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- Lawrence Livermore National Lab, Physical Sciences Division

Fluid instabilities and interfacial mixing in high energy density plasmas

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Thank you to the colleagues: Drs. Arnett WD, Remington BA, Sreenivasan KR, Meshkov EE.

Thank you to the students: Bhowmick AK, Dell ZC, Li JT, Naveh A, Pandian A, Stanic M, Swisher NC, Li JT, Williams KR, Wright CE, Wright J.

Some references: 2023 SNAS, 2023 PoF, 2022 PNAS, 2022 AML, 2022 FAM,
2021 PoP, 2021 PoF, 2020 PoP, 2020 PoF, 2020 AML, 2020 FDR,
2019 FDR, 2019 PNAS, 2019 PRF, 2017 PRF, 2017 PoP, 2017 PS,
2015 PoF, 2015 PoP, 2013 PTRS, 2010 PTRS,…,
2008 PS, …2005 PoF, …, 1998 PRL,… 1996 PS, 1996 JETP.

Who am I?

- University of Western Australia (since end of 2016)
    - Prof. Abarzhi SI; Dr. Hill DL, Dr. Matthews MT, Dr. Pfefferle D;
    - Williams K, Wright CE, Wright J, Naveh A, Li JT, Pandian A.
  - Caltech (USA) – Mr. Ilyin DV [Prof Goddard WA];
  - Stanford (USA) – Dr. Jain SS, Dr Hwang HC, Dr Chan WCHR [Prof. Moin P].

- SIA:
    - Carnegie Mellon, U Chicago, Stanford, SUNY Stony Brook – USA;
    - Osaka U – Japan; U Bayreuth – Germany;
    - Landau Inst Theor Phys, Academy of Sciences – Russia.
  - Key discoveries are:
    - new class of fluids instabilities, inertial mechanism of interface stabilization, resolution of Landau 1944 paradox;
    - special self-similarity class; order in Rayleigh-Taylor mixing; group theory approach for fundamentals of fluids instabilities and interfacial mixing.
  - Key contributions to the community:
    - founding program 'Turbulent Mixing and beyond';
    - organizing high profile conference and symposia [at the KITP in Oct 2023];
    - editorial work [20+ edited books with lead journals and publishers]
  - Some recognitions
    - Awards – Natl Acad Sci, Natl Sci Foundation, Japan Soc Prom Sci, A v Humboldt
    - APS Fellow ['for deep and abiding work on RT instabilities & community leadership']
    - member of APS Committee on Scientific Publications.

Mathematics, Science, Engineering

- Mathematics
  - studies things that do not exist in nature;
  - gets a precise knowledge about objects existing in our mind only.

- Science
  - studies things that do exist in nature;
  - gets an approximate knowledge about objects independent of our mind.

- Engineering
  - control things that are man-made;
  - gets information with account for many minds and many natural processes.

What do they have in common?

Opinion Independent Results

Rayleigh-Taylor / Richtmyer-Meshkov instabilities
RT / RM interfacial mixing

RT / RM dynamics controls a broad range of processes in nature and technology:
- core-collapse supernovae, thermonuclear flashes; photo-evaporated clouds;
- inertial confinement fusion, magneto-inertial fusion, Z-pinches;
- light-matter interaction, material transformation under impact.

RTI / RMI: Fluids of different densities are accelerated against their density gradient.
Classical RTI is for constant acceleration. Classical RMI is for impulsive acceleration.

RT / RM interfacial mixing ensues with time. RT / RM mixing is self-similar.

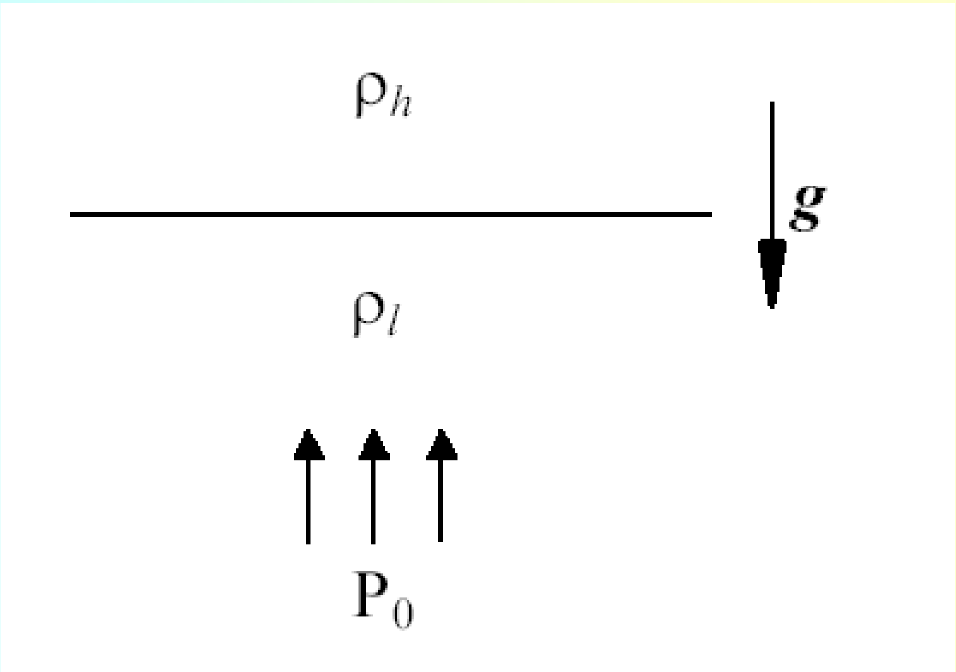
In HEDP environments, accelerations are usually variable.

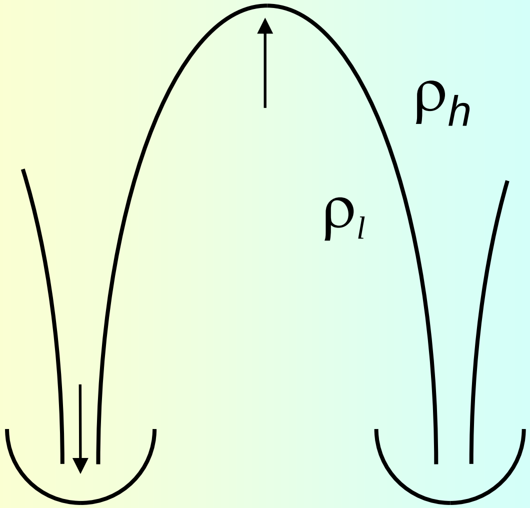
- RT / RM mixing with variable acceleration belongs to a special self-similar class.
- RT / RM self-similar dynamics can vary from super-ballistics to sub-diffusion.
- RT / RM mixing keeps memory of deterministic conditions.

- *RT / RM mixing properties depart from those of self-similar Kolmogorov turbulence & those prescribed self-similar blast waves (Sedov-Taylor, Guderley-Stanyukovich).*

Classical Rayleigh-Taylor instability

Classical RTI is for constant acceleration.





$P_0 = 10^5$Pa, $\quad P = \rho\, g\, h$

$\rho_h \sim 10^3$ kg/m^3, $\quad g \sim 10$ m/s^2

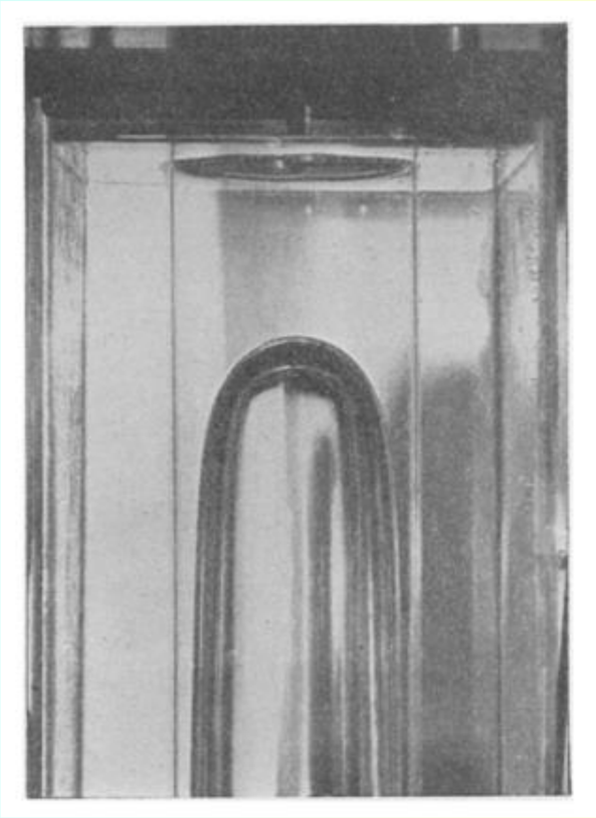
$h \sim 10$ m

Water flows out from an overturned cup

Lord Rayleigh, 1883,
Sir G.I. Taylor 1950

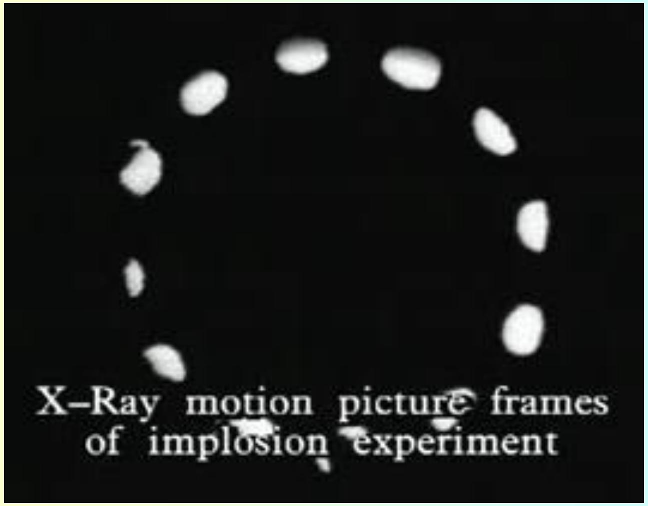
Classical Richtmyer-Meshkov instability: acceleration is shock-driven or impulsive.

‘Taylor’ problem



Experiments in a vertical tube
~2m in height and ~10 cm in diameter
Systems: water/air, ethanol/air

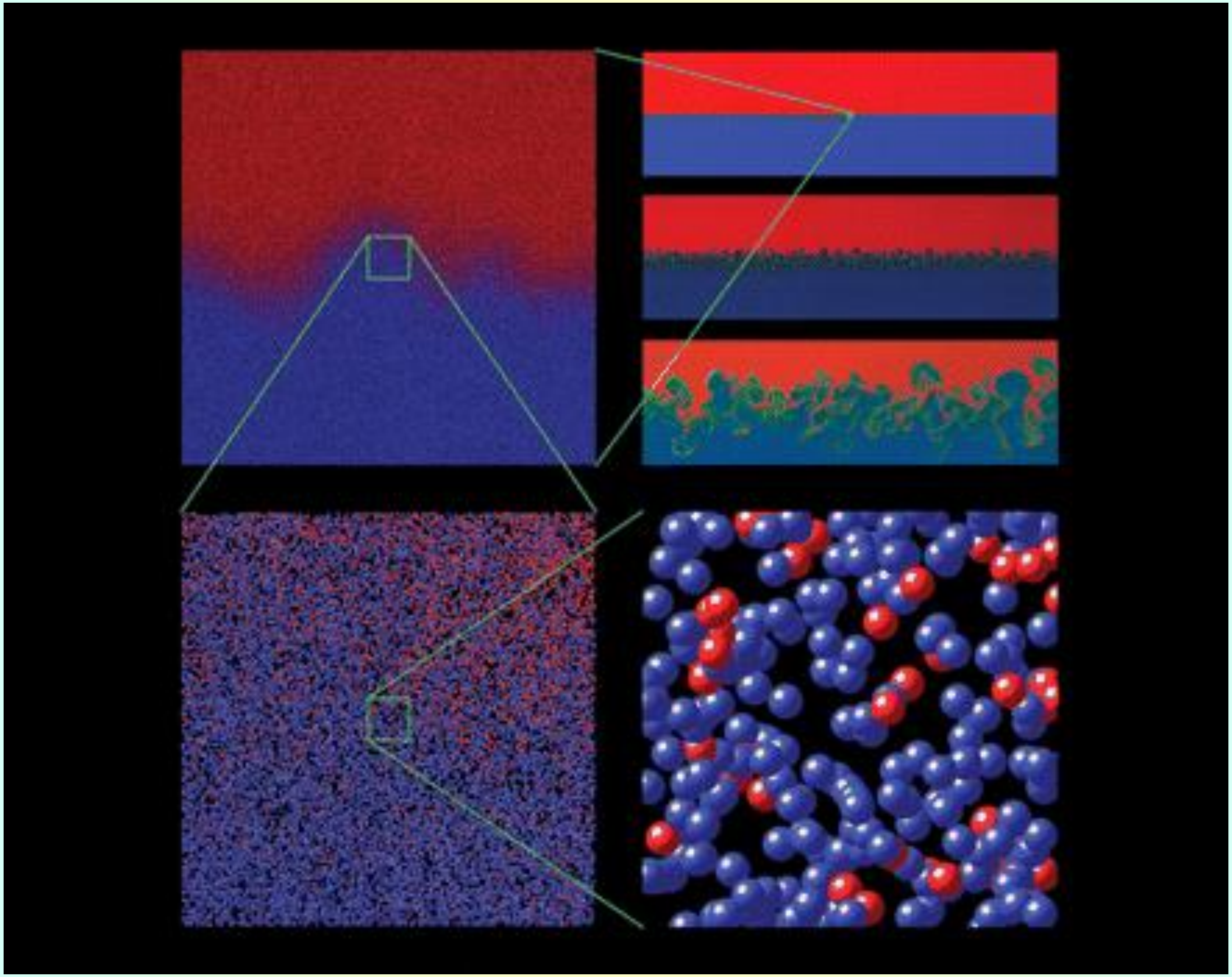
Taylor 1950; Davies & Taylor, 1950



Manhattan project.
X-ray image sequence of converging compression shock wave formed by high explosive lenses.

http://en.wikipedia.org/wiki/File:
X-Ray-Image-HE-Lens-Test-Shot.gif

Fluid instabilities and interfacial mixing from atomic to macroscopic scales



$g \sim 10^{11}\,\mathrm{m\,s^{-2}}$

Molecular dynamics simulations of Rayleigh-Taylor interfacial mixing

$\mathrm{Re} \sim 10^5$

[Kadau et al. 2010]

Fluid instabilities and interfacial mixing

Why are they important in high energy density plasmas?

Photo-evaporated molecular clouds

Birth of a star

Stalactites?
Stalagmites?

Eagle Nebula.

The fingers protrude from the wall of a vast cloud of molecular hydrogen.

The gaseous tower are light-years long.

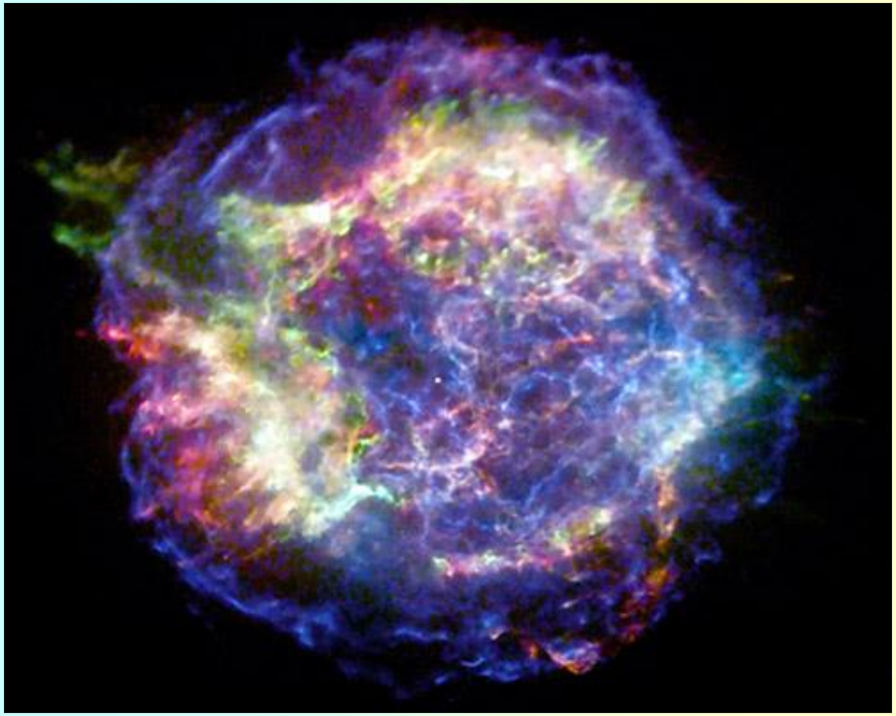
Hester and Cowen, Hubble pictures, 1995

Cloud stiffness may be due to:

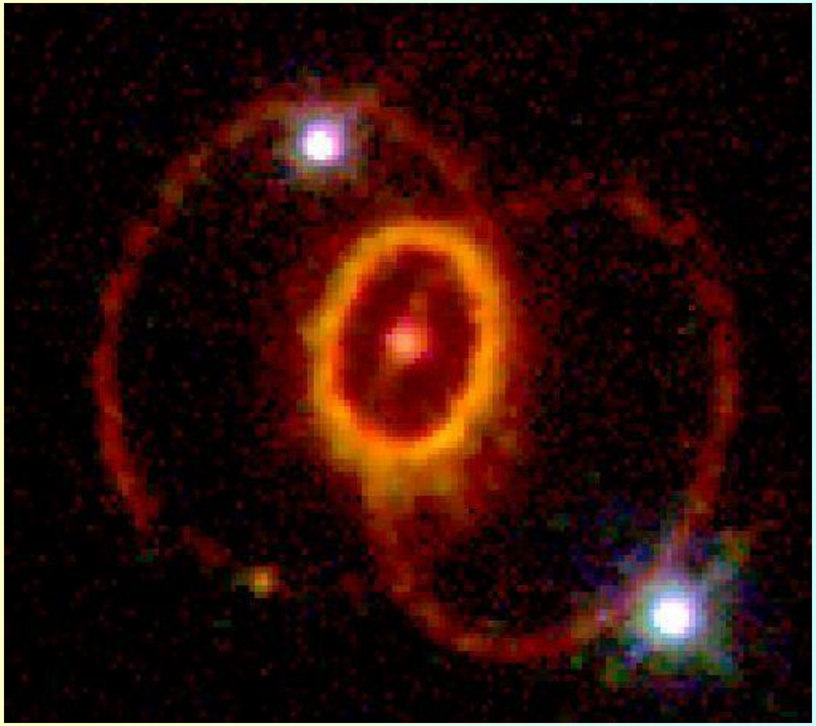
- magnetic pressure by a large-scale primordial magnetic field (Ryutov et al. 2004)
- ablation pressure by ionizing radiation of nearby stars (Spitzer, 1978)

Supernovae

- Supernovae are a central problem in astrophysics.
- Data of supernova remnants encapsulate information on the processes of stellar evolution and nucleosynthesis.



