluid
Inerfacial shear	Shear-free	Shear-free
Formal properties	Non-degenerate; 4 fundamental solutions and 4 degrees of freedom	Degenerate; 3 fundamental solutions and 4 degrees of freedom
Stability	Stable; stabilized by inertial effect	Unstable

What is usually observed?



Accelerated dynamics

Conservative accelerated dynamics

- Boundary conditions

$$[j_n]=0,\ [(p+2J_n j_n/\rho)\mathbf{n}_0]=0,\ [J_n(\mathbf{J}\cdot\boldsymbol{\tau}_1+\mathbf{j}\cdot\boldsymbol{\tau}_0)/\rho]=0,\ [J_n(w+(\mathbf{J}\cdot\mathbf{j})^2/\rho^2)]=0$$

- Solutions

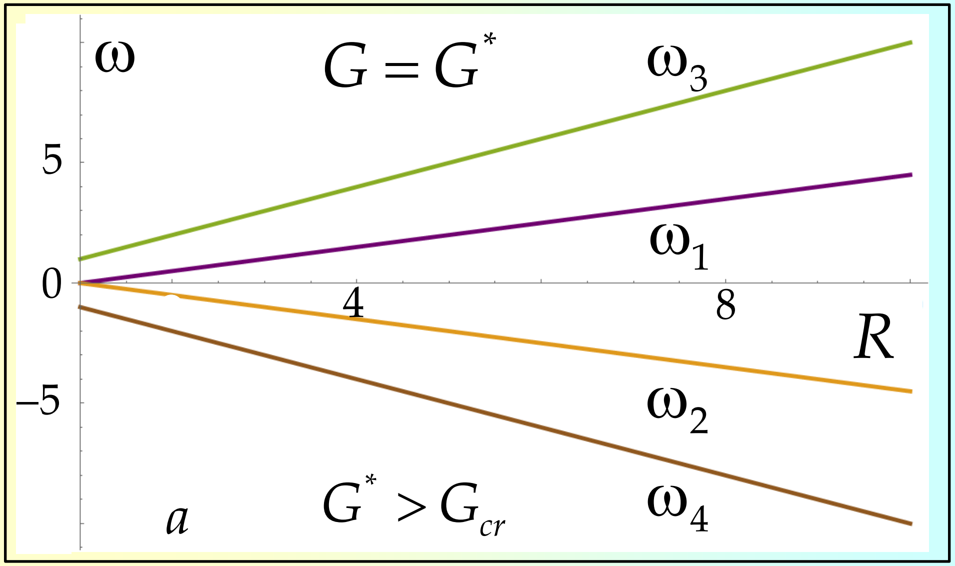
$$G^* = (R^2-1)/4$$

$$\omega_{1(2)} = \pm i\sqrt{R}\sqrt{1-G/G_{cr}},$$

$$\mathbf{e}_{1(2)} = (*,*,1,0)^{\mathrm{T}};$$

$$\omega_3 = R,\ \mathbf{e}_3 = (0,*,0,1)^{\mathrm{T}}$$

$$\omega_4 = -R,\ \mathbf{e}_4 = (*,*,0,1)^{\mathrm{T}}$$



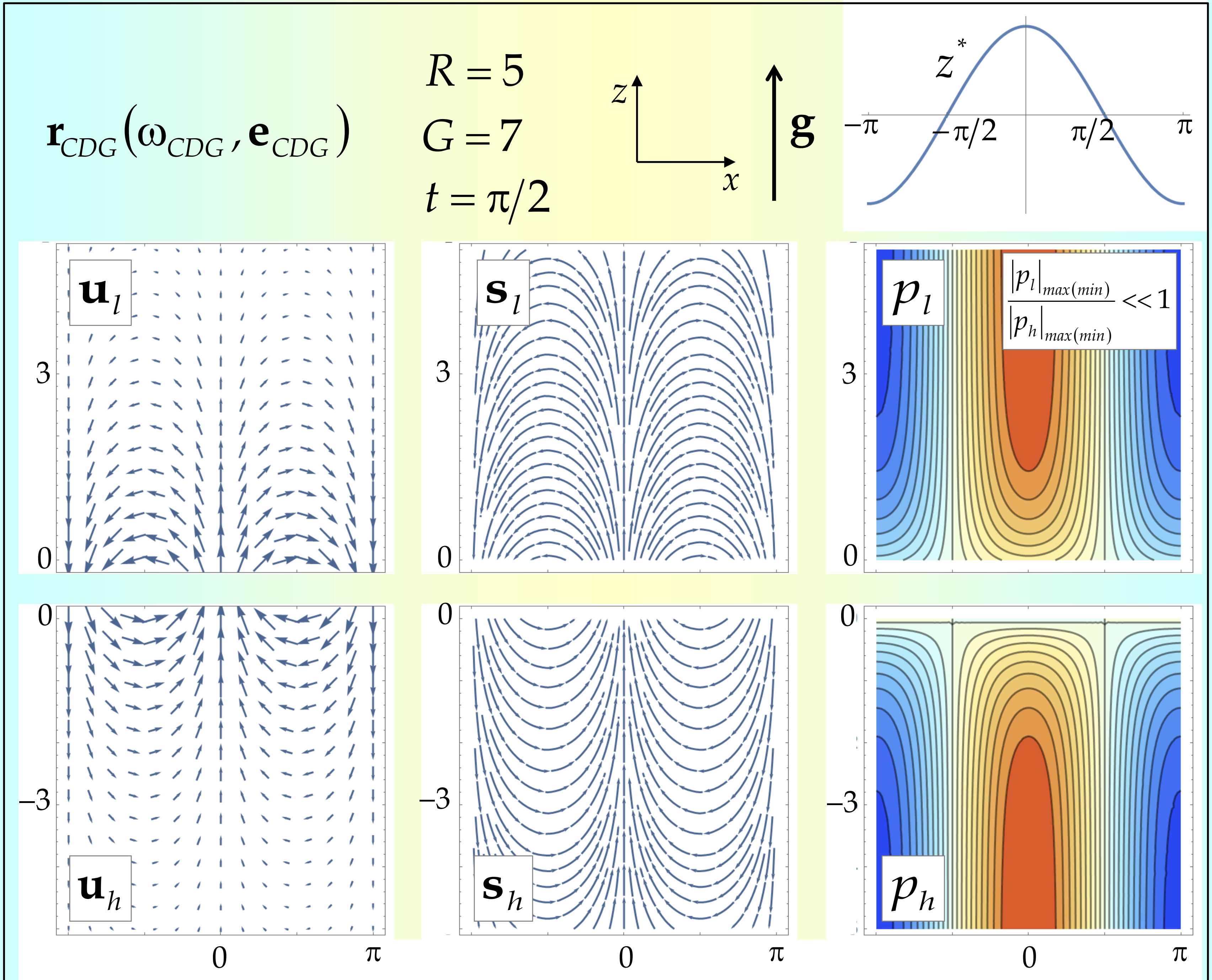
- Integration constants

$$C_1, C_2 = ic,\ C_3 = any,\ C_4 = 0$$

- Stability properties

$$G_{cr} = R(R-1)/(R+1)$$

$G < G_{cr}$	$\omega_{CDG} = \pm i\sqrt{R}\sqrt{1-G/G_{cr}}$	$\mathbf{r}_{CDG} = (\mathbf{r}_1+\mathbf{r}_2)/2$
$G > G_{cr}$	$\omega_{CDG} = \sqrt{R}\sqrt{G/G_{cr}-1}$	$\mathbf{r}_{CDG} = \mathbf{r}_1$



$\mathbf{r}_{CDG}(\omega_{CDG}, \mathbf{e}_{CDG})$
$R = 5$
$G = 7$
$t = \pi/2$
z
x
$\mathbf{g}$
z^*
$-\pi$
$-\pi/2$
$\pi/2$
π
$\mathbf{u}_l$
$\mathbf{s}_l$
p_l
$\frac{|p_l|_{max(min)}}{|p_h|_{max(min)}} << 1$
$\mathbf{u}_h$
$\mathbf{s}_h$
p_h
3
0
−3
0
π