Kepler's supernova [discovered in 1604]

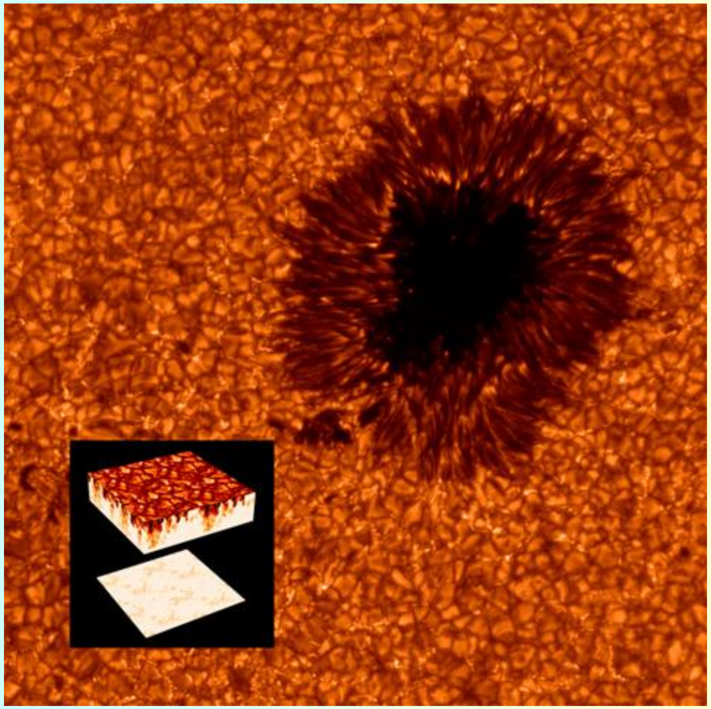


1987 supernova [Burrows, NASA,1994]

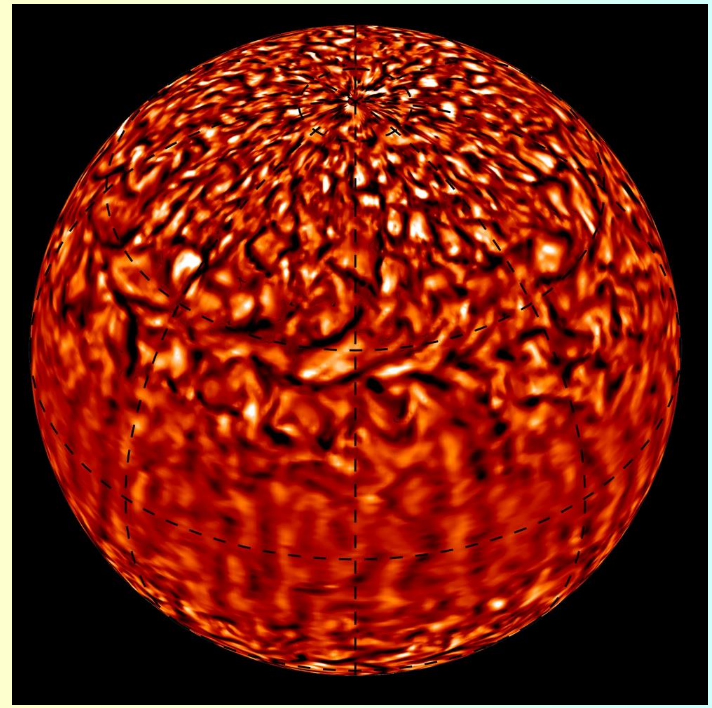
- type Ia: RT mixing dominates propagation of the nuclear flame front and provides conditions for synthesis of intermediate mass and iron peak elements.

- type II: RT mixing of the outer and inner layers of the star provides conditions for synthesis of heavy mass elements.

Convection in stellar interiors and in the Sun



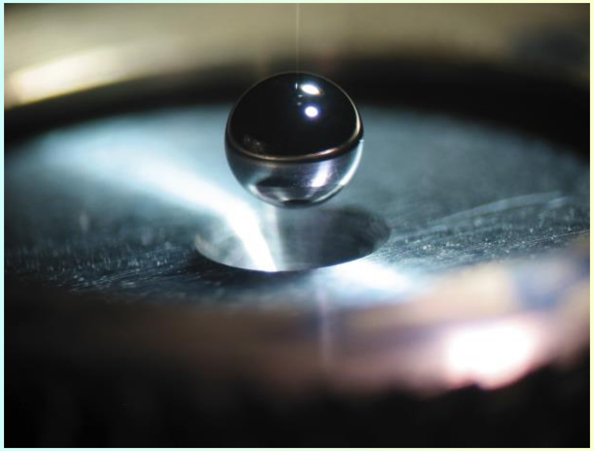
Solar surface, LMSAL, 2003



Solar convection, NASA simulations, 2015

Observations indicate:
- dynamics at the Solar surface depends on convection in the interior;
- Solar convection is influenced by downdrafts and by interfacial mixing.

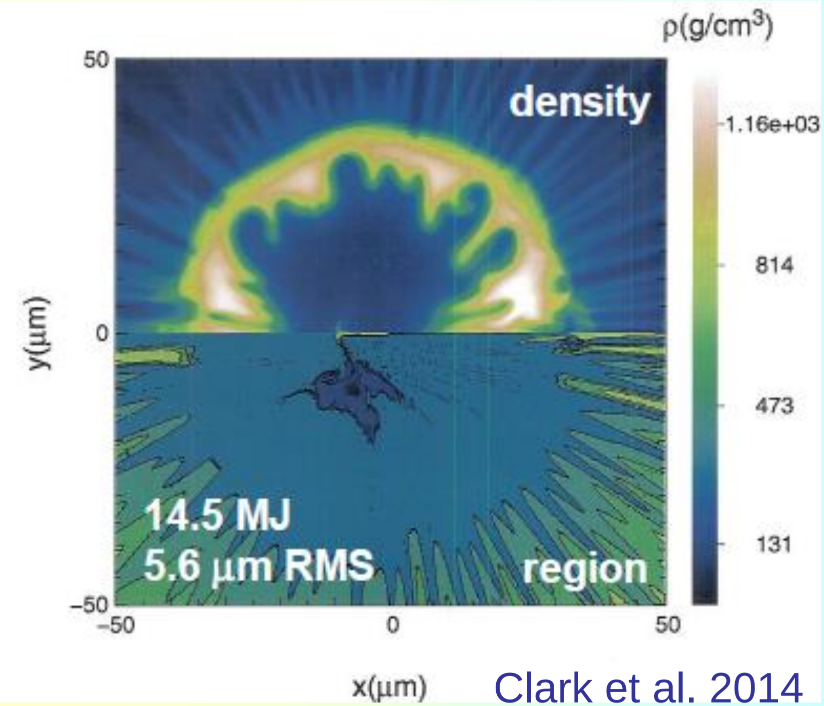
Fusion in Plasmas



NIF target is ~2mm

- For nuclear fusion reaction, the 'fuel' should be hot and dense plasma.
- For plasma compression, one applies
  - magnetic implosion (ITER);
  - laser implosion (NIF), 2022 success.
- Fluid instabilities
  - occur during the implosion process;
  - prevent the formation of 'hot spot'.

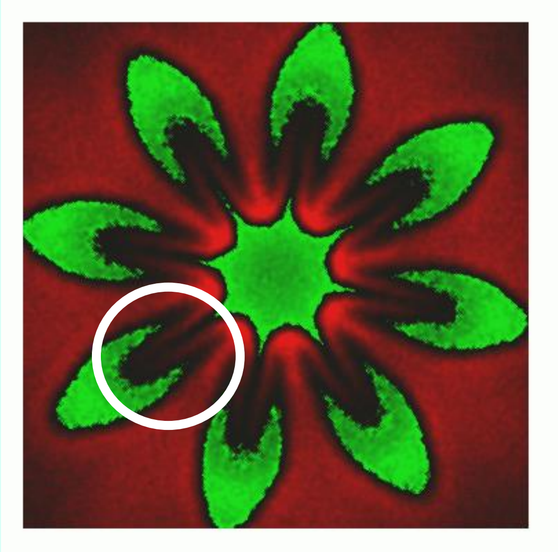
Nishihara et al. 1994

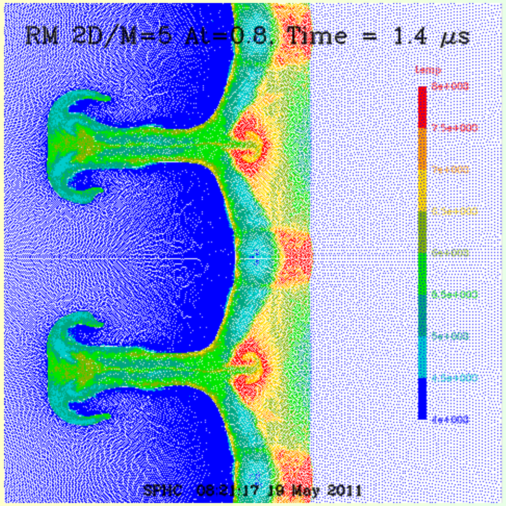


Clark et al. 2014

Nanofabrication: Materials processing

MD simulations of material melting ~4 10^6 LJ atoms (bottom), ~50 nm, 0.2 um, ps





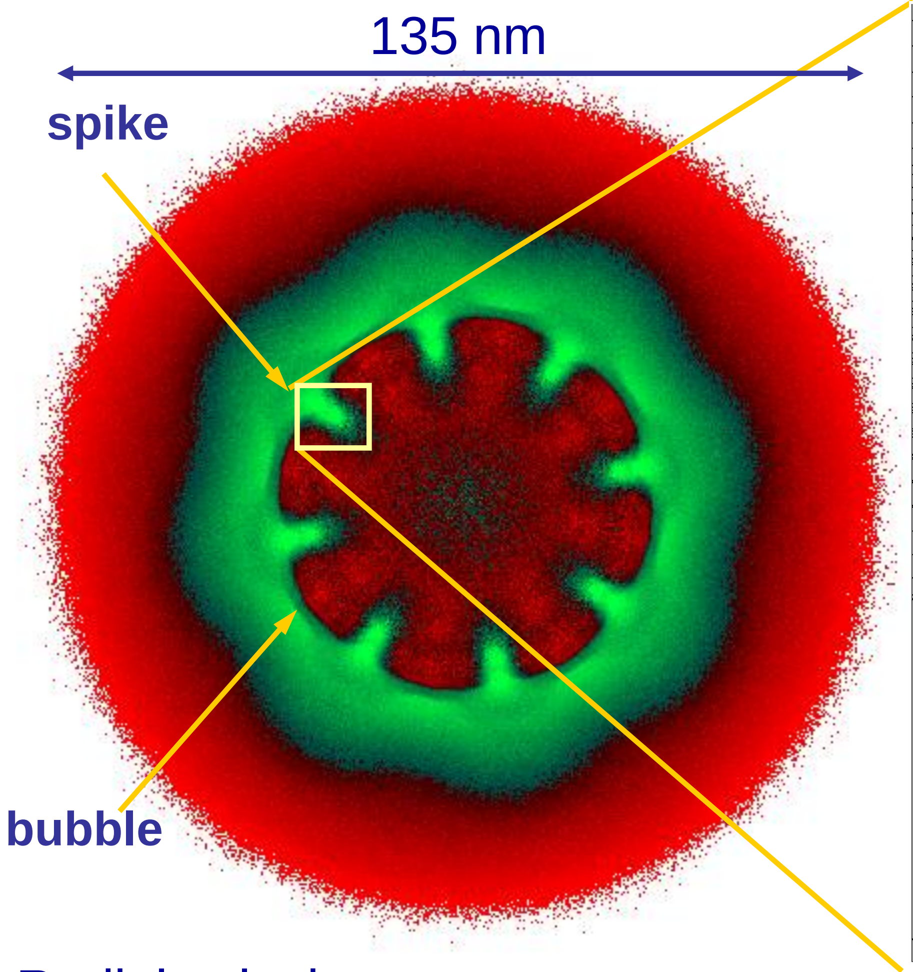
MD simulations ~ 2 x 10^8 LJ atoms (left) and SPH simulations ~ 10^5 - 10^6 particles of the Richtmyer-Meshkov instability

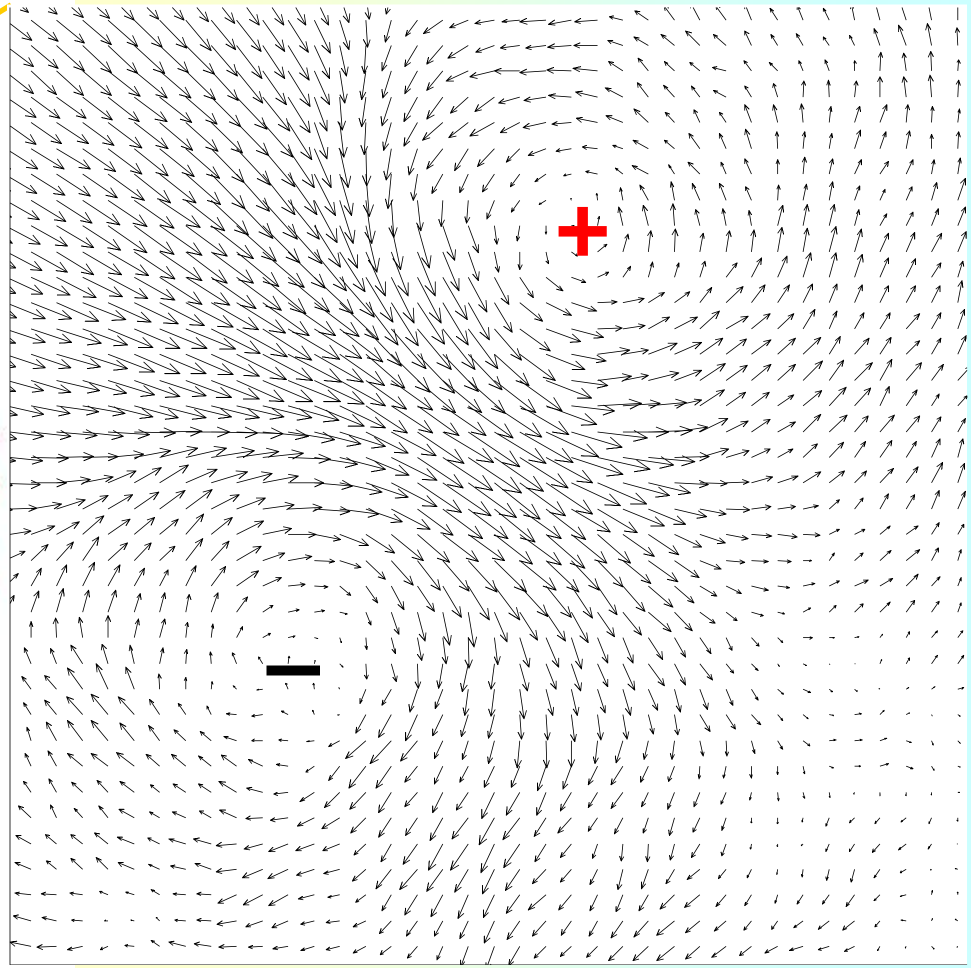
[Zhakhovsky et al. 2019, Dell et al. 2017; Pandian et al. 2017; Stanic et al.2012]

RMI: MD, LJ liquids, interfacial vortical structures

~ 4 x 10^6 LJ atoms

Two Lennard-Jones liquids: $\rho_1/\rho_2 = 16$





Radial velocity map:

green – to center
red – from center

Velocity field:

each arrow is an average of ~100 atoms

Zhakhovskii, Zybin, Bringa, Abarzhi, Remington, Nishihara 2006

Rayleigh-Taylor dynamics

What is known and unknown?

Rayleigh-Taylor dynamics

$$g = G$$

- linear regime

$$h \sim h_0 \exp(t/\tau)$$

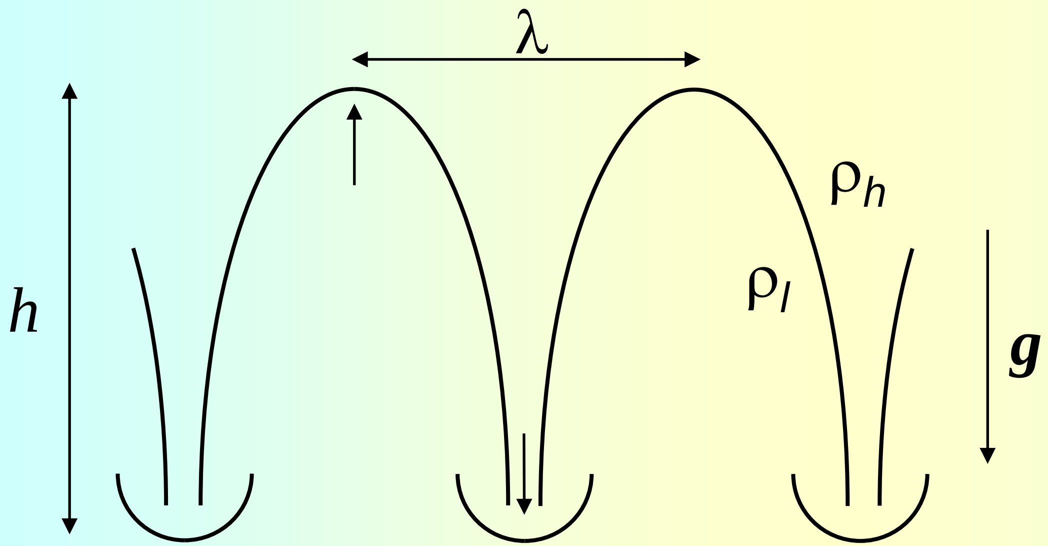
$$\tau \sim \sqrt{\lambda(\rho_h + \rho_l)/G(\rho_h - \rho_l)} \qquad \lambda \sim (\nu^2/g)^{1/3}$$

- nonlinear regime
  light (heavy) fluid penetrates
  heavy (light) fluid in bubbles (spikes)

$$h \sim \lambda(t/\tau)$$

- self-similar mixing

$$h \sim Gt^2$$



RT flow is characterized by:

- ✓ large-scale structure
- ✓ small-scale structures
- ✓ energy transfers to large and small scales

Theoretical problem

- **Conservation of mass, momentum and energy in the bulk**

$$\partial\rho/\partial t + \partial\rho v_i/\partial x_i = 0 \qquad \partial\rho v_i/\partial t + \partial\rho v_i v_j/\partial x_j + \partial P/\partial x_i = 0$$

$$\partial E/\partial t + \partial(E+P)v_i/\partial x_i = 0 \qquad P = s\rho e$$

$$(x_i, t) = (x, y, z, t) \qquad (\rho, \mathbf{v}, P, E) \qquad E = \rho\left(e + \mathbf{v}^2/2\right) \qquad W = e + P/\rho$$

$$P \to P - \rho g z \qquad g = G t^a \qquad \left(-\nu\, \partial^2 v_i/\partial x_j^2\right)$$

- **Freely evolving interface**

$$\theta(x, y, z, t) = 0$$

- **Boundary conditions at the interface**

$$\theta = -z + z^*(x, y, t)$$

- **Boundary conditions at the outside boundaries of the domain**
- **Boundary conditions set by initial conditions defining symmetries and scales**
- **The problem is more challenging than the Millennium's Navier-Stokes problem.**

Boundary value problem

$$(\rho, \mathbf{v}, P, E) = (\rho, \mathbf{v}, P, E)_h H(\theta) + (\rho, \mathbf{v}, P, E)_l H(-\theta)$$

$$\theta(x, y, z, t) = 0 \qquad \mathbf{n} = \nabla\theta / |\nabla\theta| \qquad \boldsymbol{\tau} \cdot \mathbf{n} = 0 \qquad \theta = -z + z^*(x, y, t)$$

- Boundary conditions at the freely evolving interface

$$[\mathbf{v} \cdot \mathbf{n}] = 0$$
$$[P] = 0$$
$$[\mathbf{v} \cdot \boldsymbol{\tau}] = any$$
$$[W] = any$$

- Boundary conditions at the outside boundaries

$$\mathbf{v}\big|_{z \to -\infty(+\infty)} = 0$$

- Boundary conditions set by the initial conditions

$$\mathbf{v} \cdot \mathbf{n}_{\mathrm{B}}\big|_{\mathbf{x} = (\mathbf{r}_{\mathrm{B}}, z)} = 0$$

- Initial conditions are initial perturbations; they define symmetries and scales.

Group theory

- **Group is a set of elements**

$$\forall A,B \in \mathbf{G}: \quad \exists C \in \mathbf{G} \quad A \cdot B = C$$

$$\forall A,B,C \in \mathbf{G}: \quad A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

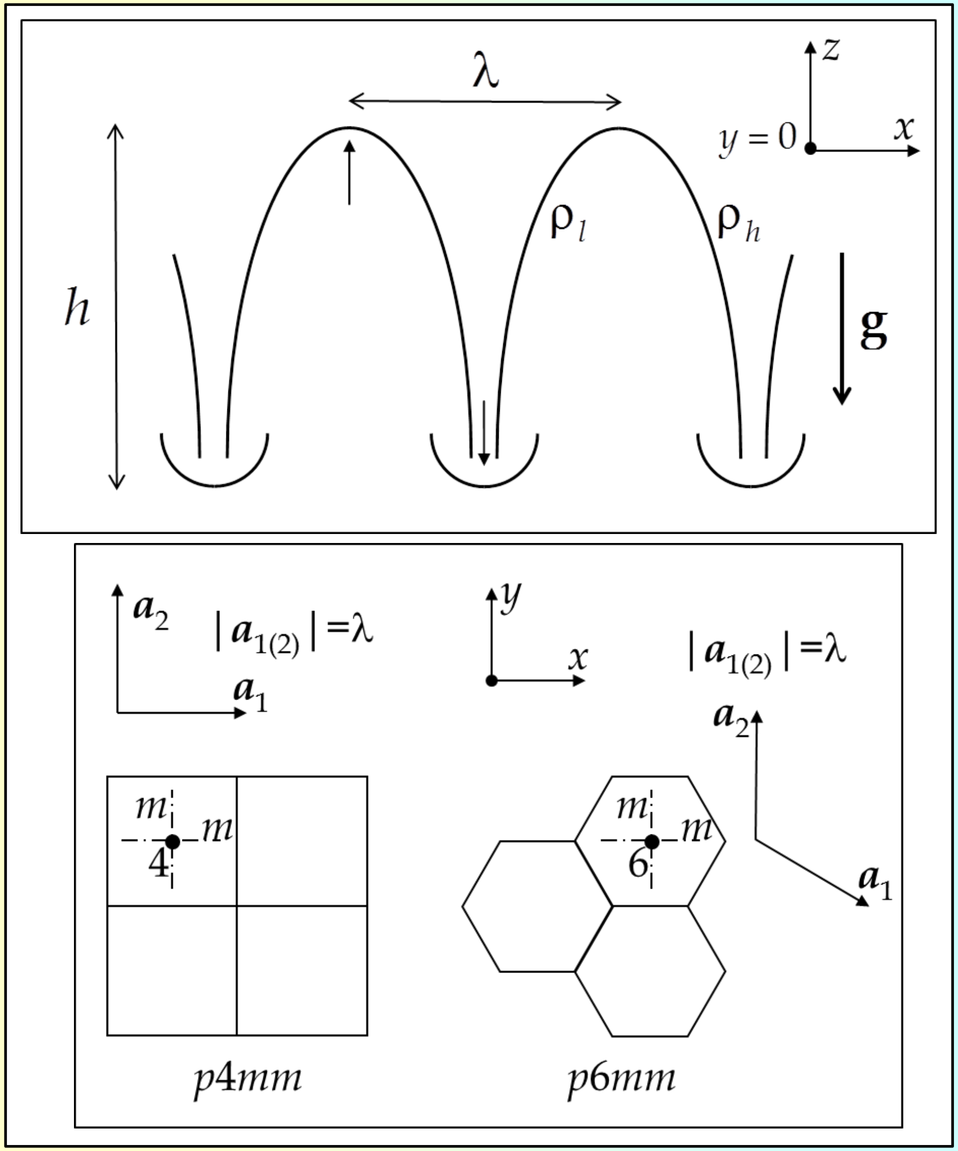
$$\exists E \in \mathbf{G}: \quad A \cdot E = E \cdot A = A$$

$$\forall A \in \mathbf{G}: \quad \exists A^{-1} \in \mathbf{G}: \quad A \cdot A^{-1} = E$$

- **Group theory approach in RT/RM dynamics**

- Scale-dependent dynamics:
- Space groups are applied to reduce governing equations to a dynamical system

- Scale-dependent & self-similar dynamics:
- Scaling transformations are realized in the momentum model.



Space groups with symmetries of square and hexagon

Group theory: dynamical system methodology

- Derive the structure of flow fields $\left(\Phi, p, \bar{\rho}, \Psi\right) = \left(\hat{\Phi}, \hat{p}, \hat{\rho}, \hat{\Psi}\right) exp\left(ikx - Kz + \Omega t\right)$

$$M_Z \mathbf{Z} = 0 \qquad \mathbf{Z} = \left(\Phi, p, \bar{\rho}, \Psi\right)^{\mathrm{T}}$$

$$M_Z : 4 \times 4$$

$$K = \pm\sqrt{k^2 + \gamma\Omega^2/c^2}, \mathbf{Z} = \hat{\Phi}\left(1, -\rho_0\Omega, *, 0\right)$$

$$K = \pm k, \quad \mathbf{Z} = \hat{\Phi}\left(1, -\rho_0\Omega, 0, 0\right)$$

- Flow fields in the bulk are $\mathbf{v}_{h(l)} = \nabla\Phi_{h(l)}$
- Solve boundary value problem $\mathbf{R} = \left(\Phi_h, \Phi_l, \left(\Omega/k\right)z^*\right)^{\mathrm{T}}$
- Use irreducible presentations of the group for the fields' global expansion $p6mm$

$$\Phi_h\left(\mathbf{r}, z, t\right) = \sum_{n=0}^{\infty} \Phi_n(t)\left(z + \frac{exp\left(-mkz\right)}{3mk}\sum_{i=1}^{3} cos\left(m\mathbf{k}_i\mathbf{r}\right)\right) + f_h(t) + cross\ terms$$

- And for the local expansion in a vicinity of a regular point – tip of the bubble or spike

$$\theta = -z + z^*\left(x, y, t\right) \qquad z^* = \sum_{N=1}^{\infty} \zeta_N(t)\mathbf{r}^{2N} + cross\ terms$$

- Define moments $M_n = \sum_{m=1}^{\infty} \Phi_n k^n m^n + cross\ terms$

Group theory: dynamical system

- Derive dynamical system in terms of momentum and surface variables $N=1$

$$\rho_h\left(\dot{\zeta}-2\zeta M_1-\frac{M_2}{4}\right)=0,\quad \rho_l\left(\dot{\zeta}-2\zeta\tilde{M}_1+\frac{\tilde{M}_2}{4}\right)=0,\quad M_1-\tilde{M}_1=arbitrary$$

$$\rho_h\left(\frac{\dot{M}_1}{4}+\zeta\dot{M}_0-\frac{M_1^2}{8}+\zeta g\right)=\rho_l\left(\frac{\dot{\tilde{M}}_1}{4}-\zeta\dot{\tilde{M}}_0-\frac{\tilde{M}_1^2}{8}+\zeta g\right),\quad M_0=-\tilde{M}_0=-v.$$

- Find solutions in the early and late time, investigate their convergence and stability.

- Variable acceleration $g=Gt^a$

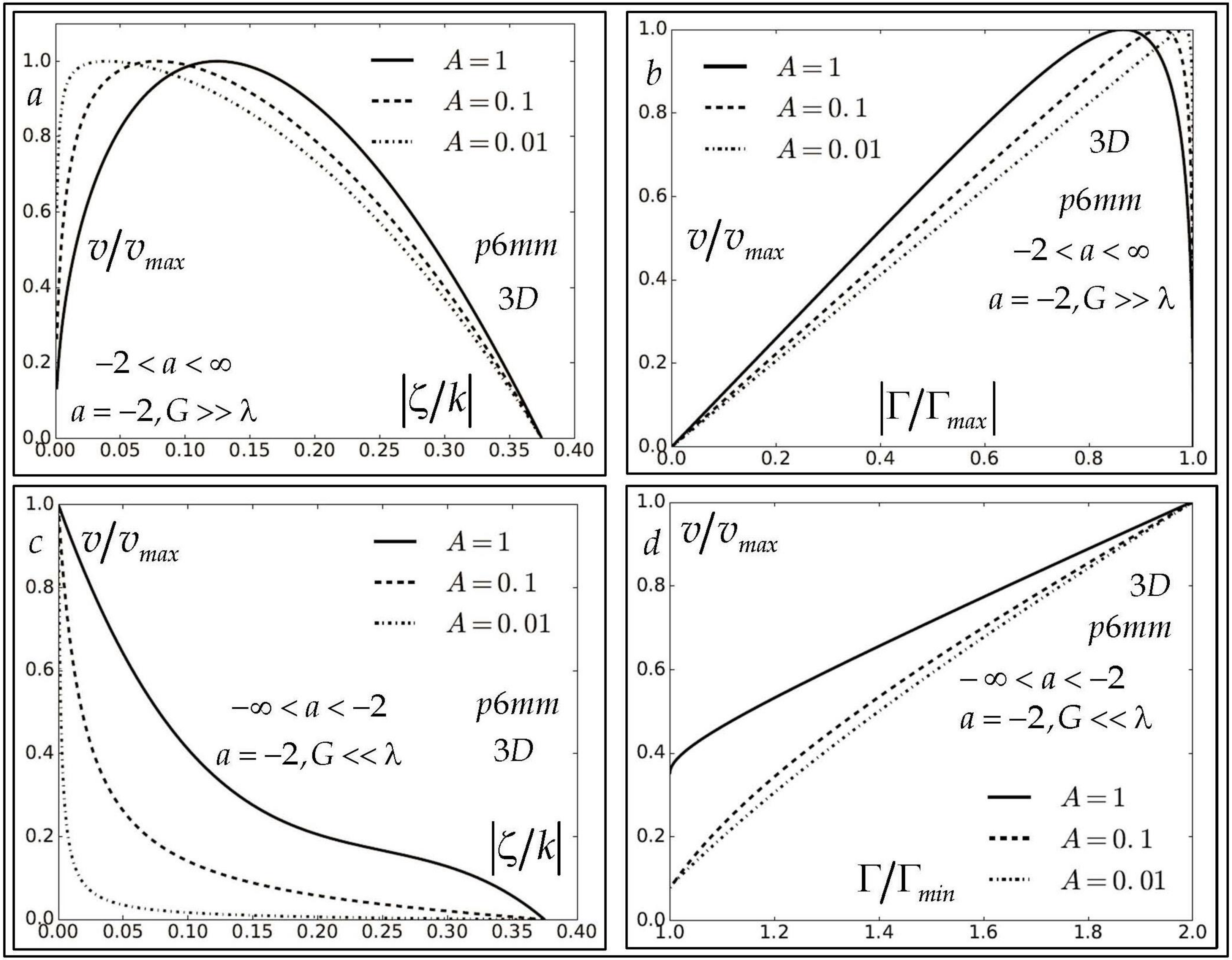
- For the early time linear dynamics, the solutions are

$$RT:\quad a>-2:\quad z_0=c_\pm\frac{1}{k}\left(\frac{t}{\tau_G}\right)^{1/2}I_{\pm 1/2s}\left(\frac{\sqrt{A}}{s}\left(\frac{t}{\tau_G}\right)^s\right),\quad \tau_G=(kG)^{-1/(a+2)},\quad v=\dot{z}_0\ ;$$

$$RM:\quad a<-2:\quad z_0=\frac{1}{2Ak}ln\left(c_+\left(\frac{t}{\tau_0}\right)+c_-\right),\quad \tau_0=(kv_0)^{-1},\quad v=\dot{z}_0.$$

Group theory: dynamical system

For the late time nonlinear dynamics, the RT (top) and RM (bottom) solutions are:

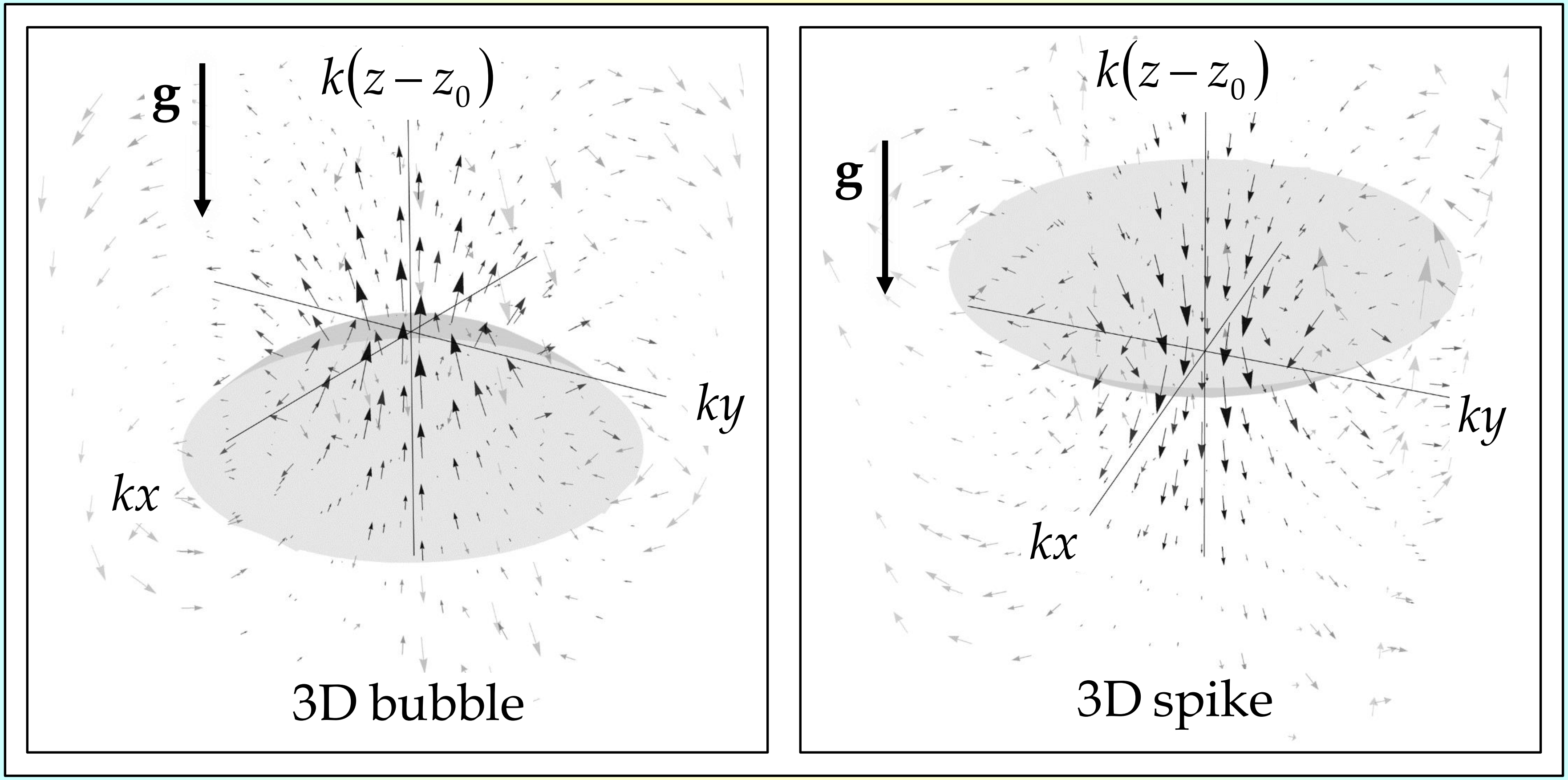


2019
PNAS

[2021 – 1995]

Group theory: flow fields in RT/RM dynamics

RT/RM dynamics is essentially interfacial.



RT/RM dynamics is essentially interfacial with:

- intense fluid motion near the interface;
- effectively no motion far away from the interface;
- shear-driven vortical structures at the interface.

In HEDP RT/RM dynamics is the superposition of 2 motion;

- background motion of the bulk;
- growth of interface perturbations.

2023 Abarzhi et al.; 2019 Abarzhi et al., since 1995

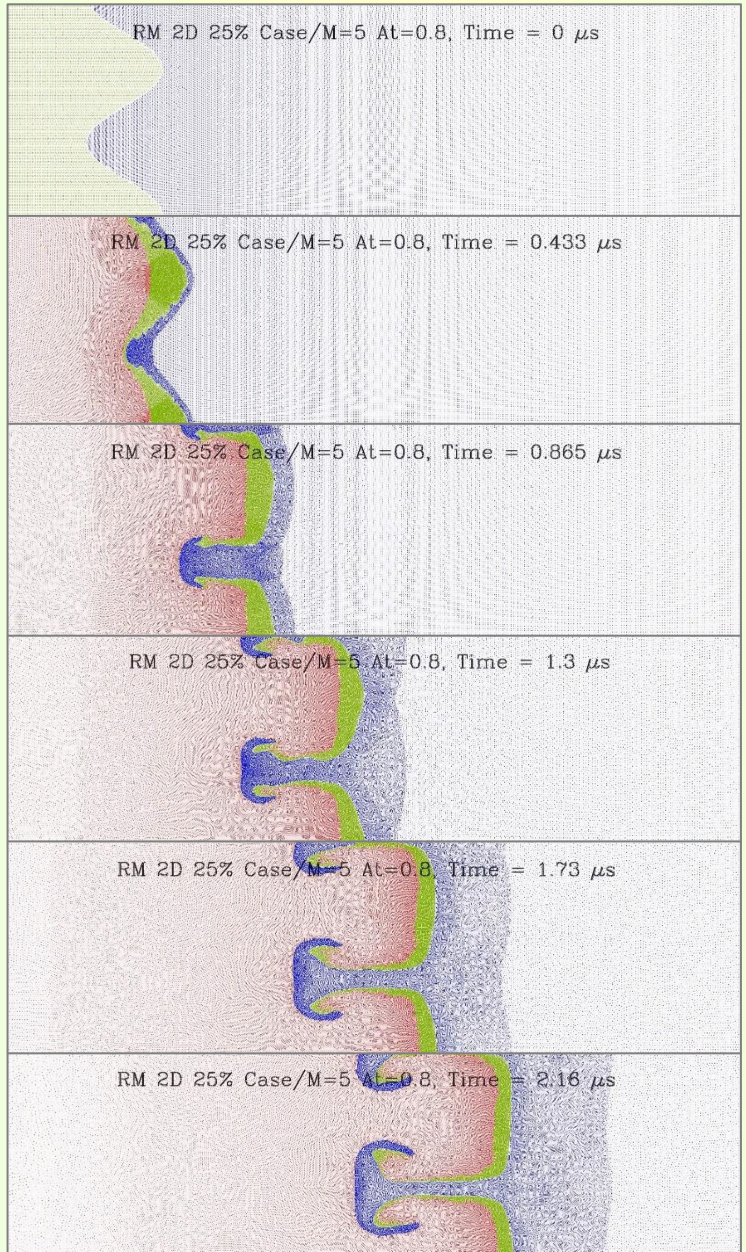
2019 Remington et al.; 2012 Stanic et al.;

Strong shock driven RM (RT) dynamics

Post-shock dynamics is the superposition of the:
- Back-ground motion
- Interfacial growth

As an estimate:
- The background velocity is ~ 10% shock velocity.
- The initial growth-rate is ~10% of background velocity.
- The nonlinear / mixing velocity is <10% of the initial growth-rate

SPH simulations



✓ Back-ground motion:
- occurs even for planar interface;
- is super (hyper) sonic for strong shocks.