Mechanism of stabilization and destabilization

- Stability of accelerated conservative dynamics is set by
  - the interplay of the inertial and buoyant effects
  - the reactive force and gravity.

$$\Omega_{CDG} = kV_h\sqrt{(\rho_h/\rho_l)(g/g_{cr}-1)} \qquad g_{cr} = kV_h^2(\rho_h/\rho_l)(\rho_h-\rho_l)/(\rho_h+\rho_l)$$

$$g/g_{cr} < 1 \qquad\qquad g/g_{cr} > 1$$

$$\Omega_{CDG} = ikV_h\sqrt{(\rho_h/\rho_l)(1-g/g_{cr})} \qquad \Omega_{CDG} = kV_h\sqrt{(\rho_h/\rho_l)(g/g_{cr}-1)}$$

- Stability of accelerated conservative dynamics is set by the interplay of the inertial and buoyant effects or by the reactive force and gravity.

$$k = k_{cr}: \Omega_{CDG}\big|_{k=k_{cr}} = 0 \qquad k = k_{max}: \partial\Omega_{CDG}/\partial k\big|_{k=k_{max}} = 0, \partial^2\Omega_{CDG}/\partial k^2\big|_{k=k_{max}} < 0$$

$$k_{cr} = \left(\frac{g}{V_h^2}\right)\left(\frac{\rho_l}{\rho_h}\right)\left(\frac{\rho_h+\rho_l}{\rho_h-\rho_l}\right) \qquad k_{max} = \frac{1}{2}\left(\frac{g}{V_h^2}\right)\left(\frac{\rho_l}{\rho_h}\right)\left(\frac{\rho_h+\rho_l}{\rho_h-\rho_l}\right)$$

$$\frac{k_{cr}}{k_{max}} = 2$$

Landau's accelerated dynamics

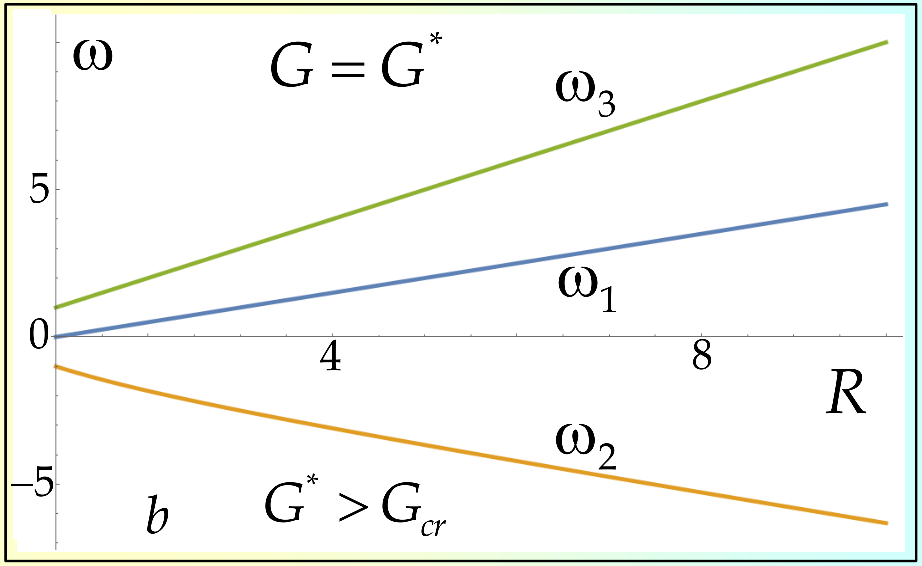
- Boundary conditions

$$[j_n]=0,\ [(p+2J_n j_n/\rho)\mathbf{n}_0]=0,\ [J_n(\mathbf{J}\cdot\boldsymbol{\tau}_1+\mathbf{j}\cdot\boldsymbol{\tau}_0)/\rho]=0,\ [\mathbf{u}\cdot\mathbf{n_0}]=0$$

- Solutions

$$\omega_{1(2)}=\left(-R\pm\sqrt{(R^3+R^2-R)+G(R^2-1)}\right)\Big/(1+R),\ \mathbf{e}_{1(2)}=(*,*,*,1)^{\mathrm{T}};$$