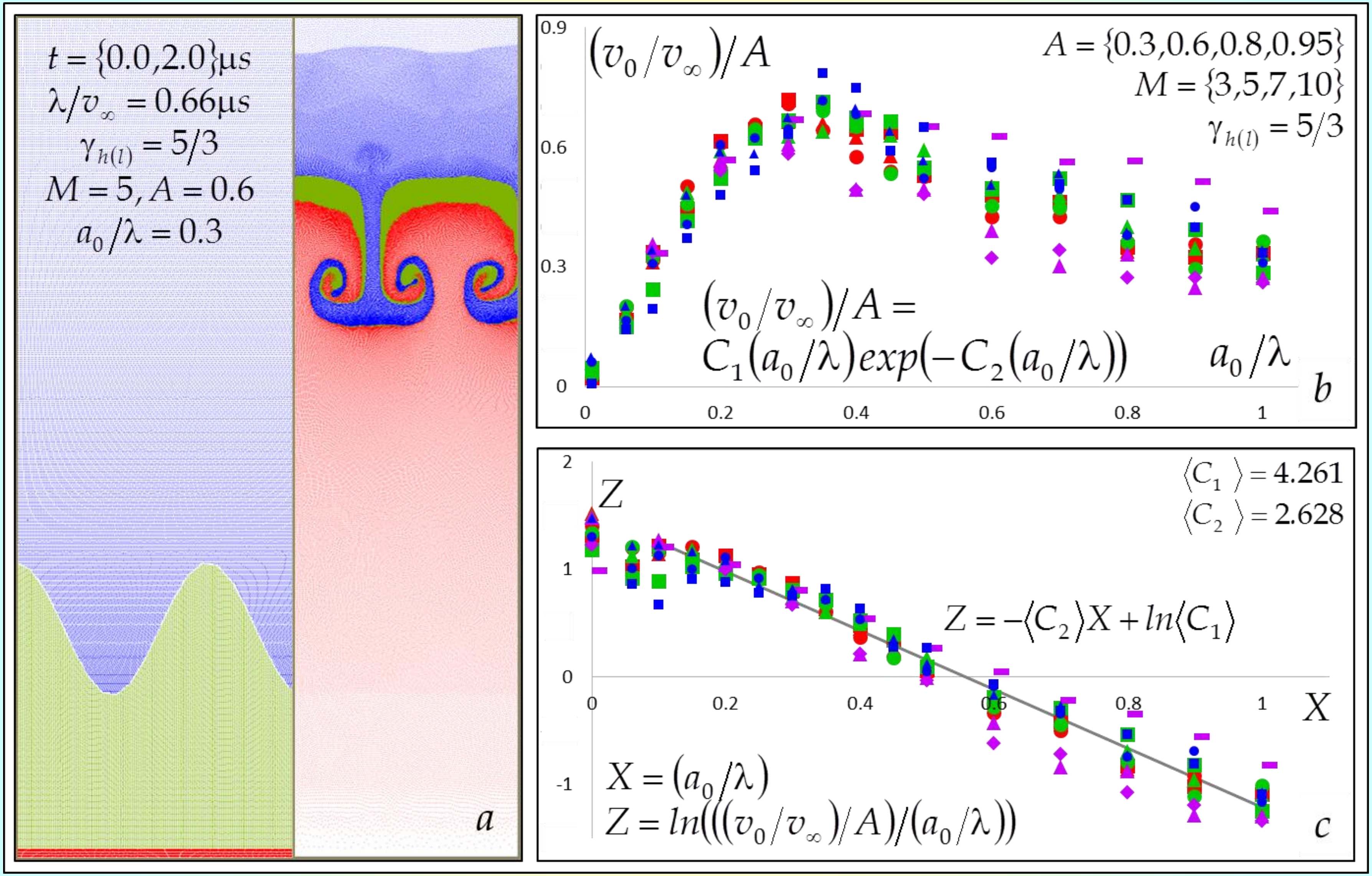
✓ Interfacial growth:
- occurs only for perturbed interface (fields)
- is sub-sonic (nearly incompressible)

$$M = 5$$
$$A = 0.8$$
$$(a_0/\lambda) = 0.25$$

2021 Wright & Abarzhi; 2017 Dell et al.; 2015 Dell et al.; 2013 Stanic et al.; 2012 Stanic et al.;

RMI growth-rate for a finite initial amplitude

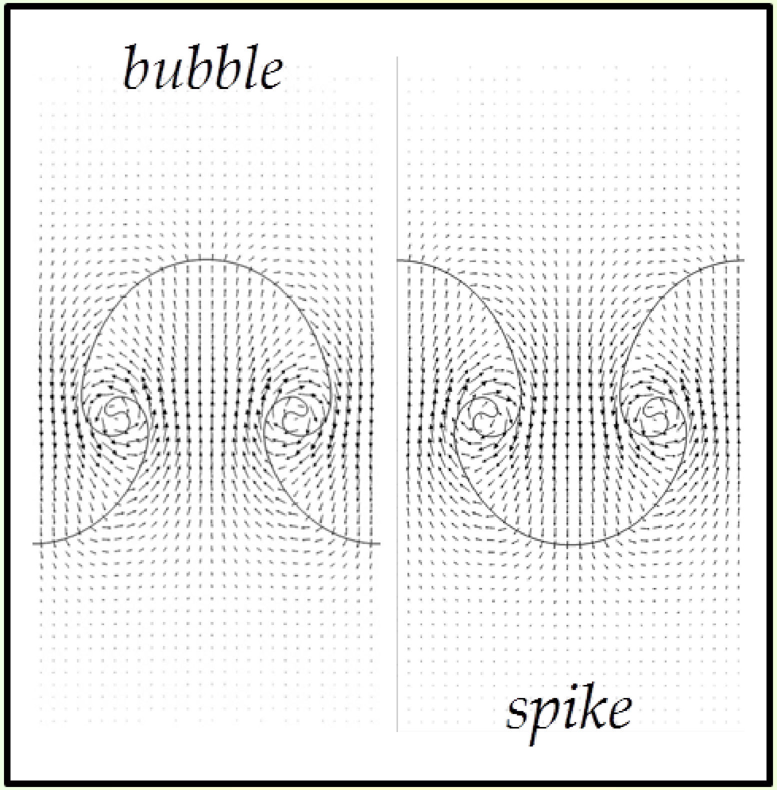
In a broad parameter regime the model agrees (within 3 signif. digits) with experiments.



2019 Abarzhi et al.; 2017 Dell et al.

Velocity fields in nonlinear RT/RM dynamics

RT/RM velocity field for variable accelerations



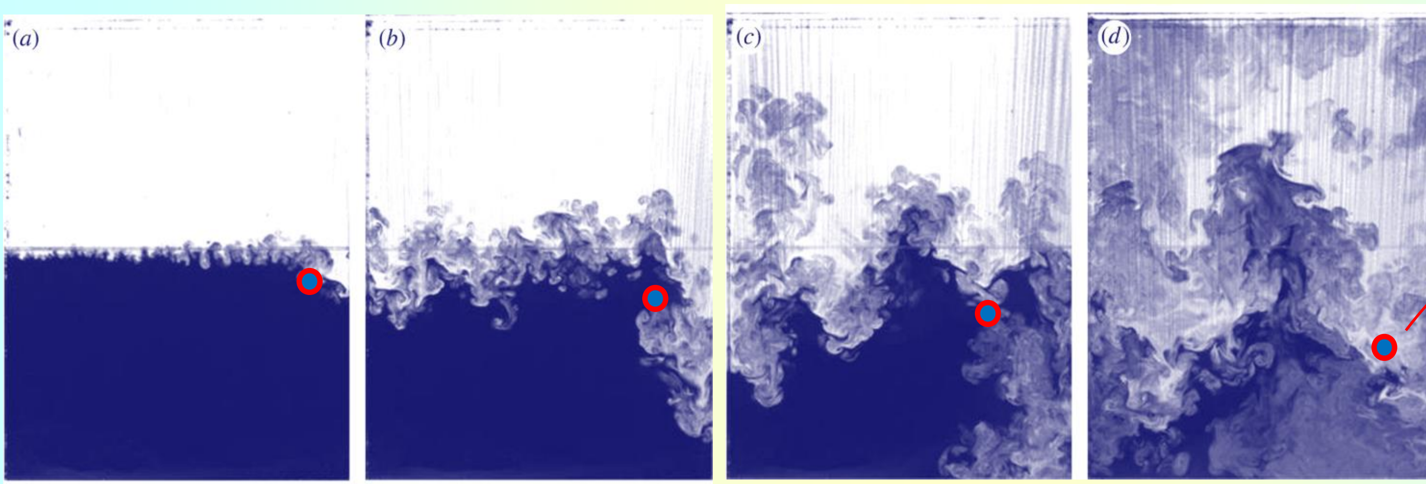
The group theory finds: RT / RM dynamics is interfacial:
intense motions & vortical structures at interface.
The simulations [DNS (DI, VoF), SPH] confirm the theory.

2023 Chan et al.; 2019 Abarzhi et al.; 2015 Dell et al.; 2012 Stanic et al.

Physics of RT / RM dynamics

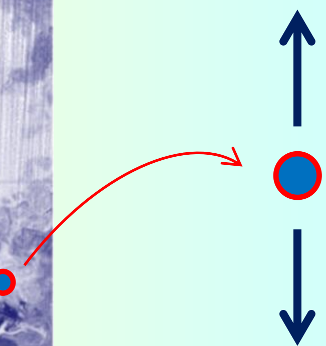
Dynamics of a parcel of fluid is driven by the specific balance per unit mass of the rate of momentum gain and the rate of momentum loss.

Andrews & Dalziel 2010



initial → final

Momentum is lost due to dissipation.



Momentum is gained due to buoyancy.

rate of change of momentum = force

rate of change of energy = power = force x velocity

rate of momentum gain = buoyant force = rate of energy gain / velocity

rate of momentum loss = ‘dissipation’ force = rate of energy dissipation / velocity

Group theory: momentum model of RT/RM dynamics

RT / RM dynamics is driven by the specific balance per unit mass of the rate of momentum gain and rate of momentum loss.

The rates are projections of vectors along the gravity.

$$\frac{dh}{dt} = v \qquad \frac{dv}{dt} = \tilde{\mu} - \mu \qquad \frac{dv}{dt} = \frac{\tilde{\varepsilon}}{v} - \frac{\varepsilon}{v}$$

rate of momentum gain	$\tilde{\mu} = \tilde{\varepsilon}/v$	buoyant force
rate of energy gain	$\tilde{\varepsilon} = v\,Bg$	$Bg \to g$
rate of momentum loss	$\mu = \varepsilon/v$	dissipation force
rate of energy dissipation dimensional & Kolmogorov	$\varepsilon = C\,v^3/L$	L - length-scale
Power and force are related	$\tilde{\varepsilon} = v\,\tilde{\mu},\ \varepsilon = v\,\mu$	$B, C > 0\ (< 0)$

Momentum model has the same symmetries as conservation laws.
Its parameters are directly derived from the governing equations.

Momentum model parameters

Group theory approach precisely derives the model parameters from the governing equations.

$$\dot{h} = v, \quad \dot{v} = \tilde{\mu} - \mu \qquad \tilde{\mu} = \frac{\tilde{\varepsilon}}{v}, \quad \mu = \frac{\varepsilon}{v}, \quad \tilde{\varepsilon} = Bgv, \quad \varepsilon = C\frac{v^3}{L}$$

$$L \sim \{\lambda, \lambda, |h|\}, \quad B = \{B_l, B_n, B_m\}, \quad C = \{C_l, C_n, C_m\}$$

- Linear regime $\quad B_l = Akz_0 = -4A\frac{\zeta}{k}, \quad C_l = A/2$
- Nonlinear regime $\quad B_n = B_n(A, \xi), \quad C_n = C_n(A, \xi)$
- Mixing regime $\quad B_m, \quad C_m$ are free parameters,
  can be independent stochastic processes

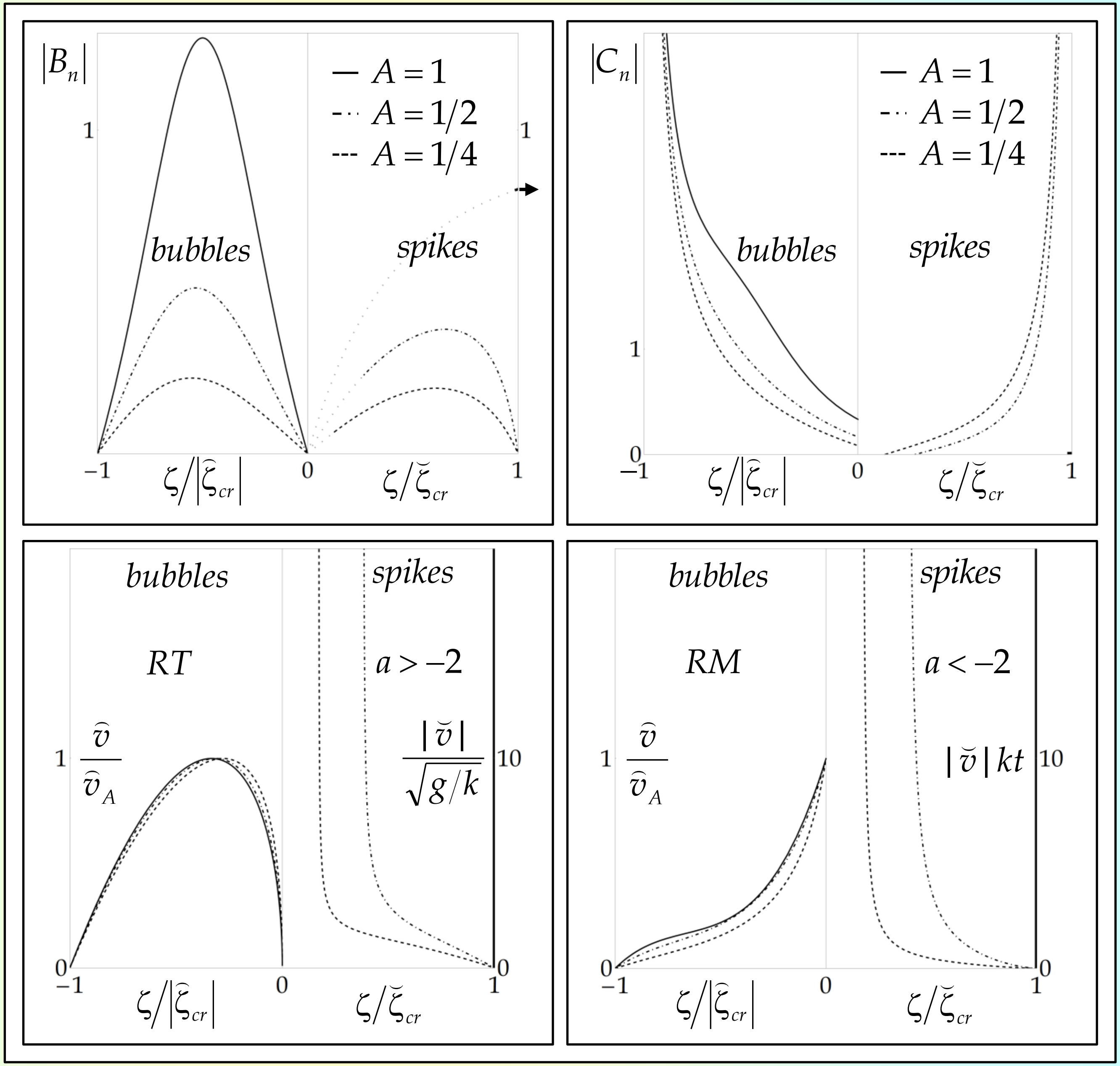
[Abarzhi et al. 2005; Abarzhi et al. 2022; Abarzhi et al. 2023]

Link between the two group theory implementations

- Theory solves analytically RT/RM evolution of bubbles/spikes in the linear, nonlinear, and mixing regimes.

- Buoyancy and drag parameters are precisely derived for RT/RM bubbles and spikes.

- RT/RM flow is interfacial. It has intense motions and vortical structures at the interface.



[Abarzhi et al. 2022; Abarzhi et al. 2023]

Buoyancy and drag parameters

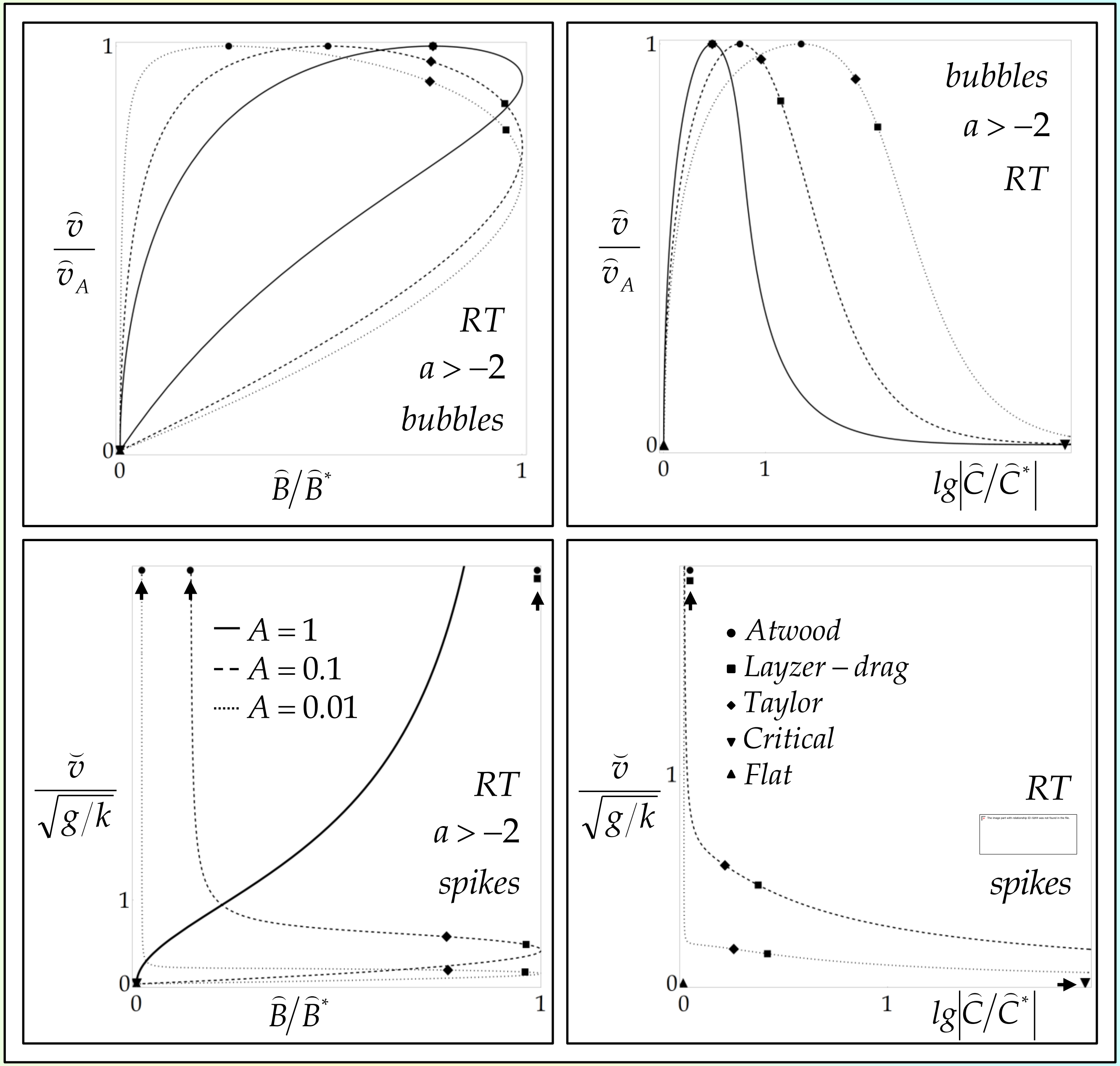
Nonlinear RT dynamics

- Buoyancy and drag parameters for bubbles and spikes are more complex than it traditionally believed.