$$\omega_3=R,\ \mathbf{e}_3=(0,*,0,1)^{\mathrm{T}}$$

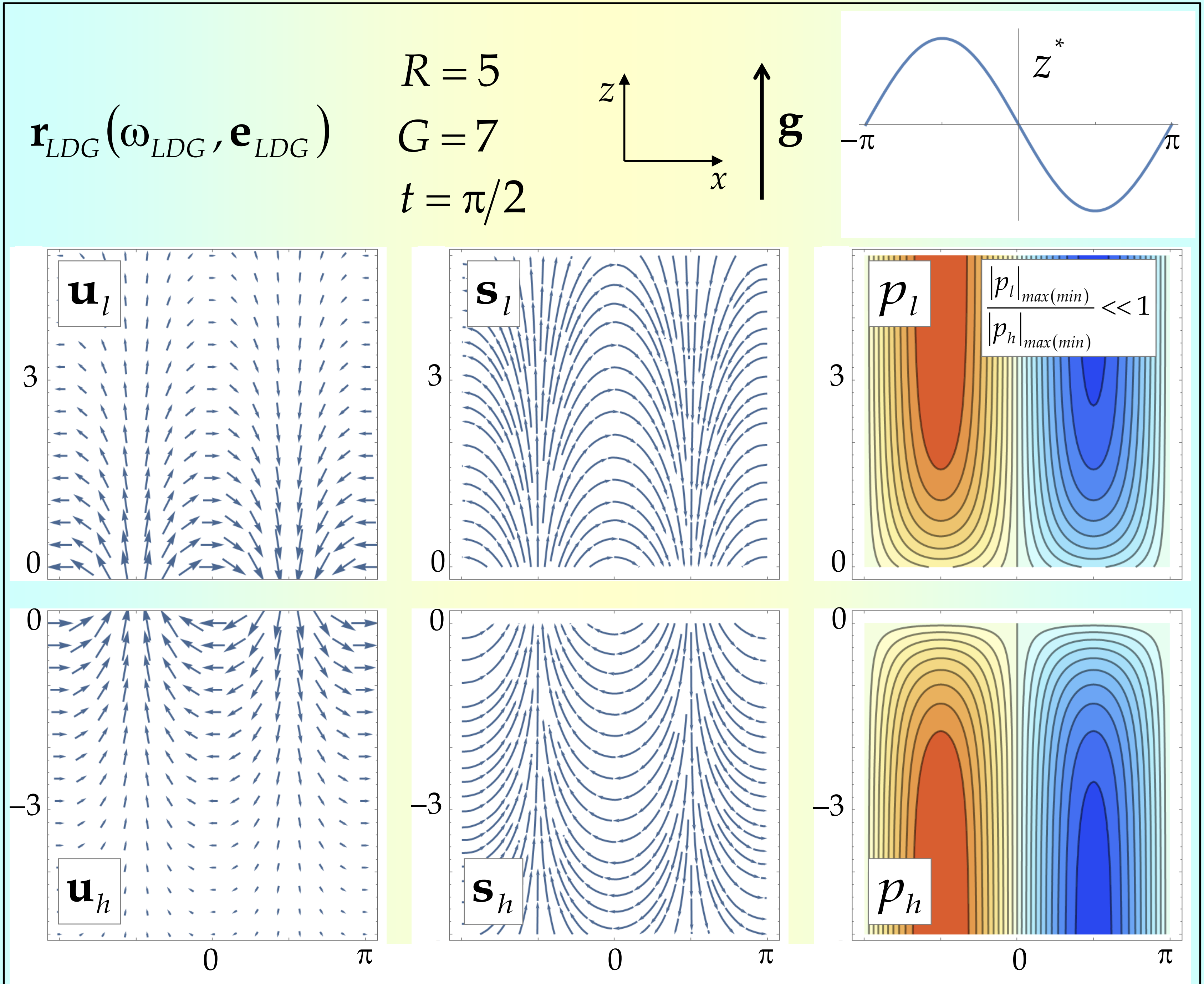


- Integration constants

$$C_1=ic,\ C_2=0,\ C_3=any$$

- Stability properties

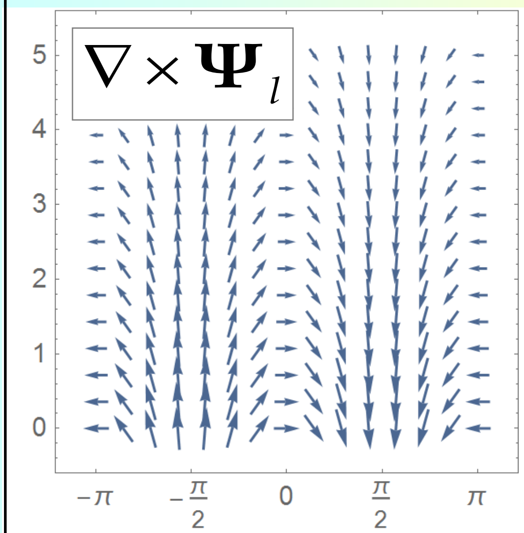
$$G\ge 0 \qquad \omega_{LDG}=\left(-R+\sqrt{(R^3+R^2-R)+G(R^2-1)}\right)\Big/(1+R)>0 \qquad \mathbf{r}_{LDG}=\mathbf{r_1}$$

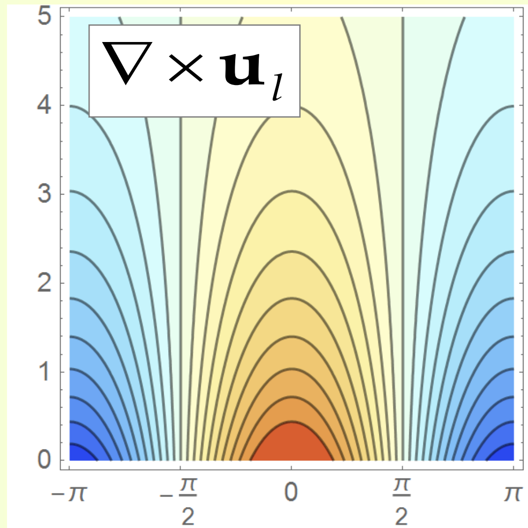


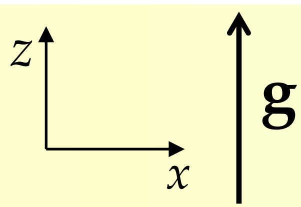
$\mathbf{r}_{LDG}(\omega_{LDG}, \mathbf{e}_{LDG})$
$R = 5$
$G = 7$
$t = \pi/2$
z
x
g
z^*
−π
π
$\mathbf{u}_l$
$\mathbf{s}_l$
p_l
$\frac{|p_l|_{max(min)}}{|p_h|_{max(min)}} << 1$
3
0
$\mathbf{u}_h$
$\mathbf{s}_h$
p_h
−3
0
π

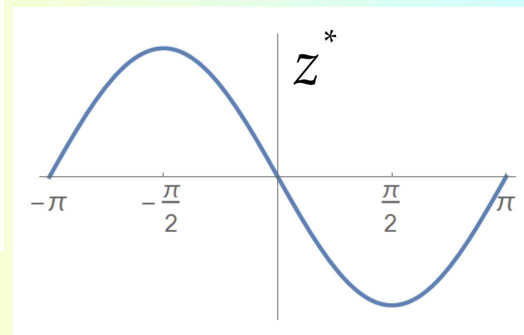
LDG

$$\mathbf{r}_{LDG}(\omega_{LDG}, \mathbf{e}_{LDG})$$









$R = 5$

$G = 7$

$t = \pi/2$

Rayleigh-Taylor dynamics

- Boundary conditions

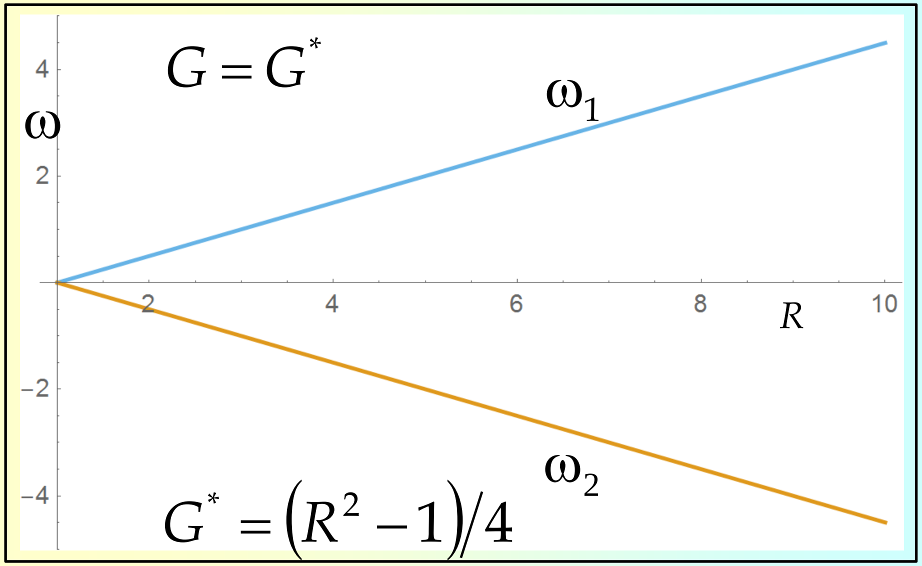
$$\left[\tilde{j}_n\right]=0,\ [P\mathbf{n}]=0,\ [\mathbf{v}\cdot\mathbf{n}]=0,\ [\mathbf{v}\cdot\boldsymbol{\tau}]=any,\ [W]=any,\ \mathbf{v}_h\big|_{z\to-\infty}=0,\ \mathbf{v}_l\big|_{z\to+\infty}=0$$

$$\mathrm{M}\,\mathbf{r}=0 \qquad \mathbf{r}=\left(\Phi_h,\Phi_l,V_h z^*\right)^{\mathrm{T}}$$

- Solutions

$$\omega_{1(2)}=\pm\sqrt{G(R-1)/(R+1)},$$

$$\mathbf{e}_{1(2)}=(*,*,1)^{\mathrm{T}}$$



- Integration constants

$$C_{1(2)}=ic$$

- Stability properties

$$G\geq 0 \qquad \omega_{RT}=\sqrt{G(R-1)/(R+1)}>0 \qquad \mathbf{r}_{RT}=\mathbf{r}_1$$

RT

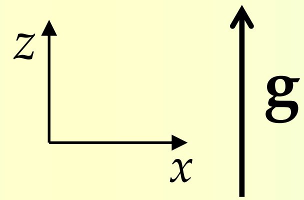
$\mathbf{r}_{RT}(\omega_{RT}, \mathbf{e}_{RT})$