- There are special solutions, including the Atwood / Taylor / Layzer-drag bubble / spike.

- In RT, these special solutions are consistent 'in numbers' with traditional models.

- The special solutions have interfacial dynamics.



Self-similar RT mixing and self-similar turbulence

- In RT mixing with constant acceleration $g = G$

velocity $v \sim Gt$ 'integral' scale $L \sim Gt^2$

Reynolds number $\mathrm{Re} \sim (vL/\nu) \sim G^2 t^3/\nu$

rate of energy dissipation $\varepsilon \sim v^2/(L/v) \sim G^2 t$

'viscous' scale $l_\nu \sim (\nu^3/\varepsilon)^{1/4} \sim (\nu^3/G^2 t)^{1/4}$

span of scales $L/l_\nu \sim G^{3/2} t^{9/4}/\nu^{3/4}$

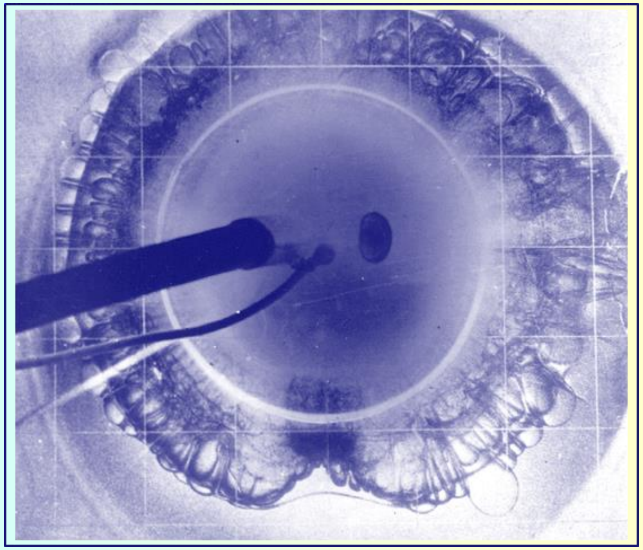
- One may model turbulence effect (assuming it may develop) on RT mixing.

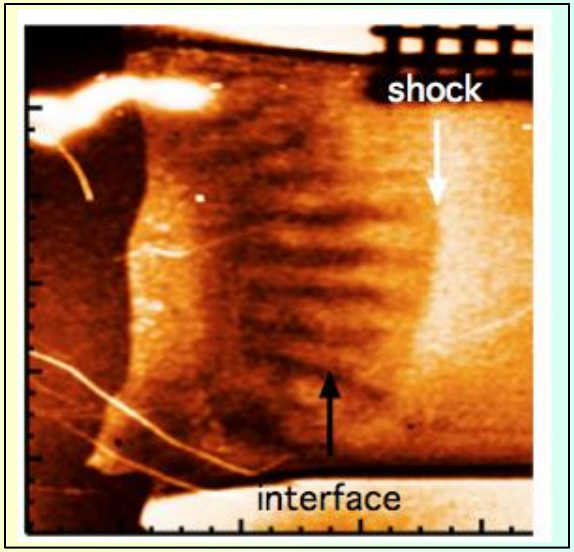
2000s: Chertkov, Zhou, Dimotakis
1990s: Shvarts, Clark & Ristorcelli
1980s: Youngs, Harlow
1970s: Neuvazhaev, Gamalii
1960s: Belenkii & Fradkin

- May RT mixing with constant acceleration be a 'super-turbulence'?

Self-similar RT mixing may keep order

Puzzle of RT mixing was observed in fluids and plasmas.





In fluids, RT mixing is heterogeneous,
RT interface has a density jump.
Reynolds number is ~3.2 x 10^6
[Meshkov 2019, 2013, 2006,1990].

In high energy density plasmas, RT mixing
may keep order.
Reynolds number is >10^6
[Kuranz et al. 2018, 2010; Robey et al. 2003].

Accelerated flows may laminarize.
[Sreenivasan 1982, 1973; Taylor 1929]

Group theory analysis shows for constant acceleration: $g = G$

- RT mixing can exhibit order;
- RT mixing has strong correlations, weak fluctuations, and steep spectra;
- RT mixing differs from Kolmogorov turbulence.

[Abarzhi 2010]

Rayleigh-Taylor interfacial mixing with variable acceleration

Our theoretical approach

- finds special class of self-similar solution for RT mixing with variable acceleration;
- identifies new invariant, scaling and spectral properties of RT mixing;
- explains (qualitatively and quantitatively) existing experiments and simulations;
- suggests new approaches for diagnostics and control in experiment.

Asymptotic dynamics for RT / RM mixing

$$g = Gt^{a}$$

• characteristic length-scale is the amplitude $L = h$ $h = Bt^{b}$

RT	$a > -1$	$b_{RT} = a + 2$	$B_{RT} = \frac{G(a_{cr} + 2)}{(a+2)(a - a_{cr})}$	$\dot{v} > 0$
	$a = -1$	$b_{RT} = 1$	$B_{RT} = \frac{G}{C} = \frac{G(a_{cr} + 2)}{(-a_{cr} - 1)}$	$\dot{v} = 0$
	$a_{cr} < a < -1$	$b_{RT} = a + 2$	$B_{RT} = \frac{G(a_{cr} + 2)}{(a+2)(a - a_{cr})}$	$\dot{v} < 0$
RM	$a < a_{cr}$	$b_{RM} = a_{cr} + 2$	$B_{RM} = H_0\left((1+C)\left(\frac{V_0}{H_0}\right)\right)^{1/(1+C)}$	$\dot{v} < 0$

- critical exponent is $a_{cr} = -2 + (1+C)^{-1}$ $a_{cr} \in (-2, -1)$ $C \in (0, +\infty)$
- deterministic conditions are H_0, V_0 $G_{cr} = H_0 (V_0 / H_0)^{(a_{cr}+2)}$

[Abarzhi 2021; Abarzhi et al. 2019, Pandian et al. 2017, Swisher et al. 2015]

RT / RM mixing with variable acceleration

- For variable acceleration, self-similar mixing can be either RT mixing or RM mixing.

  - RT mixing is driven by acceleration: $a > a_{cr}$
    - exponent is set by acceleration;
    - pre-factor depends on acceleration parameters and on drag

  - RM mixing is driven by drag / initial growth-rate: $a < a_{cr}$
    - exponent is set by drag
    - exponent is independent of acceleration parameters;
    - pre-factor is set by deterministic (initial) conditions

- RM mixing has quicker dynamics than the acceleration prescribes.

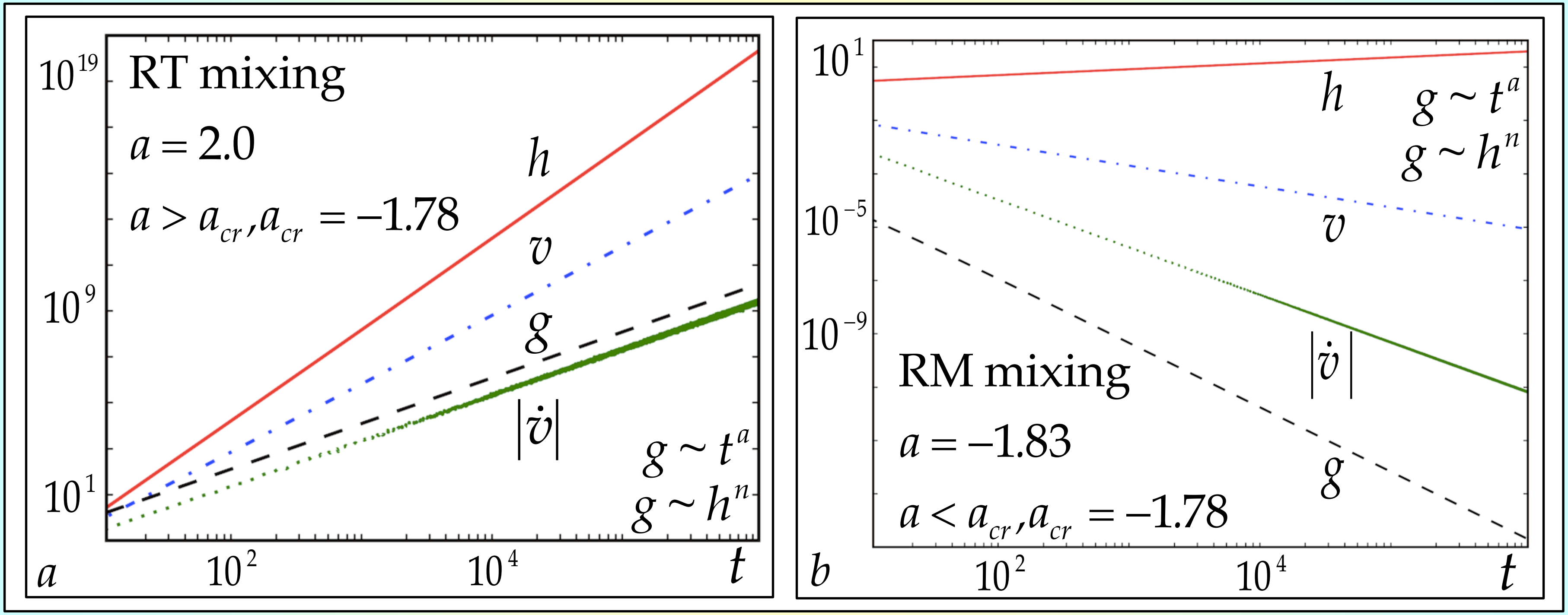
  - Transition from RT mixing to RM mixing is singular in nature.

$$g = Gt^{a} \qquad a_{cr} = -2 + (1 + C)^{-1} \qquad a_{cr} \in (-2, -1)$$

$$G \sim G_{cr} \qquad a \sim a_{cr}$$

  - RT/RM mixing have properties different from other self-similar processes.

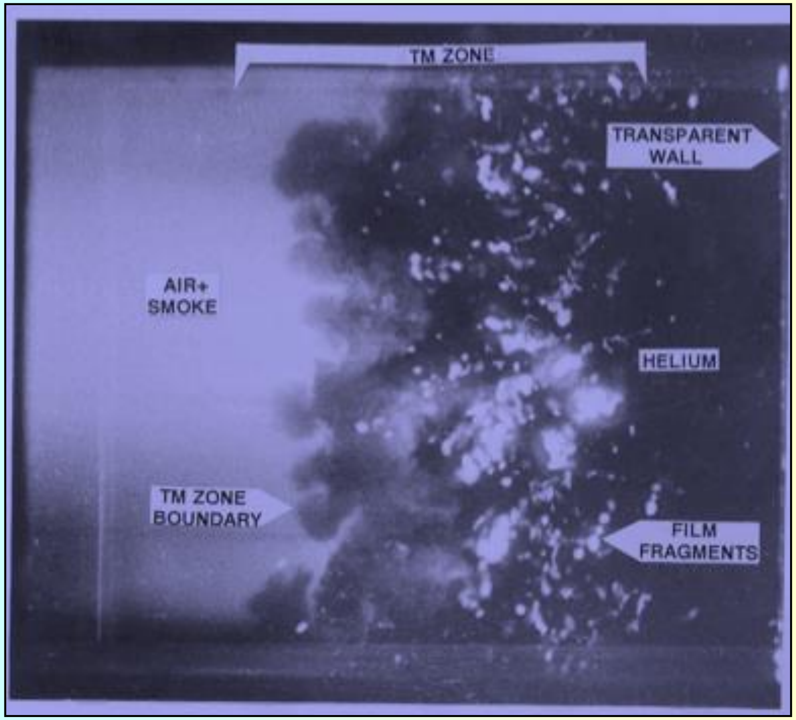
Self-similar RT / RM mixing



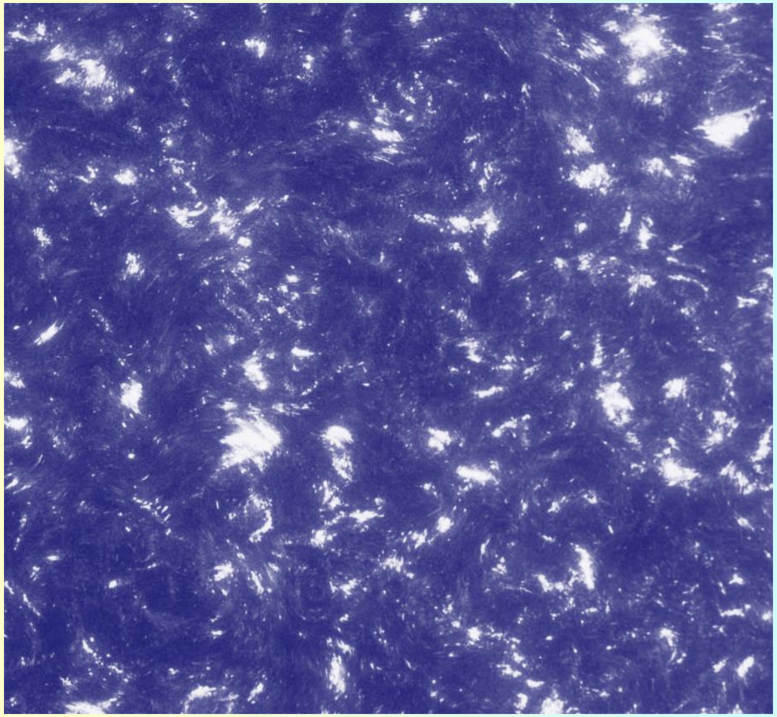
[Abarzhi et al. 2019, Pandian et al. 2017, Swisher et al. 2015]

RT/RM versus Kolmogorov turbulence

Are they any similar?



Anisotropic heterogeneous RT mixing
Meshkov 1990



Isotropic homogeneous turbulence
[Sreenivasan 1999]

Group theory approach finds that
RT/RM mixing and Kolmogorov turbulence are principally different

Scale-invariant turbulence: Kolmogorov theory

Isotropy, homogeneity, locality:

$$\rho = const$$

$$\mathrm{Re} = \frac{vL}{\nu} \to \infty$$

Conservation laws: Navier-Stokes

$$\partial v_i / \partial x_i = 0$$

$$\partial v_i / \partial t + \partial v_i v_j / \partial x_j + \partial \left(P/\rho \right) / \partial x_i = 0 \qquad \left(- \nu \, \partial^2 v_i / \partial x_j^2 \right)$$

- no boundaries
- no memory of initial conditions

Invariance properties:
- Galilean transformation
- temporal translations
- spatial translations, rotations, reflections
- scaling invariance (Kolmogorov 1941)

Turbulence is driven by an external energy source with constant power.

$$L \to LK \quad t \to tK^{1-n} \quad v \to vK^n \quad \nu \to \nu K^{1+n}$$

$$n = 1/3 \qquad \varepsilon = \nu \left(\partial v_i / \partial x_j \right)^2$$

- rate of change of specific kinetic energy $\varepsilon \sim v^3 / L$

specific energy per unit time

Scale-invariant RT/RM mixing: group theory

Conservation laws: three-dimensional Navier-Stokes equations

$$\partial\rho/\partial t + \partial\rho v_i/\partial x_i = 0 \qquad \partial\rho v_i/\partial t + \partial\rho v_i v_j/\partial x_j + \partial P/\partial x_i = 0$$

$$\partial E/\partial t + \partial(E+P)v_i/\partial x_i = 0 \qquad P \to P - \rho g z \qquad \left(-\nu\,\partial^2 v_i/\partial x_j^2\right)$$

+ boundary conditions at the interface + initial conditions

Invariance properties of RT mixing: $\quad g = Gt^a$

- non-inertial
- translations, rotations and reflections in plane
- scaling invariance

$$L \to LK \qquad t \to tK^{1-n} \qquad v \to vK^n \qquad \nu \to \nu K^{1+n} \qquad G \to GK^{n(a+2)-(a+1)}$$

$$n = (a+1)/(a+2) \qquad \mathrm{M} \qquad \partial^a/\partial t^a \qquad \mu_i = \nu\left(\partial^2 v_i/\partial x_j^2\right)$$

$$\mathrm{M} \sim \mu/t^a$$

- modified rate of change of specific momentum $\quad \mathrm{M} \sim v^{(a+2)}/L^{(a+1)}$

Scaling invariance of self-similar dynamics

For self-similar Kolmogorov turbulence
the measure of scaling invariance is the rate of energy dissipation:

$$\varepsilon \sim \frac{v^3}{L} \sim \frac{v_l^3}{l} \qquad n = \frac{1}{3}$$

For self-similar RT mixing
the measure of scaling invariance is the modified rate of momentum:

$$\mathrm{M} \sim \frac{v^{a+2}}{L^{a+1}} \sim \frac{v_l^{a+2}}{l^{a+1}} \qquad n = \frac{a+1}{a+2}$$

For self-similar RM mixing
the measure of scaling invariance is the critical modified rate of momentum:

$$\mathrm{M}_{cr} \sim \frac{v^{a_{cr}+2}}{L^{a_{cr}+1}} \sim \frac{v_l^{a_{cr}+2}}{l^{a_{cr}+1}} \qquad n = \frac{a_{cr}+1}{a_{cr}+2}$$

RT / RM mixing and canonical turbulence have different scaling properties.