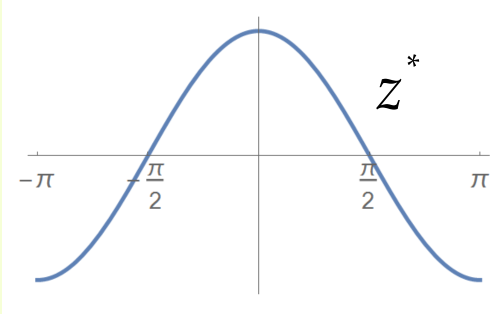
$\mathbf{r}_{RT} = \mathbf{r}_1$

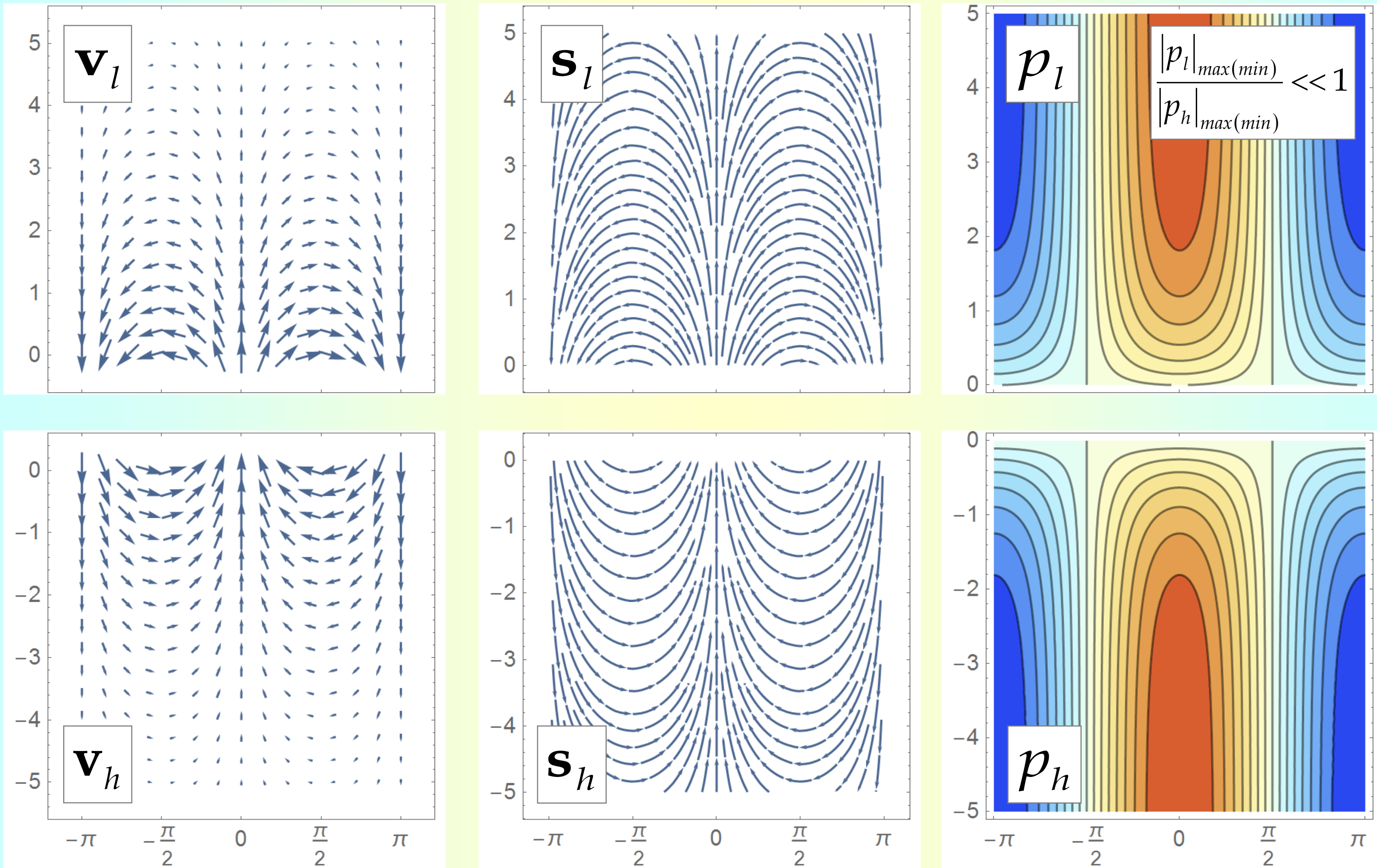
$R = 5$

$G = 7$

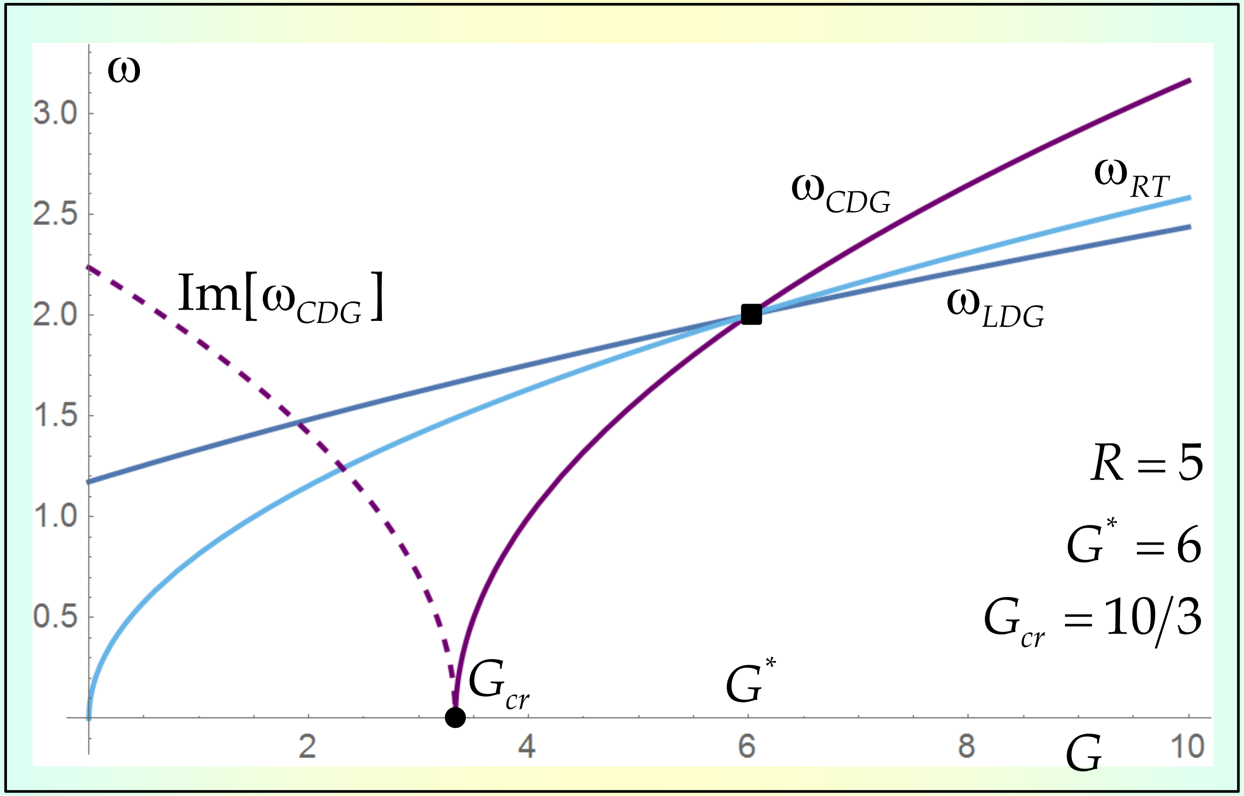
$t = \pi/2$







Interface dynamics under destabilizing acceleration



$G = g/kV_h^2$

$G < G_{cr}$ $\quad \omega_{CDG} = \pm i\sqrt{R}\sqrt{1 - G/G_{cr}}$

$G > G_{cr}$ $\quad \omega_{CDG} = \sqrt{R}\sqrt{G/G_{cr} - 1}$

$G_{cr} = R(R-1)/(R+1)$

$G^* = (R^2 - 1)/4$

$G \geq 0$ $\quad \omega_{LDG} = \left(-R + \sqrt{(R^3 + R^2 - R) + G(R^2 - 1)}\right)/(1+R) > 0$

$G > 0$ $\quad \omega_{RT} = \sqrt{G(R-1)/(R+1)} > 0$

Properties of accelerated dynamics

	CDG	LDG	RT
Conservation properties	Conserves mass, momentum and energy at the interface	Conserves mass and momentum and has zero perturbed mass flux at the interface	Conserves mass and momentum and has zero mass flux at the interface
Interface velocity	Time-dependent	Constant	Zero
Flow field	Potential velocity fields	Vortical field is present	Potential velocity fields
Interfacial shear	Shear-free	Shear-free	Interfacial shear
Formal properties	Non-degenerate; 4 solutions, 4 degrees of freedom	Degenerate; 3 solutions, 4 degrees of freedom	Degenerate; 2 solutions, 3 degrees of freedom
Stability	Stability is set by the interplay of effects of inertia (reactive force) and buoyancy (gravity).	Unstable for any gravity value, including zero gravity value.	Unstable for any gravity value; neutrally stable for zero gravity

Interface dynamics in realistic fluids

- Reveal structure of flow fields and their link to interfacial conditions.
- Identify control parameters for three regimes.
- Discover fluid instabilities never earlier discussed.
- Define the interface as the balance place.