Reynolds number, viscous scale, span of scales

Kolmogorov turbulence

RT mixing $g = Gt^{a}, a > a_{cr}$

- Reynolds number

$$\mathrm{Re} \sim (vL/\nu) \sim const$$

$$\mathrm{Re} \sim G^{2} t^{(2a+3)}/\nu \quad a > -3/2$$

$$\mathrm{Re}_{l}/\mathrm{Re} \sim (l/L)^{4/3}$$

$$\mathrm{Re}_{l}/\mathrm{Re} \sim (l/L)^{(2a+3)/(a+2)}$$

- viscous scale

$$l \sim (\nu^{3}/\varepsilon)^{1/4}$$

$$l_{\nu} \sim (\nu^{(a+2)}/G)^{1/(2a+3)}$$

- span of scales

$$L/l_{\nu} \sim L(\varepsilon/\nu^{3})^{1/4}$$

$$L/l_{\nu} \sim t^{a+2}(G^{2}/\nu)^{(a+2)/(2a+3)}$$

In RT mixing, viscous scale is finite and is set by acceleration.

Spectral properties

Kolmogorov turbulence:

• spectrum of kinetic energy (velocity)

kinetic energy = $\int_k E(k)dk \sim \int_k \varepsilon^{2/3} k^{-5/3} dk \sim \frac{\varepsilon^{2/3}}{k^{2/3}} \sim (\varepsilon l)^{2/3} \sim v_l^2$

$$E(k) \sim \varepsilon^{2/3} k^{-5/3} \qquad E(\omega) \sim \varepsilon \omega^{-2}$$

RT mixing $g = Gt^a,\ a > a_{cr}$

• spectrum of kinetic energy

$$E(k) \sim \mathrm{M}^{2/(a+2)} k^{-(3a+4)/(a+2)} \qquad a > a_{cr}$$

$$E(\omega) \sim \mathrm{M}^2 \omega^{-(2a+3)}$$

- steeper than Kolmogorov's spectrum $a > -1/2$
- same as Kolmogorov's spectrum $a = -1/2$
- more gradual than Kolmogorov's spectrum $a < -1/2$

RT mixing has steeper (more gradual) spectra than canonical turbulence.

Spectral properties

Spectrum of density

- Kolmogorov turbulence

$$\rho \sim \int_k E(k)dk \sim \int_k \rho_0 \varepsilon^0 k^{-1} dk \qquad E(k) \sim \rho_0 \varepsilon^0 k^{-1}$$

- RT mixing

$$\rho \sim \int_k E(k)dk \sim \int_k \rho_0 \mathrm{M}^0 k^{-1} dk \qquad E(k) \sim \rho_0 \mathrm{M}^0 k^{-1}$$

- RM mixing

$$\rho \sim \int_k E(k)dk \sim \int_k \rho_0 \mathrm{M}_{cr}^0 k^{-1} dk \qquad E(k) \sim \rho_0 \mathrm{M}_{cr}^0 k^{-1}$$

RT/RM mixing & Kolmogorov turbulence have the same power-law in the density spectra.
This is due to the specificity (per unit mass) dynamics.

Properties of fluctuations

Kolmogorov turbulence

RT mixing

$g = Gt^a,\ a > a_{cr}$

• velocity fluctuations

Two fluid parcels are entrained in the motion with a time-delay $\tilde{\tau}$

Their relative velocity is

$\sim (\varepsilon\tilde{\tau})^{1/2}$ $\sim M\tilde{\tau}^{(a+1)}$

much smaller than | same order as

turbulence velocity fluctuations.

$\sim (\varepsilon \upsilon \tilde{\tau})^{1/3}$ $\sim M\tilde{\tau}^{(a+1)}$

Their ratios to mean velocity is

$\sim \left(\varepsilon\tilde{\tau}/\upsilon^2\right)^{1/3}$ $\sim \left(\tilde{\tau}/t\right)^{(a+1)}$

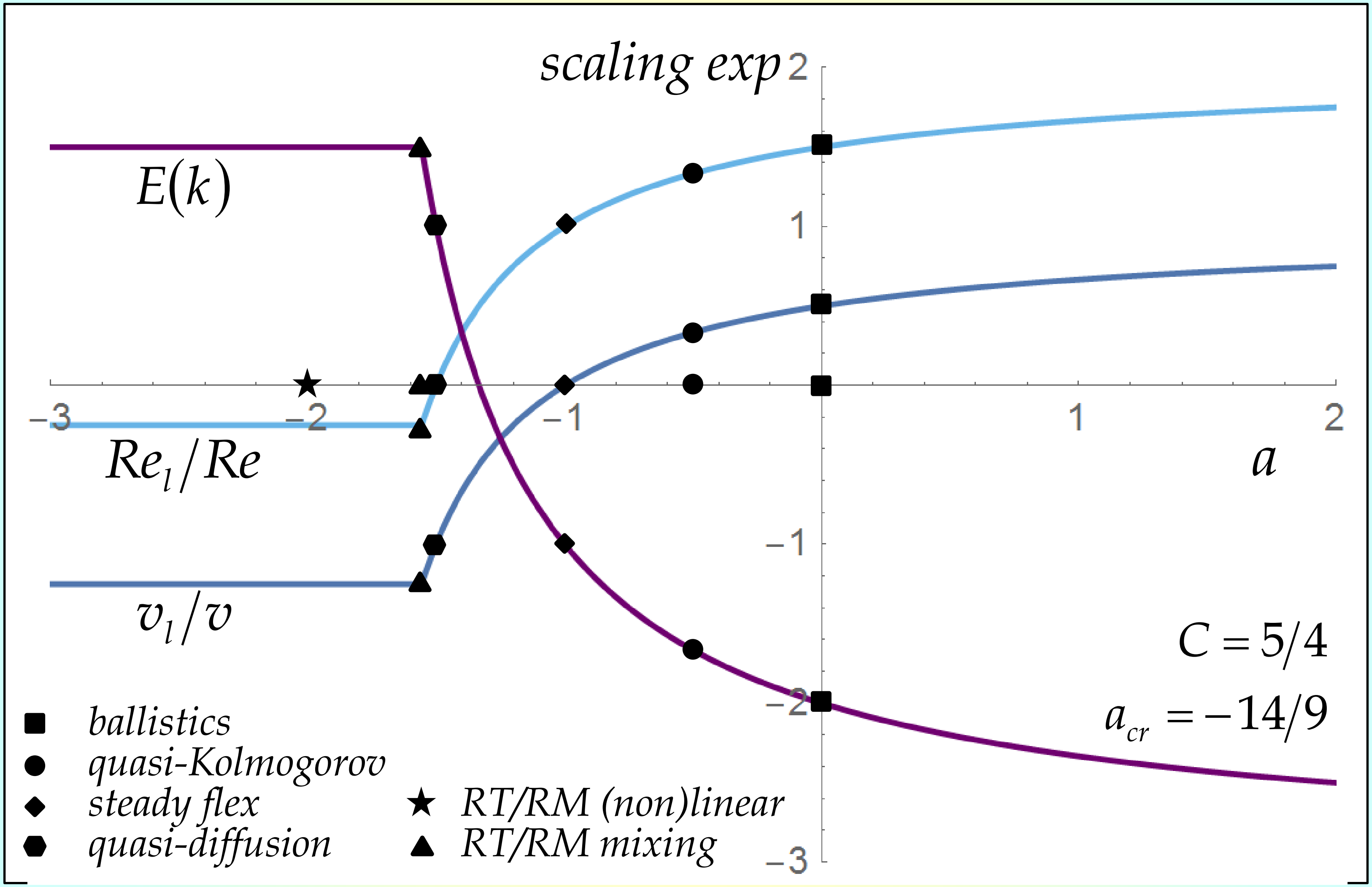
In Kolmogorov turbulence: deterministic conditions do not play a role; fluctuations are self-generated.

In RT mixing: fluctuations are set by deterministic (initial) conditions.

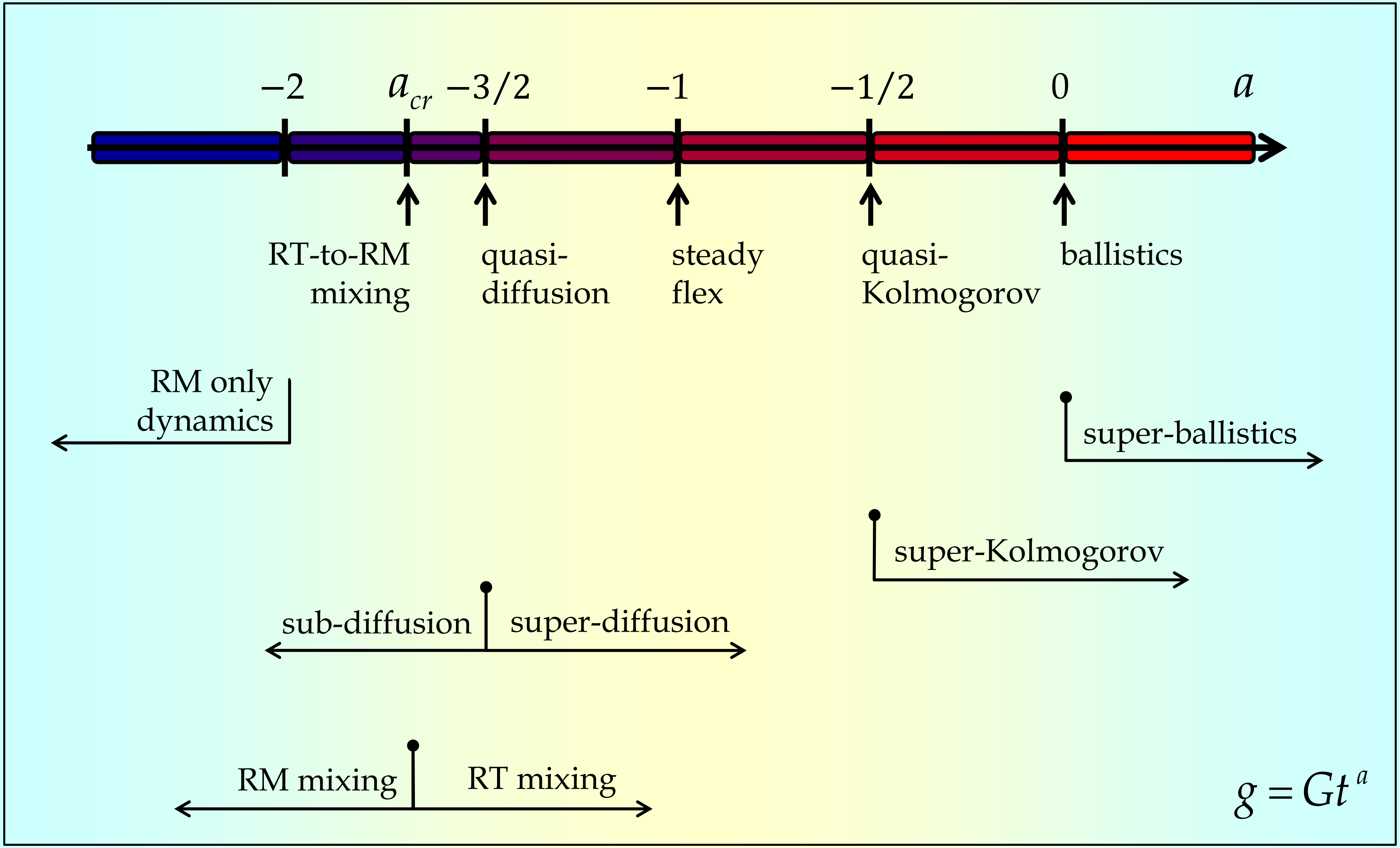
Their contribution decays with time for $a > -1$

Their contribution increases with time for $a < -1$ leading to RM mixing for $a \to a_{cr}$

Scaling exponents for RT mixing with variable acceleration

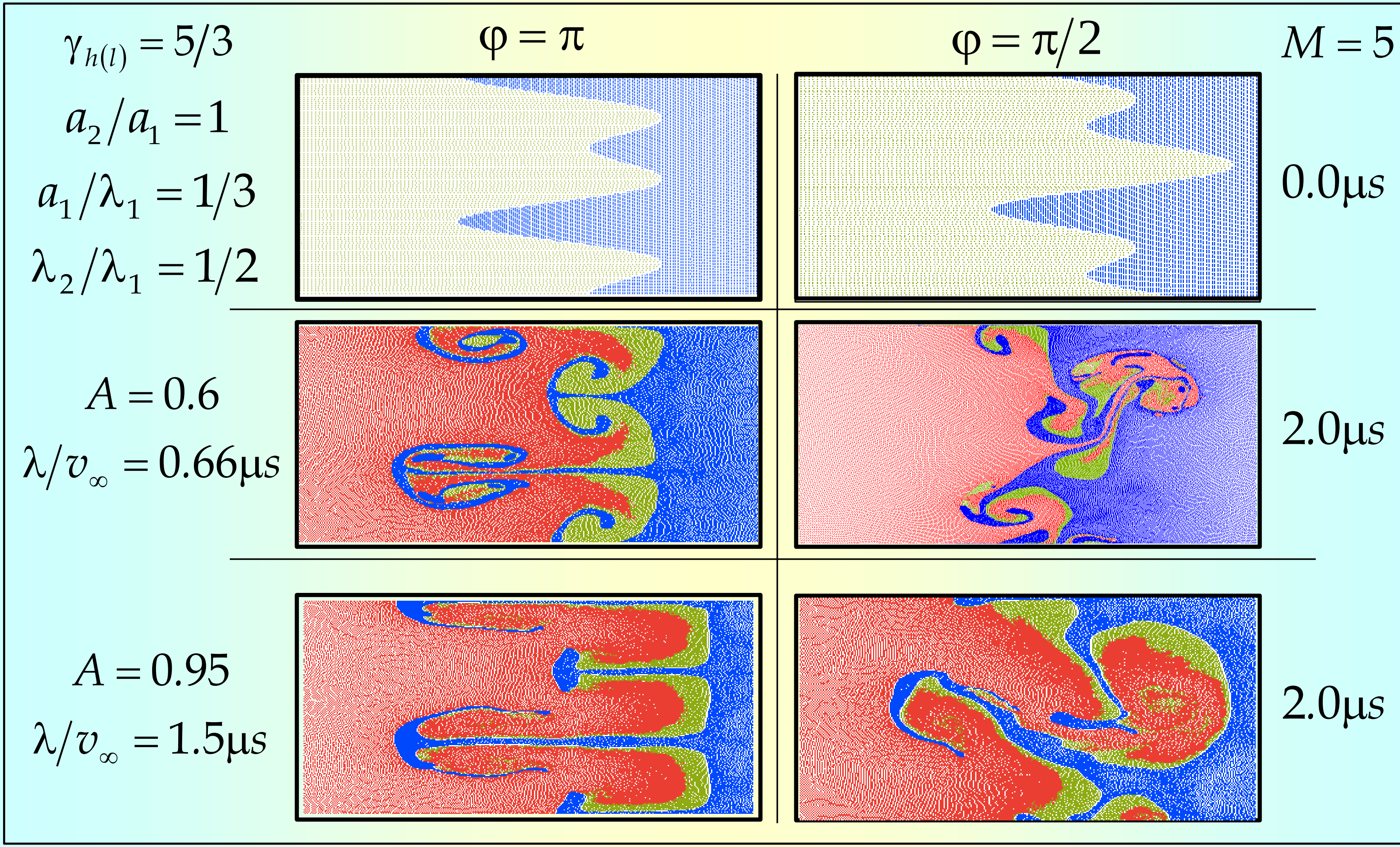


Special self-similar class
RT/RM mixing with variable acceleration



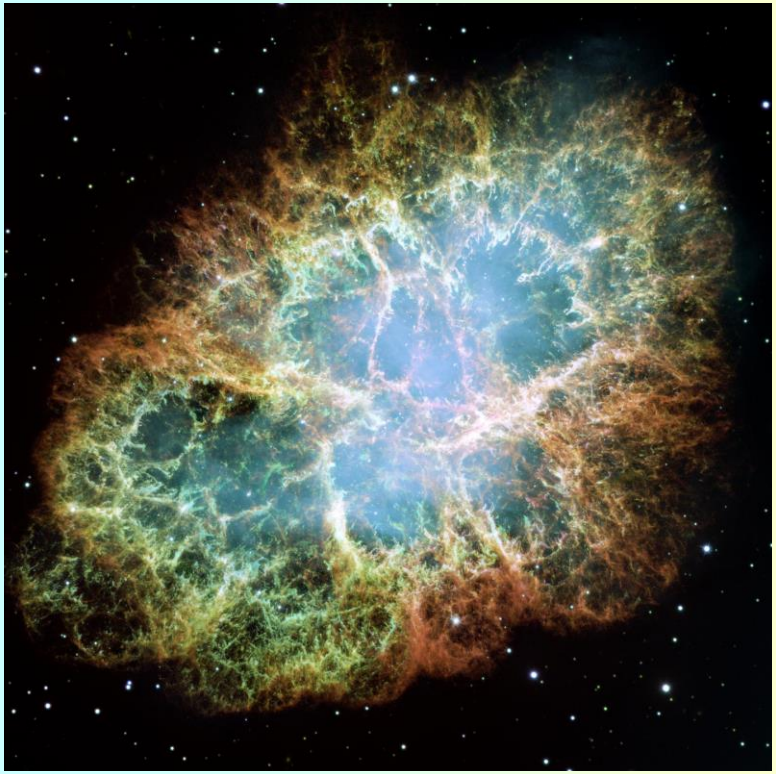
Sensitivity RT/RM mixing to initial conditions

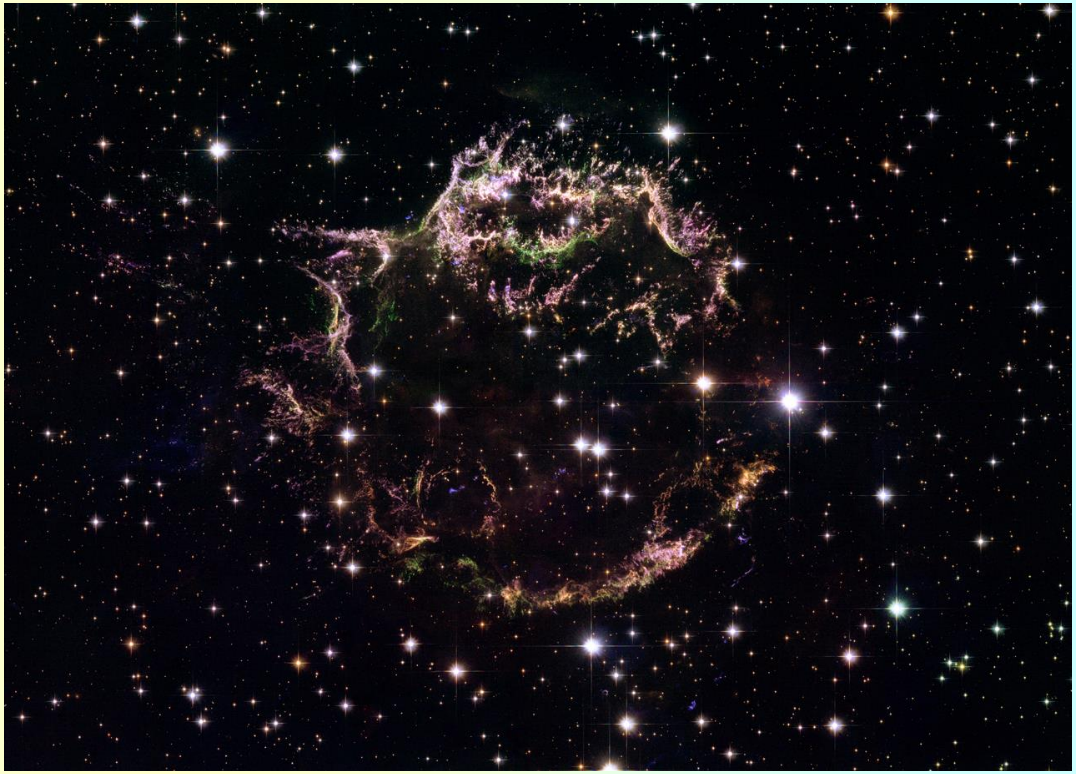
Smoothed Particle Hydrodynamics simulations of strong-shock-driven RMI illustrate the strong sensitivity of the highly nonlinear RM dynamics to the deterministic conditions.



Order / disorder depends on the relative phase of initial perturbation waves.

Supernova Remnants

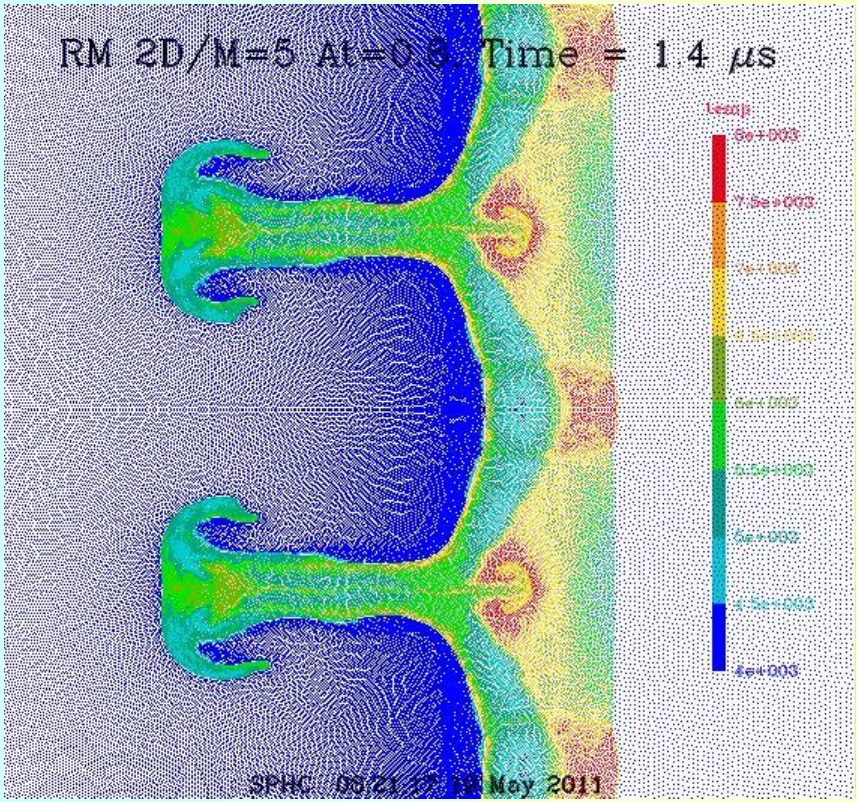




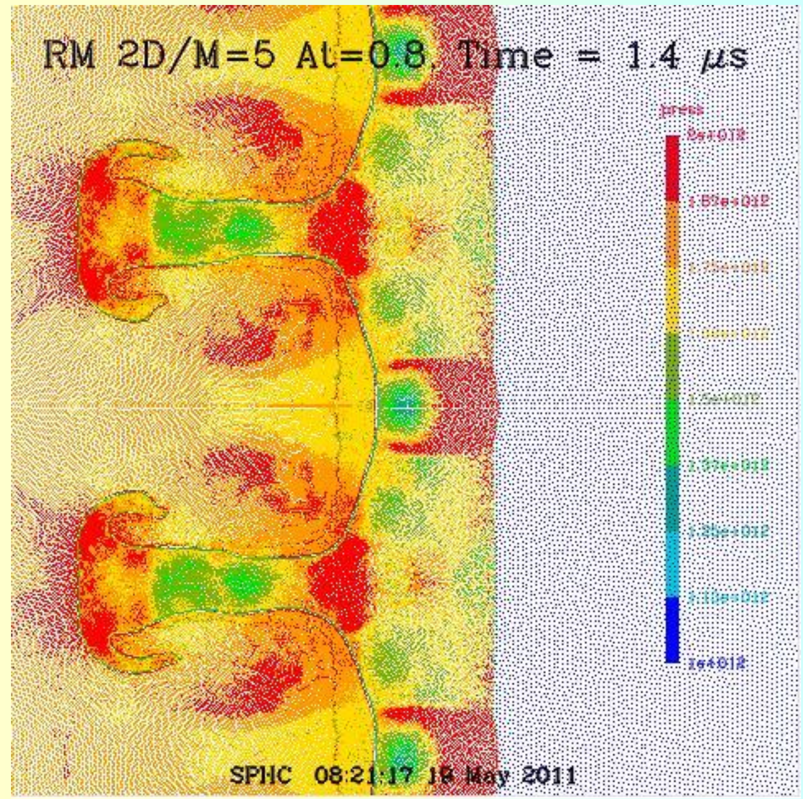
- Supernovae remnants in Crab Nebula (left) and Cassiopea A (right) are filled with filaments that are caused by RT/RM mixing developing at a supernova blast.
- Our theory is consistent with and explains the observations:
  - Filaments can move at a speed higher than a blast wave prescribes.
  - Energy transfer at small scales can occur via localization and trapping, thus enabling conditions for synthesis of heavy and intermediate mass elements.
  - Dynamics at large and at small scales can be sensitive to deterministic conditions.
- Supernovae can indeed be the astrophysical initial value problem.

Non-uniform volumetric structures at small scales

Smoothed Particle Hydrodynamics simulations of strong-shock-driven RMI find intense formation of non-uniform structures in flow fields at small scales in the bulk:
cumulative jets, hot (cold) spots, high (low) pressure regions



Temperature



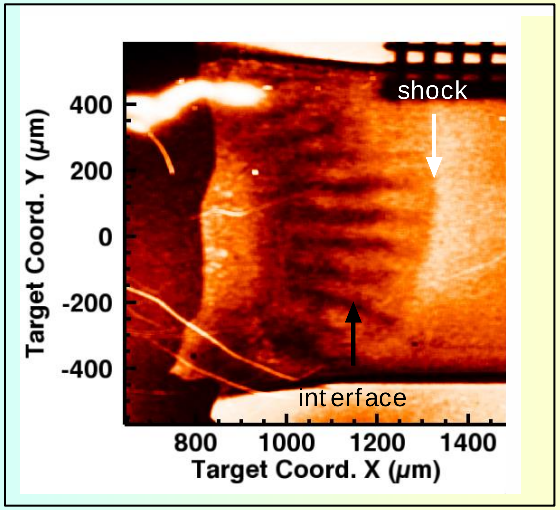
Pressure

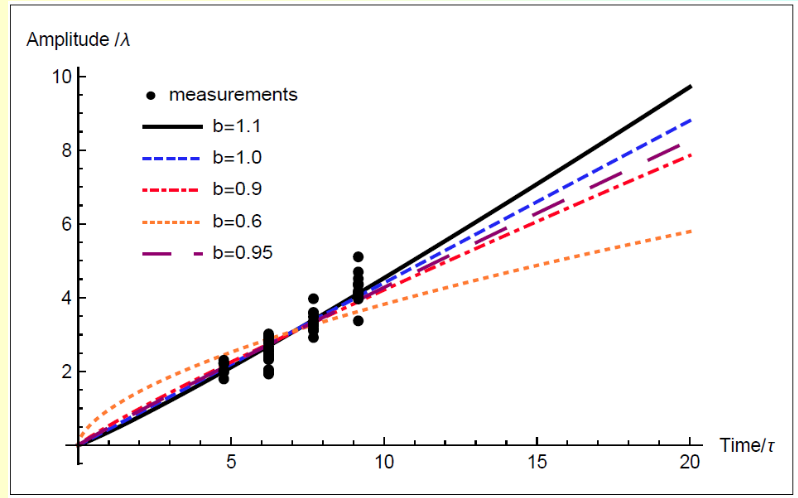
[Abarzhi et al. 2018, Dell et al. 2017; Pandian et al. 2017; Dell et al. 2015; Stanic et al.2012]

Experiments in Plasmas

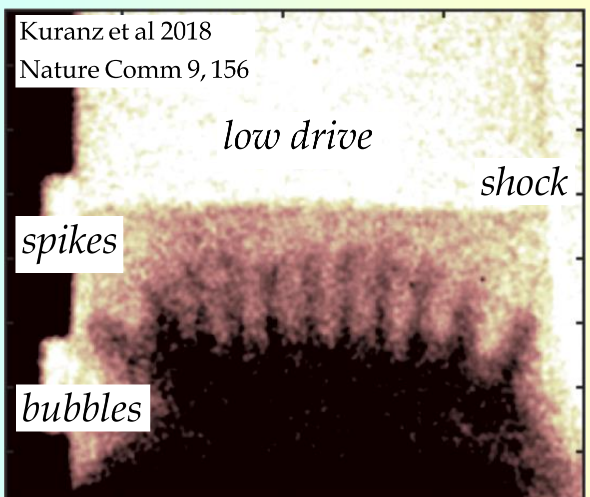
The theory achieves good agreement with the data in experiments in plasmas at the Omega facility (top) and the NIF (bottom).

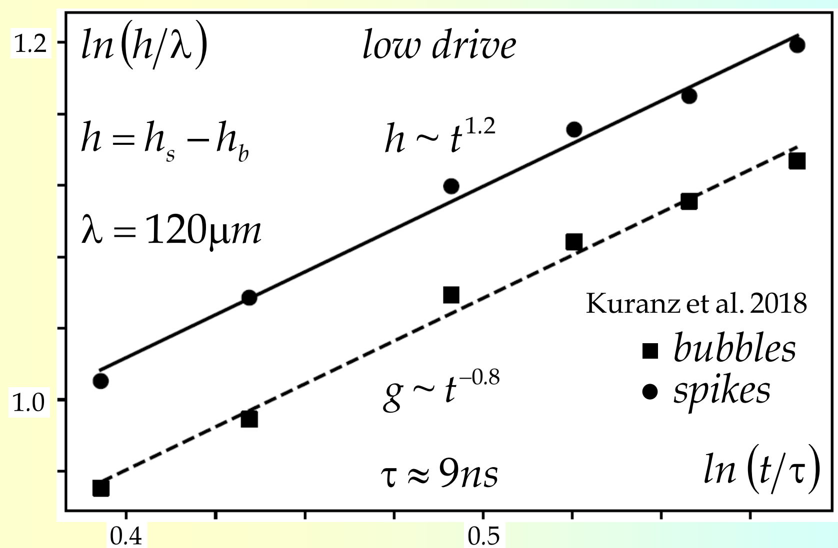
$$g \sim 1.0 \times 10^{14}\,\mathrm{ms}^{-2} \qquad t \sim 40 \times 10^{-9}\,\mathrm{s} \qquad \mathrm{Re} \sim 1.0 \times 10^{6}$$





Swisher et al. 2015
Kuranz et al. 2008
Robey et al. 2003

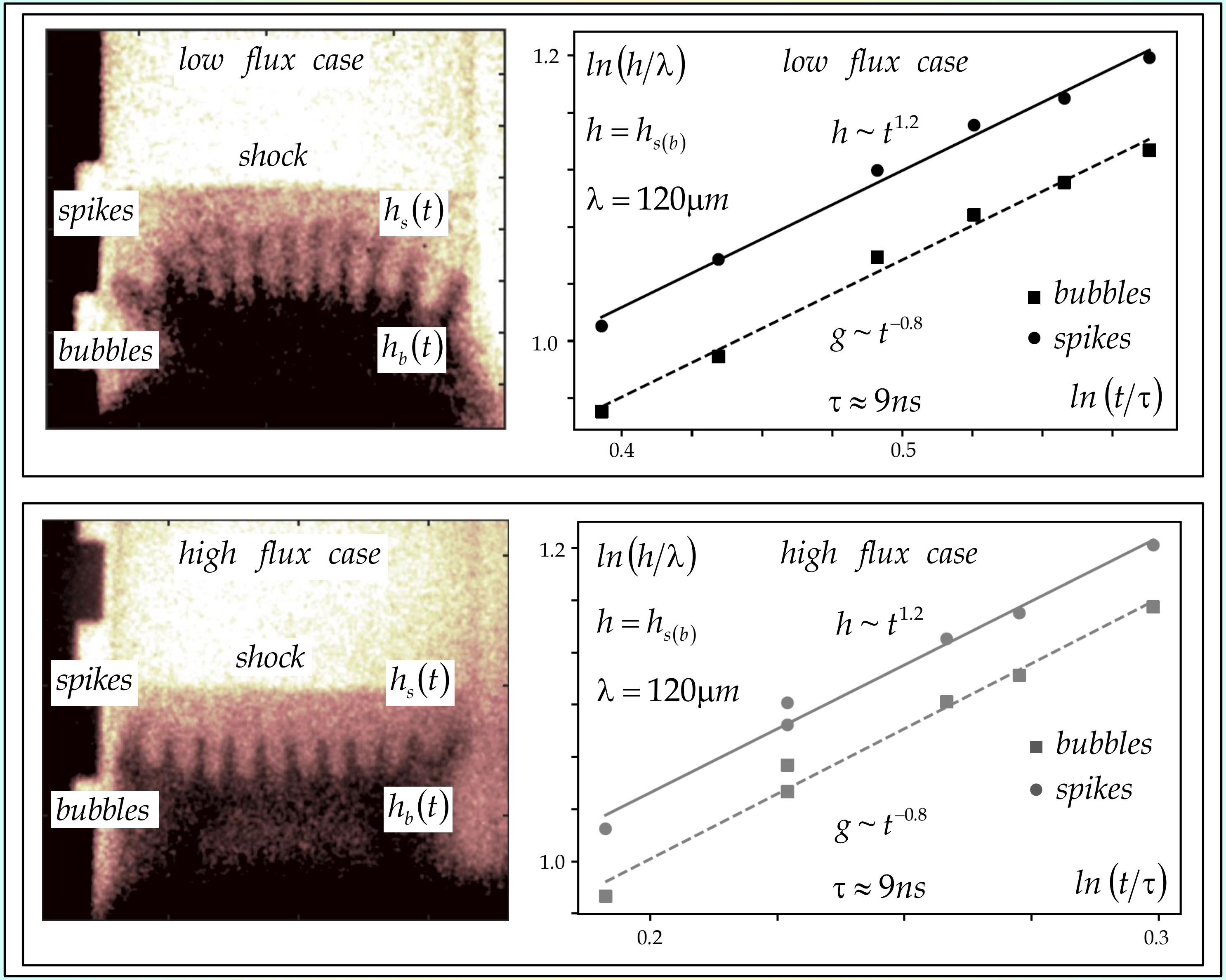




Kuranz et al. 2018

Supernova experiments at the NIF

The theory explains the difference in the interface morphology in the low flux case (top) and the high flux case (bottom) observed in supernova experiments at the NIF.

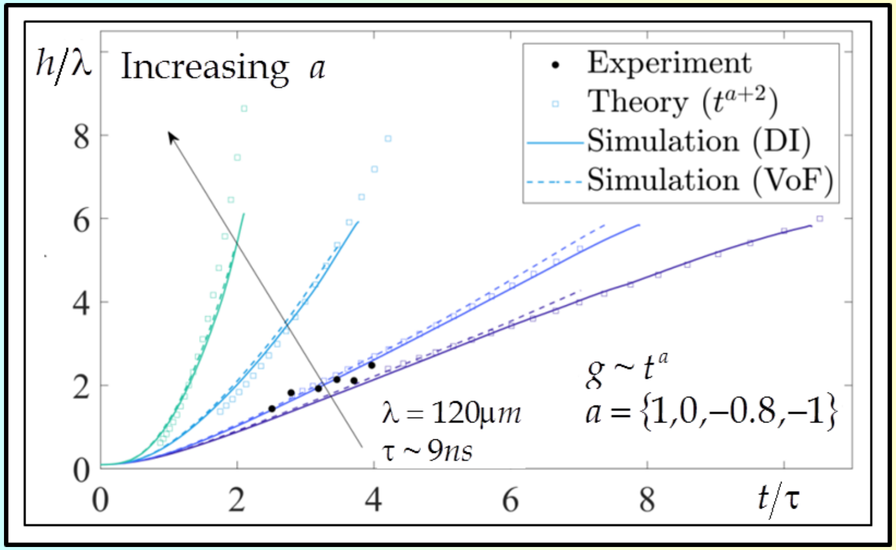


Kuranz et al. 2018

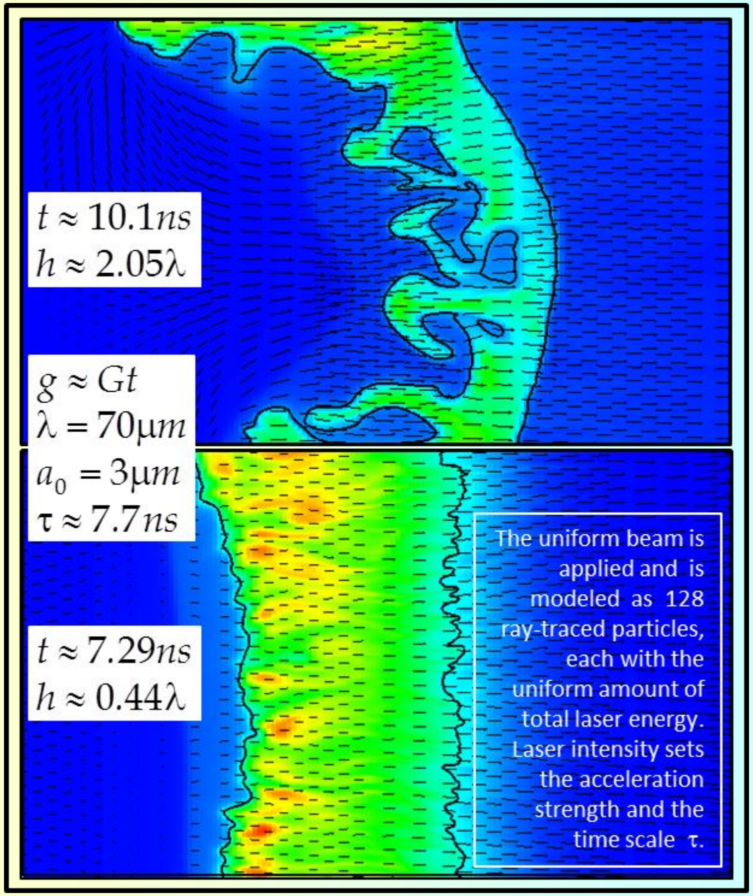
Future NIF experiments: RT with variable g

- Theory, simulations, prior NIF experiments agree with one another.

DI & VoF [DNS]: RT mixing with variable accelerations



FLASH [multi-physics]: RT mixing / HEDP



- RT growth strongly depends on the accelerations.

- The difference is large; it can be confidently detected in the NIF experiments.