Conservative sub-sonic dynamics in realistic fluids

- Leading order:

$$\left(\rho, \mathbf{v}, P, e, W, \tilde{\mathbf{j}}, \mathbf{Q}\right)_{h(l)} = \left(\rho_0, \mathbf{V}, P_0, e_0, W_0, \mathbf{J}, \mathbf{Q}_0\right)_{h(l)}$$

$$[J_n] = 0, \quad \left[\left(P_0 + \frac{J_n^2}{\rho_0}\right)\mathbf{n}_0\right] = 0, \quad \left[J_n\left(W_0 + \frac{\mathbf{J}^2}{2\rho_0^2}\right) + (\mathbf{Q}_0 \cdot \mathbf{n}_0)\right] = 0$$

$$\left(\mathbf{Q}_0 \cdot \boldsymbol{\tau}_0\right)\Big|_{\theta=0^+} = 0, \quad \left(\mathbf{Q}_0 \cdot \boldsymbol{\tau}_0\right)\Big|_{\theta=0^-} = 0$$

- First order:

$$(\partial/\partial t + \mathbf{V}\cdot\nabla)\bar{\rho} + \rho_0 \nabla\cdot\mathbf{u} = 0, \;\; \rho_0(\partial/\partial t + \mathbf{V}\cdot\nabla)\mathbf{u} + \nabla p = 0,$$

$$\rho_0(\partial/\partial t + \mathbf{V}\cdot\nabla)\bar{e} + \nabla\cdot\mathbf{q} + P_0\nabla\cdot\mathbf{u} = 0, \;\; \mathbf{q} + \nabla(\chi\bar{e}) = 0, \;\; p/P_0 = \bar{\rho}/\rho_0 + \bar{e}/e_0$$

$$\left[(\mathbf{j} + \bar{\mathbf{j}})\cdot\mathbf{n}_0\right] = 0, \; \left[\left(p + \frac{2(\mathbf{J}\cdot\mathbf{n}_0)(\mathbf{j}\cdot\mathbf{n}_0)}{\rho_0} + \frac{(\mathbf{J}\cdot\mathbf{n}_0)(\bar{\mathbf{j}}\cdot\mathbf{n}_0)}{\rho_0}\right)\mathbf{n}_0\right] = 0,$$

$$\left[(\mathbf{J}\cdot\mathbf{n}_0)(\mathbf{J}\cdot\boldsymbol{\tau}_1 + \mathbf{j}\cdot\boldsymbol{\tau}_0)\frac{\boldsymbol{\tau}_0}{\rho_0}\right] = 0, \; \left[(\mathbf{J}\cdot\mathbf{n}_0)\left(w + (\mathbf{J}\cdot\mathbf{j})/\rho_0^2\right) + (\mathbf{q}\cdot\mathbf{n}_0)\right] = 0,$$

$$\left(\mathbf{Q}_0\cdot\boldsymbol{\tau}_1 + \mathbf{q}\cdot\boldsymbol{\tau}_0\right)\Big|_{\theta=0^-} = 0, \; \left(\mathbf{Q}_0\cdot\boldsymbol{\tau}_1 + \mathbf{q}\cdot\boldsymbol{\tau}_0\right)\Big|_{\theta=0^-} = 0$$

$$\left(\bar{\rho}, \mathbf{u}, p, \bar{e}, \mathbf{j}, \bar{\mathbf{j}}, \mathbf{q}, w\right)\Big|_{z\to\pm\infty} = 0 \qquad \hat{\mathbf{j}} = \mathbf{j} + \bar{\mathbf{j}}, \; \mathbf{j} = \rho_0(\mathbf{u} + \mathbf{n}_0\dot{\theta}), \; \bar{\mathbf{j}} = (\bar{\rho}/\rho_0)\mathbf{J}$$

Interface and fields in realistic fluids

- Fields in the bulk are: $\left(\Phi, p, \Psi, \bar{\rho}, \bar{e}\right) = \left(\hat{\Phi}, \hat{p}, \hat{\Psi}, \hat{\rho}, \hat{e}\right) exp\left(ikx - Kz + \Omega t\right)$

$$M_Z \mathbf{Z} = 0 \qquad det\, M_Z = \left(K - \Omega/V\right)\left(K^4 + c_3 K^3 + c_2 K^2 + c_1 K + c_0\right)$$

- Scales and parameters are: $K_m = \rho_0 V/\chi$ $\quad K_m/k \quad \rho_0 V^2/P_0$

$$k_m = K_m/k \qquad Ma^2 = \rho_0 V^2/P_0$$

- Three regimes exist: advection diffusion low Mach

- Three regimes have in common the four waves (mechanical waves).

$$\left(\Phi, p\right)_h = \hat{\Phi}_h \left(1, -k\rho_0 V\left(1 - \Omega/kV\right)\right)_h e^{ikx + kz + \Omega t}$$

$$\left(\Phi, p\right)_l = \hat{\Phi}_l \left(1, k\rho_0 V\left(1 + \Omega/kV\right)\right)_l e^{ikx - kz + \Omega t}$$

$$z^* = Z^* e^{ikx + \Omega t}, \quad \Psi_l = \hat{\Psi} e^{ikx - \left(\Omega/V_l\right) z + \Omega t}$$

- Other two wave are regime specific (energetic waves).

- Seeds of energy perturbations are: $\left(\mathbf{Q}_0\right)_{h(l)} = \left(\mathbf{J} e_0 \varepsilon_0\right)_{h(l)}$

- In each regime the interface velocity is: $\tilde{\mathbf{V}} = \tilde{\mathbf{V}}_0 + \tilde{\mathbf{v}} \qquad \tilde{\mathbf{v}} \cdot \mathbf{n}_0 = -\left(\mathbf{u} \cdot \mathbf{n}_0 + \dot{\theta}\right)\Big|_{\theta = 0}$

Advection

- Parameters are: $$K_m/k \to \infty, \quad P_0/\rho_0 V^2 \to \infty$$

- Energetic waves are:

$$\left(\bar{e}, \bar{\rho}_e\right)_h = \hat{e}_h \left(1, \left(\rho_0^2 s/P_0\right)\right)_h e^{ikx + K_{mh}(1+s_h)z + \Omega t}$$

$$\left(\bar{e}, \bar{\rho}_e\right)_l = \hat{e}_l \left(1, \left(\rho_0^2 s/P_0\right)\right)_l e^{ikx - (\Omega/V_l)z + \Omega t}$$