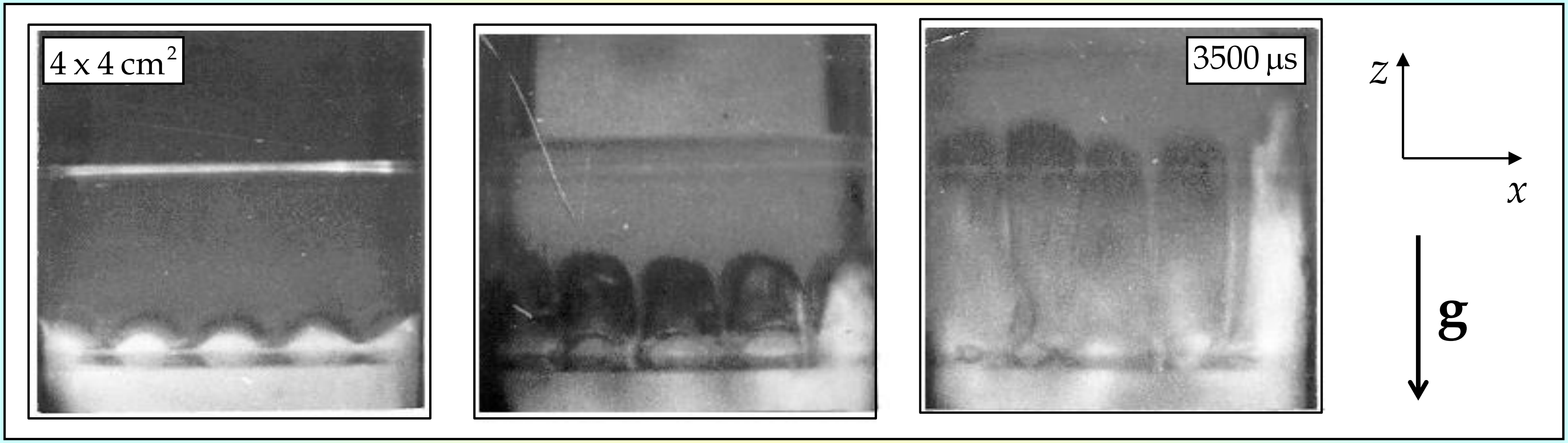
DI & VoF: RT growth, variable accelerations

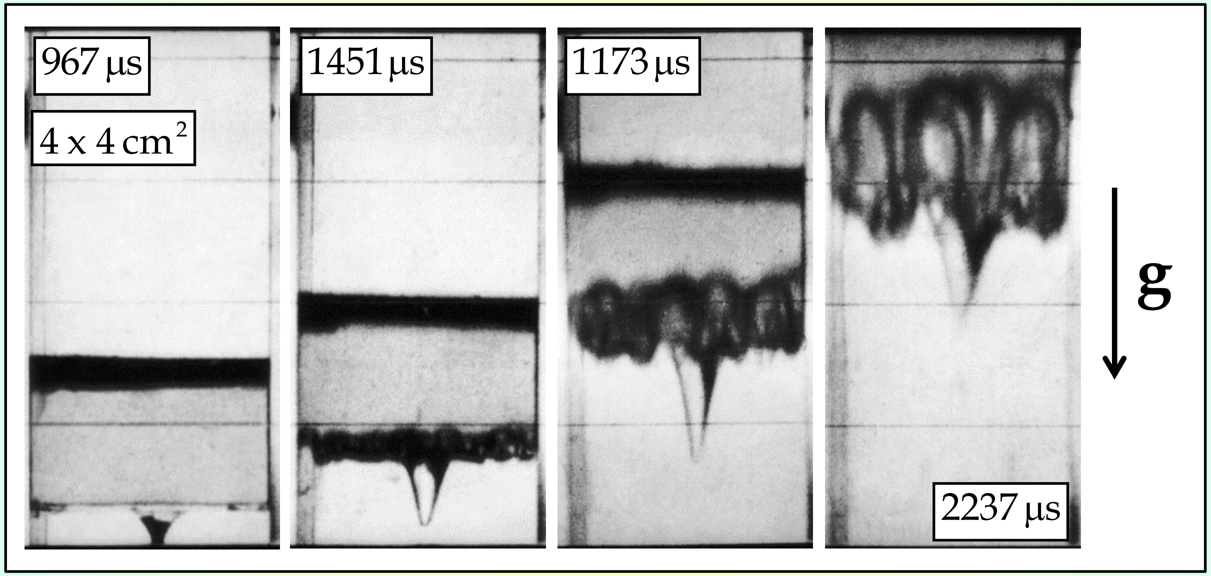
a	t/τ	h/λ	t/τ	h/λ	t/τ	h/λ
−1	1.40	0.55	2.28	1.05	5.41	3.04
0	1.40	1.05	1.38	1.05	2.52	3.00
1	1.40	2.42	0.98	1.07	1.55	3.01

Chan et al. 2023 PoF, Hwang et al. 2021 PoF

Experiments in Fluids

The theory explains the experiments in fluids at high Reynolds numbers.





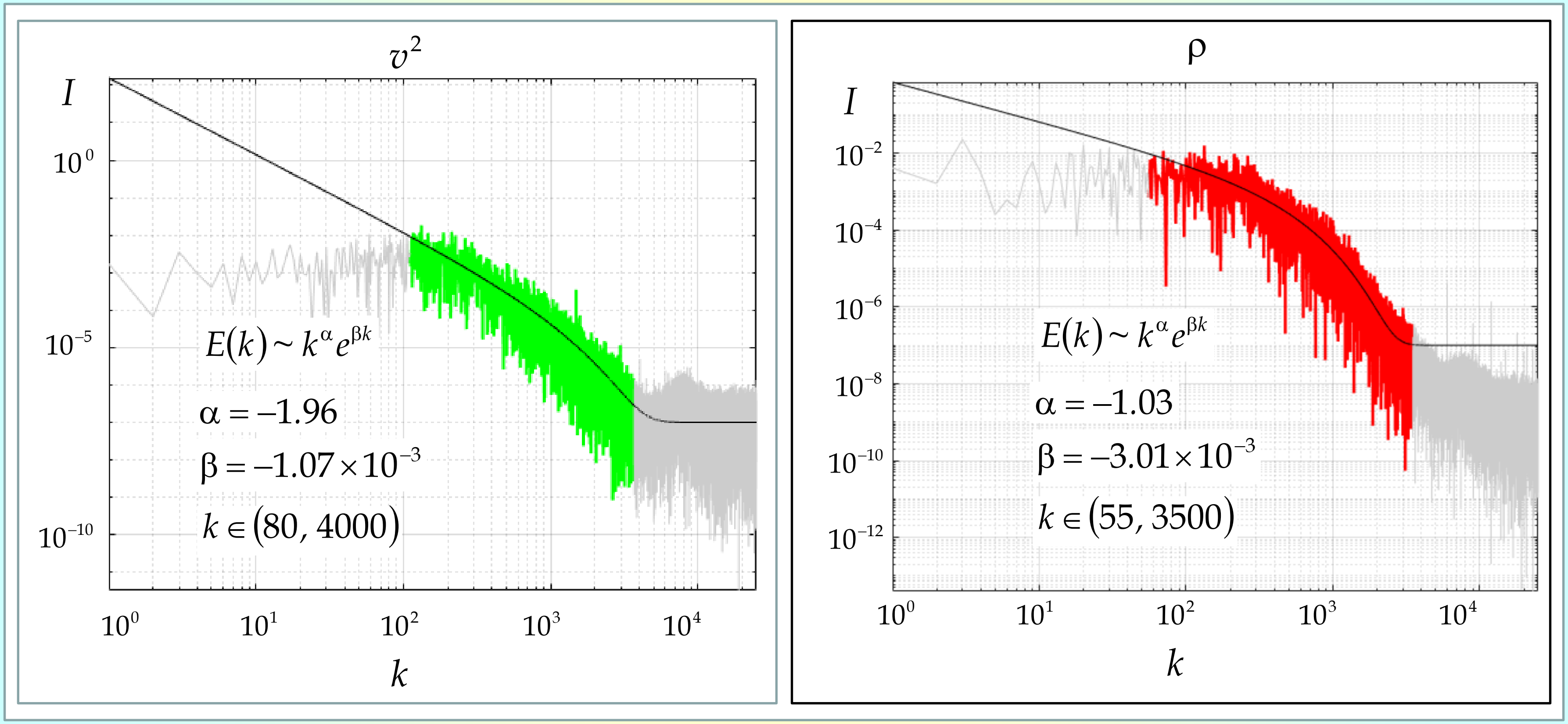
$g \sim 7.0 \times 10^{4}\,\mathrm{ms}^{-2}$

$t \sim 4.0 \times 10^{-3}\,\mathrm{s}$

$\mathrm{Re} \sim 3.2 \times 10^{6}$

[Meshkov & Abarzhi 2019; Meshkov 2013; Meshkov 1990s]

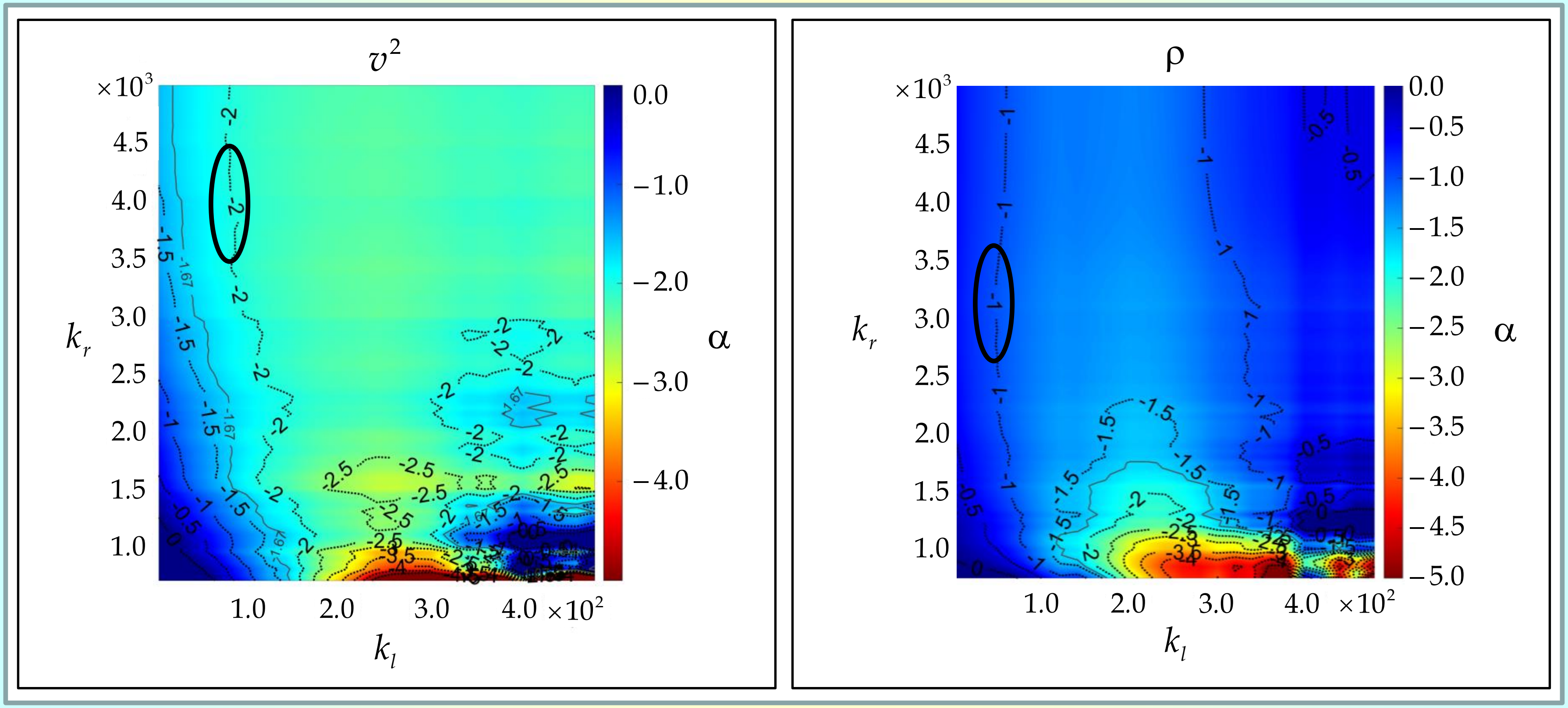
RT spectra in available experiments



- Guided by group theory, the data analysis method is developed.
- The method applies the rigorous statistical technique in order to:
  - study raw data – fitting functions, analysis of residuals, effect of the fitting interval;
  - identify the fitting function parameters – mean values, relative errors, goodness-of-fit score;
  - find the ‘best fitting interval’ and the ‘best fit
- The data analysis results are in conformity with our theory

[Williams & Abarzhi 2022; 2020 Pfefferle & Abarzhi; 2017 Akula … Ranjan]

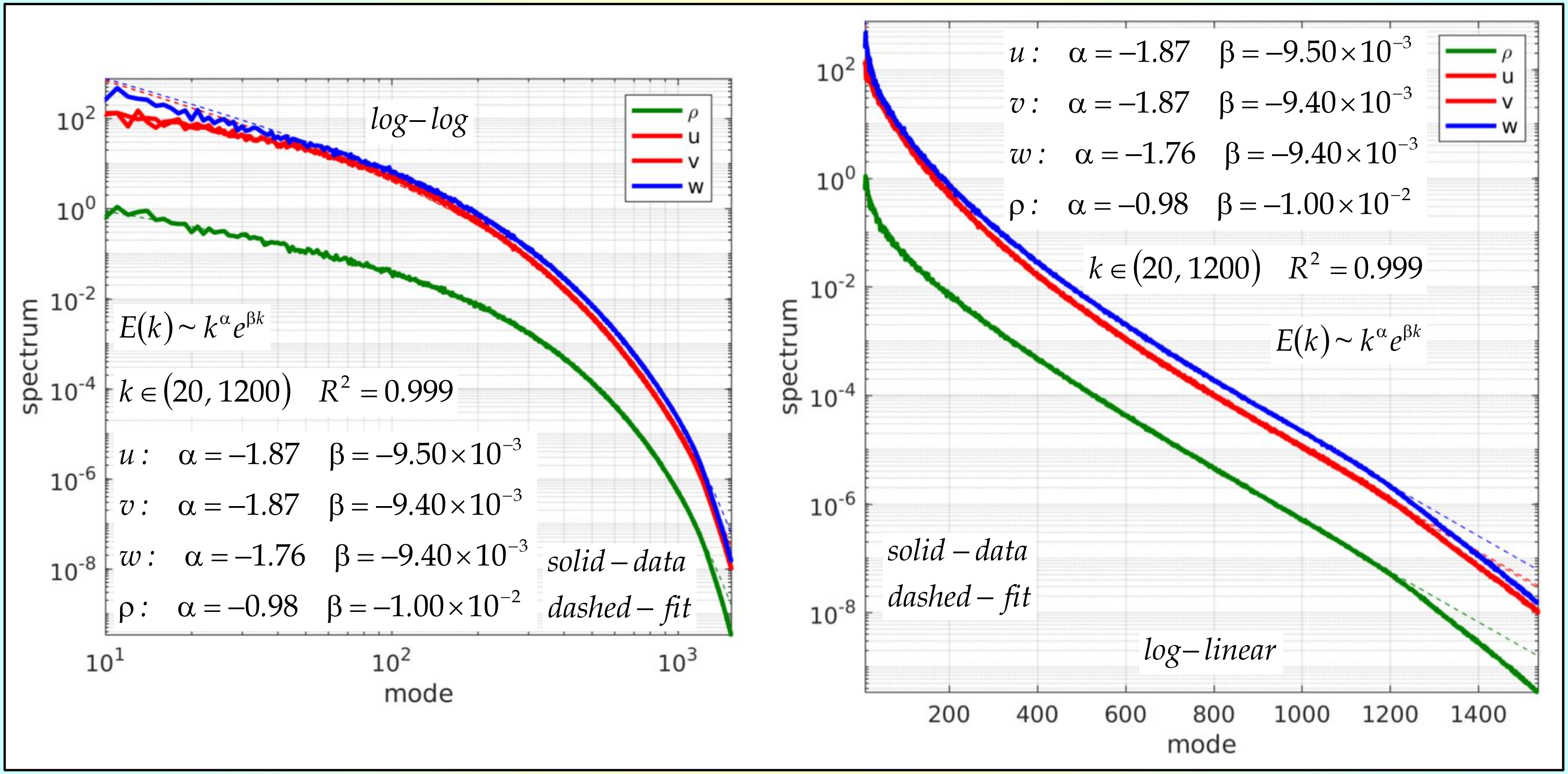
RT spectra: effect of the fitting interval



- Properties of data in RT spectra
  - Spectra are at least as complex as a compound function (power law x exponential).
  - The fit interval and left/right cut-offs influence the parameters of the fitting function.
  - In the fitting interval, where the relative errors are small (3-7%), the goodness-of-fit score is high (>50%), and the dynamic range is large,
    the data analysis results agree with group theory results.

[Williams & Abarzhi 2022; 2020 Pfefferle & Abarzhi; 2017 Akula … Ranjan]

RT spectra in available simulations



- The method is applied to study processed data in available simulations.
- The data analysis results are in conformity with our theory

[Cabot & Cook 2006 Nature Phys; 2020 Pfefferle & Abarzhi Phys Rev E]

Group theory and the classical approaches

- Group theory approach is fully consistent with the classical approaches.

- Group theory results can be can be directly linked to the classical results in Kolmogorov theory.

- This can be done by accurate accounting for the scale-dependence of the energy dissipation rate and the invariant forms of RT/RM mixing.

- Consider RT mixing with constant acceleration. $g = Gt^0 = g_0$

- Its invariant form is the rate of momentum loss. $\mathrm{M} = \mu \quad \mu \sim v^2/L \sim g_0$

- Its energy dissipation rate is scale-dependent. $\varepsilon \sim \mu^{3/2} L^{1/2} \sim g_0^{3/2} L^{1/2}$

- The spectral density is: $k \sim L^{-1}$

$$E(k) \sim \varepsilon^{2/3} k^{-5/3} \sim \left(\mu^{3/2} k^{-1/2}\right)^{2/3} k^{-5/3} \sim \mu\, k^{-2} \quad \Rightarrow$$

$$E(k) \sim \mu\, k^{-2} \sim g_0\, k^{-2}$$

Conclusions

Our theory

- Applies group theory for study non-equilibrium dynamics of interfaces and mixing.
- Finds invariants, scaling, spectra of Rayleigh-Taylor mixing with variable acceleration.
- Finds special self-similar class for Rayleigh-Taylor / Richtmyer-Meshkov dynamics.

We find

- Rayleigh-Taylor mixing is driven by transports of mass, momentum and energy,
  canonical turbulence is driven by energy transport.
- Self-similar mixing with variable acceleration can be
  acceleration-driven RT mixing and initial growth-rate and drag driven RM mixing
- RT / RM mixing have their own invariants, scaling, correlations and spectral properties;
  they depart from those in canonical turbulence and those prescribed by blast waves.

Key conclusions

- RT / RM mixing dynamics can vary super-ballistics to sub-diffusion depending on acceleration.
- RT / RM mixing are sensitive to deterministic conditions for any acceleration.
- RT mixing with strong accelerations can keep order and laminarize.