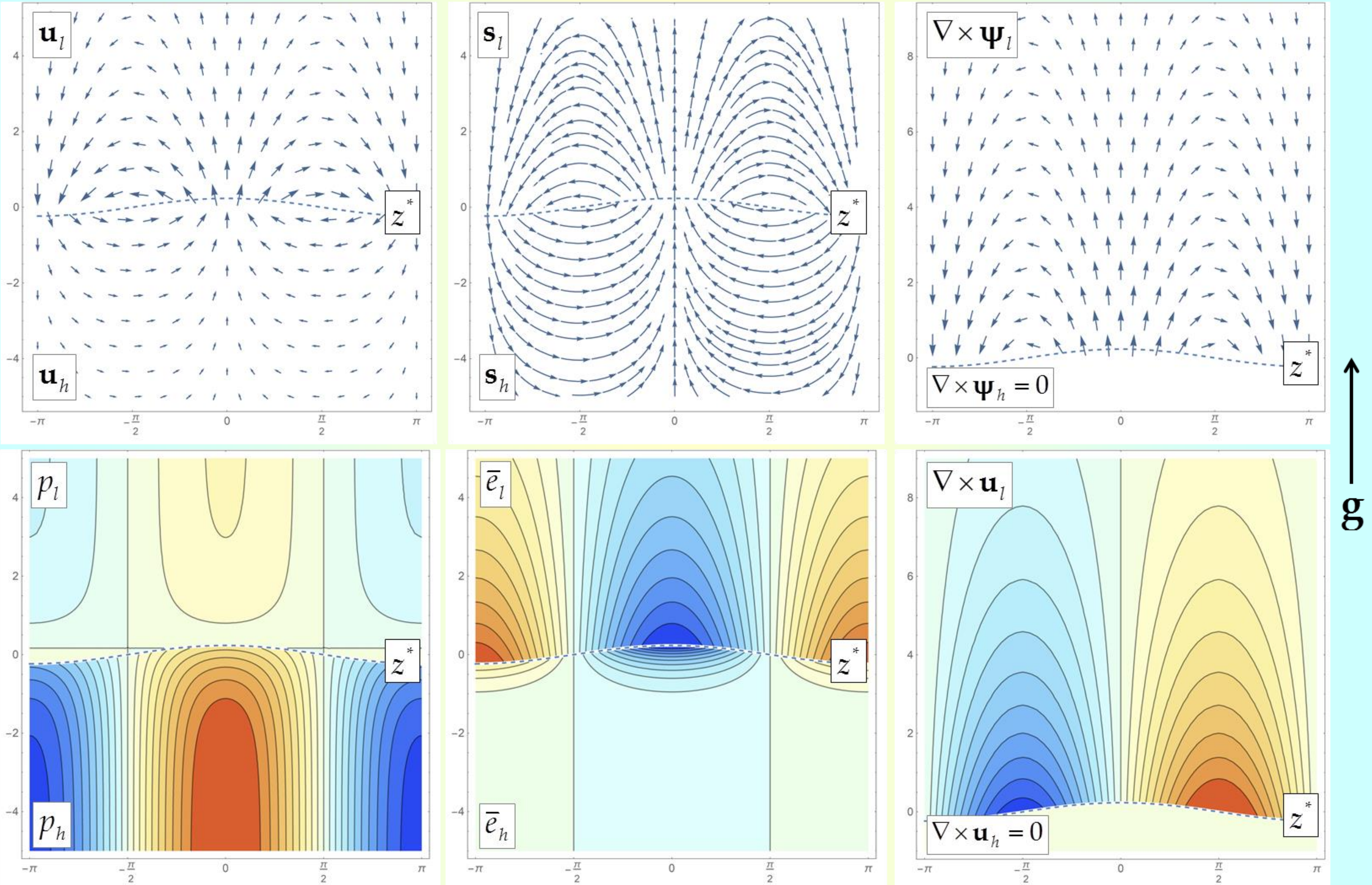
- Interface growth-rate is: $$Ma \to 0$$

$$\omega_{CDGA} = -F \pm i\sqrt{R}\sqrt{1 - G/G_{cr} - F^2/R + 2\,F\,k_{ml}\left(1 + s_l R\right)}$$

$$G_{cr} = R(R-1)/(R+1), \quad F = -\varepsilon_{0l}/(R-1)^2, F \geq 0$$

- Interface is stable (unstable): $$\mathrm{Re}\left[\omega_{CDGA}\right] \leq 0\ (> 0) \qquad G \leq \tilde{G}_{cr}\ \left(> \tilde{G}_{cr}\right)$$

$$\left(k_m/\varepsilon_0\right)_l \to 0 \qquad \omega_{CDGA} \to -F \pm i\sqrt{R}\sqrt{1 - G/G_{cr} - F^2/R} \qquad \tilde{G}_{cr} = G_{cr}$$



$\mathbf{r}_{CDGA}$ $R = 5$ $G = 7$ $F = 2$ $\varepsilon_{0h} = -20$ $k_{mh} = 2$ $k_{ml} << \varepsilon_{0l}$ $t = \pi/4$ $C = 10^{-1}$

Diffusion

- Parameters are: $K_m/k << 1, \quad \left(\rho_0 V^2/P_0\right)/\left(K_m/k\right) \to 0$

- Energetic waves are:

$$\left(\bar{e}, \overline{\Phi}_e, \bar{p}_e\right)_h = \hat{e}_h\left(1, \rho_0 V/K_m P_0, \rho_0\left(k/K_m\right)\left(\rho_0 V^2/P_0\right)\left(1-\Omega/kV\right)\right)_h e^{ikx+K_h z+\Omega t}$$

$$\left(\bar{e}, \overline{\Phi}_e, \bar{p}_e\right)_l = \hat{e}_l\left(1, \rho_0 V/K_m P_0, -\rho_0\left(k/K_m\right)\left(\rho_0 V^2/P_0\right)\left(1+\Omega/kV\right)\right)_l e^{ikx-K_l z+\Omega t}$$

$$K_{h(l)} = k - \bar{k}_{h(l)}, \bar{k}_{h(l)} \sim K_{mh} \qquad k_m = K_m/k$$

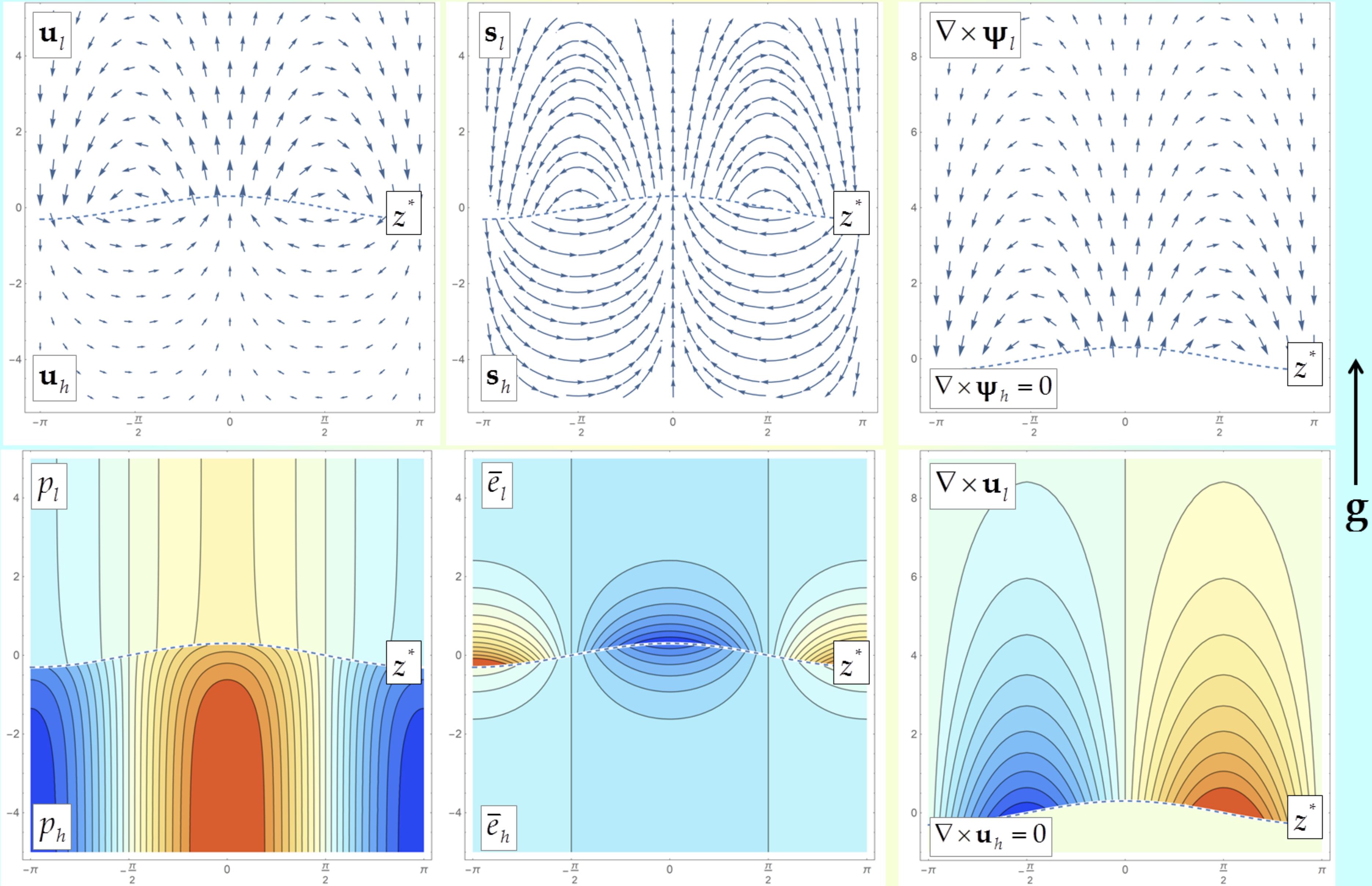
- Interface growth-rate is: $Ma \to 0$

$$\omega_{CDGD} = \pm i\sqrt{R}\sqrt{1 - G/G_{cr} - 2\left(\left(\varepsilon_0\left(1-k_m\right)\right)_h + \left(\varepsilon_0\left(1+k_m\right)\right)_l\right)/(R-1)^2}$$

$$F(R-1)^2 = -\left(\varepsilon_{0h} + \varepsilon_{0l}\right), F \geq 0$$

- Interface is stable (unstable): $\operatorname{Re}\left[\omega_{CDGD}\right] \leq 0\ (>0) \qquad G \leq \hat{G}_{cr}\ \left(> \hat{G}_{cr}\right)$

$$\left(k_m\right)_{h(l)} \to 0 \qquad \omega_{CDGD} = \pm i\sqrt{R}\sqrt{(1+2F) - G/G_{cr}} \qquad \hat{G}_{cr} = G_{cr}(1+2F)$$



$\mathbf{r}_{CDGD}$ $R=5$ $G=18$ $F=2$ $\varepsilon_{0h}=-10$ $k_{m(h,l)} << 1$ $t=\pi/4$ $C=10^{-1}$

Low Mach

- Parameters are: $\rho_0 V^2/P_0 \to 0, \quad (K_m/k)/(\rho_0 V^2/P_0) \to 0$

- Energetic waves are:

$$(\bar{e}, \overline{\Phi}_e, \bar{\rho}_e, \bar{p}_e)_h = \hat{e}_h \left(1, -\rho_0 V/2kP_0 + 2s/(kV-\Omega), \rho_0^2 s/P_0, 2\rho_0 s_h\right)_h e^{ikx+K_h z+\Omega t}$$

$$(\bar{e}, \overline{\Phi}_e, \bar{\rho}_e, \bar{p}_e)_l = \hat{e}_l \left(1, \rho_0 V/2kP_0 - 2s/(kV+\Omega), \rho_0^2 s/P_0, 2\rho_0 s\right)_l e^{ikx+K_l z+\Omega t}$$

$$K_{h(l)} = k - \tilde{k}_{h(l)} \qquad \tilde{k}_l = (kV_l - \Omega)^2 (\rho_0/2kP_0)_h, \ \tilde{k}_l = (kV_l + \Omega)^2 (\rho_0/2kP_0)_l$$

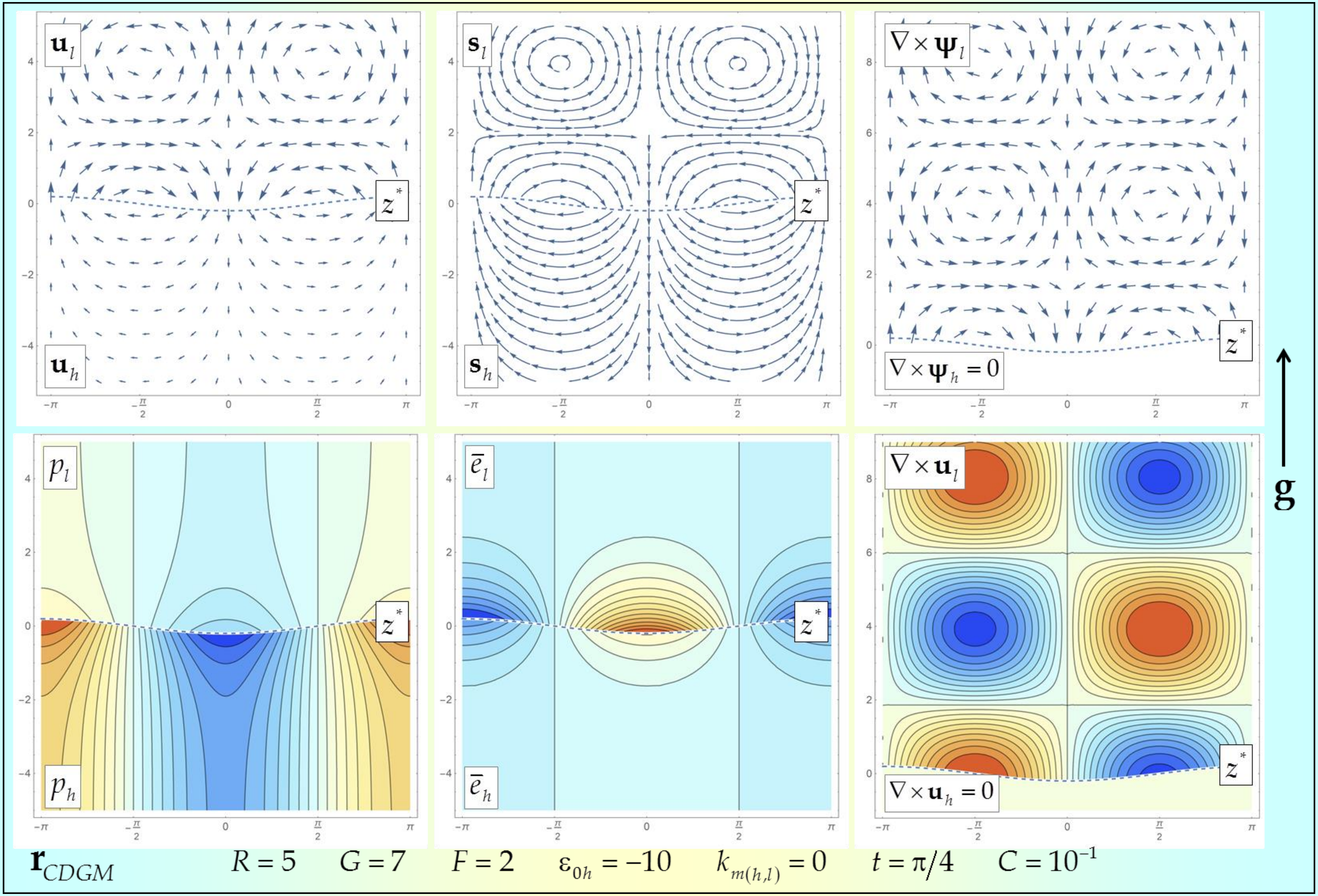
- Interface growth-rate is: $Ma \to 0$

$$\omega_{CDGM} = \pm i\sqrt{R}\sqrt{1 - G/G_{cr} - 2(\varepsilon_{0h} + \varepsilon_{0l})/(R-1)^2}$$

$$F(R-1)^2 = -(\varepsilon_{0h} + \varepsilon_{0l}), F \geq 0$$

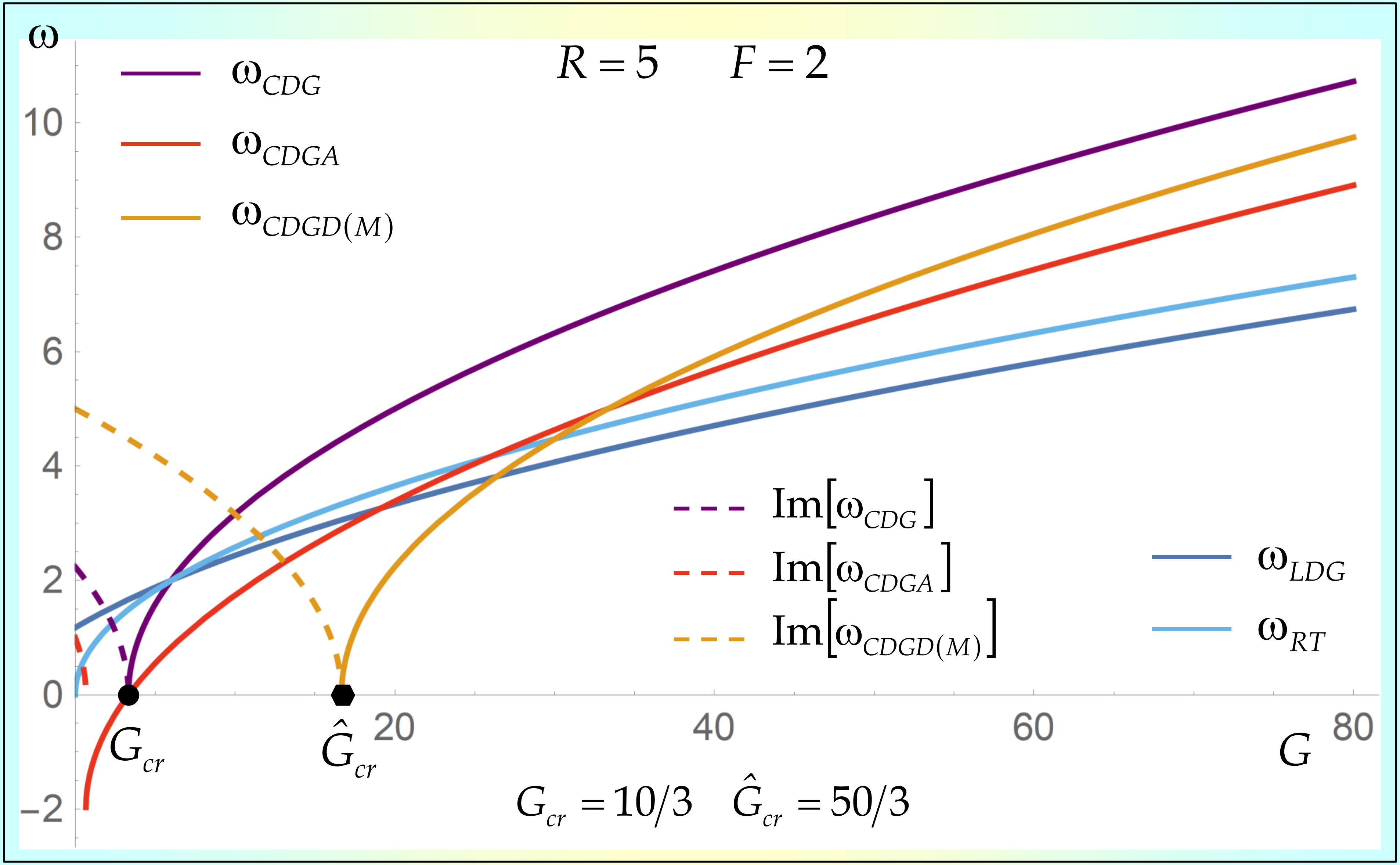
- Interface is stable (unstable): $\mathrm{Re}\left[\omega_{CDGM}\right] \leq 0 \ (>0) \qquad G \leq \overline{G}_{cr} \ \left(> \overline{G}_{cr}\right)$

$$\overline{G}_{cr} = G_{cr}(1+2F)$$



Turbulence: interface stability & energy flux are needed to created massive vortical fields in the bulk.

Interface dynamics under destabilizing acceleration



For strong accelerations, the growth-rate ω_{CDG} envelopes the growth-rates $\omega_{CDG(A)(D)(M)}$

Mechanisms of stabilization and destabilization

- Stability of accelerated conservative dynamics is set primarily by the interplay of
  - the macroscopic inertial mechanism with the destabilizing acceleration;
  - the reactive force and gravity.

$$\Omega_{CDG} = kV_h \sqrt{(\rho_h/\rho_l)(g/g_{cr} - 1)} \qquad g_{cr} = kV_h^2 (\rho_h/\rho_l)(\rho_h - \rho_l)/(\rho_h + \rho_l)$$

$$g/g_{cr} < 1 \qquad\qquad g/g_{cr} > 1$$

$$\Omega_{CDG} = ikV_h \sqrt{(\rho_h/\rho_l)(1 - g/g_{cr})} \qquad \Omega_{CDG} = kV_h \sqrt{(\rho_h/\rho_l)(g/g_{cr} - 1)}$$

- Microscopic thermodynamics provides with additional stabilization.

- Thermal heat flux and thermal conductivity set the vortical field in the bulk.

- Thermal heat flux seeds internal energy perturbations.

$$[(\mathbf{Q}_0 \cdot \mathbf{n}_0)] = -[(\mathbf{J} \cdot \mathbf{n}_0)\left(W_0 + \left(\mathbf{J}^2/2\rho_0^2\right)\right)] \qquad W_0 = \overline{W}_0 + C_P \Theta$$

- Interface is the place where balances are achieved, by linking micro to macro scales.

Conclusion

This work:

- Develops theoretical framework to study interface dynamics;
- Identifies the mechanisms of the interface stabilization and destabilization;
- Directly links the microscopic interfacial transport to macroscopic flow fields.

Theory outcomes are:

- Resolves challenges not addressed before for ideal and realistic fluids.
  - Structure of fields and the thermal heat flux;
  - Dependence of the fields coupling on the system parameters.
- Discovers novel class of fluid instabilities for ideal and realistic fluids;
  - Conservative dynamics & dynamics in advection, diffusion, low Mach regimes;
  - Interplay of inertial stabilization, thermal heat flux and conductivity with destabilizing acceleration.
- Finds mechanisms of stabilization and destabilization
  - The interface stability is achieved macroscopically, through inertial stabilization balancing destabilizing acceleration.
  - Thermal heat flux and microscopic thermodynamics create vortical fields in the bulk.
- Provides broad impact
  - For the 3rd prospect of Landau finds that Landau 1944 solution is a perfect match.
  - Suggests improvements for numerical modeling and experimental diagnostics.
  - Has broad range of applications and further developments: supernovae, fusion, stability of shocks, reactive and super-critical fluids, nanofabrication, etc.

- Interface is the place of the balances achieved by linking micro to macro scales.
- Interfaces are real. They are globally stable. They can be forced to destabilize.

Perspectives

- The classic Landau's framework is a source of inspiration for theory research.
- New properties of the interfacial dynamics are identified.
- New theory benchmarks are elaborated.
- New numerical and experimental approaches are in demand.
    - Interface tracking methods in simulations
    - Diagnostics of fields in experiments
- Realistic conditions and stabilizing effects can be systematically implemented.
    - dissipation, compressibility, radiation transport, stratification, electro-magnetic effects, non-local forces
- A broad range of application problems can now be rigorously considered.
    - D'yakov-Kontorovich instability in shocks, ablative RTI and RMI in fusion plasmas, deflagration-to-detonation transition in supernova, and dynamics of reactive and super-critical fluids
- Our new results suggest that 'complexity can self-regularize'.