Generation and dynamics of seed magnetic fields in collisionless plasmas

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Introduction

- Magnetic field *generation* and *amplification* is a central problem in astrophysics: turbulent dynamo can amplify, but a *seed magnetic field is required*.
- Self-magnetized plasmas (B~1–100 MG) occur in laser-target interactions.
- Physical mechanism for the *creation* of these fields (both astro and lasers) is thought to be the *Biermann battery*.
- For astrophysical plasmas, Biermann field is **very** small. E.g., in the ICM, field is $\sim\mu$G, but Biermann field is $\sim 10^{-20}$ G
- Understanding of Biermann battery largely rooted in fluid theory – collisionless plasmas?

Biermann battery

- MHD induction equation is linear in B, cannot generate magnetic field if B(t=0)=0

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

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- Relative magnitude of 2nd to 1st term on RHS is ρ_s/L. Usually small, but gets large when B→0. Source term.

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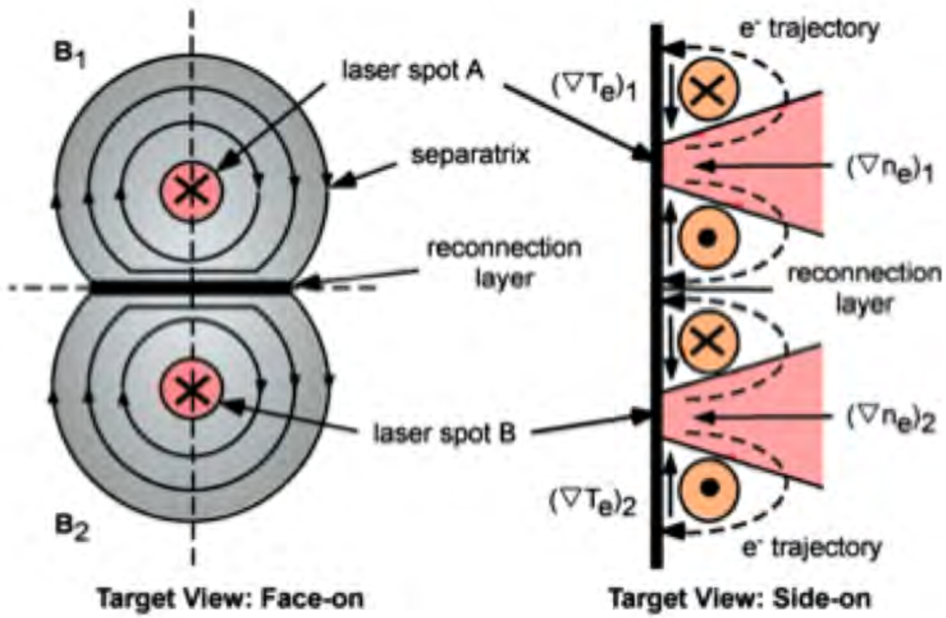
- *At early times*:

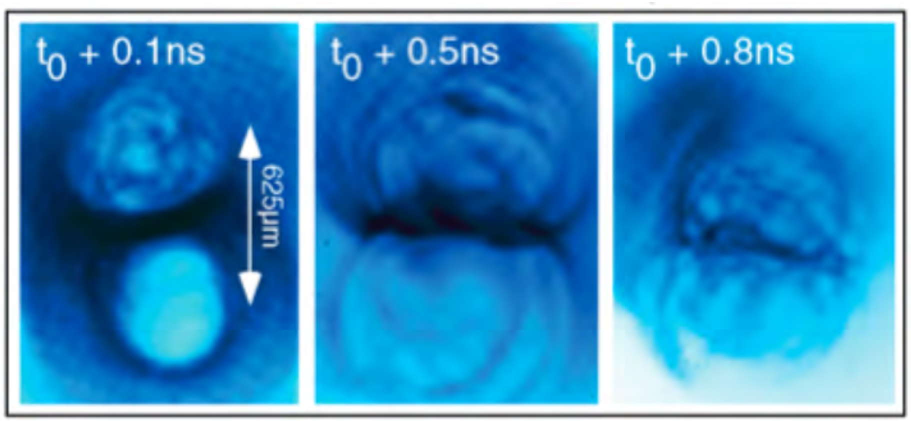
$$\frac{\partial \mathbf{B}}{\partial t} \sim c\frac{\nabla n_e \times \nabla p_e}{n_e^2 e} \Rightarrow \qquad \Omega_i = \frac{c_s^2}{L^2}t; \qquad \rho_s = \frac{c_s}{\Omega_i} = \frac{L^2}{c_s\, t}$$

- *Saturation (assuming fixed gradients):*

$$\nabla \times (\mathbf{u} \times \mathbf{B}) \sim c\frac{\nabla n_e \times \nabla p_e}{n_e^2 e} \Rightarrow \qquad \Omega_{i,sat} = \frac{c_s}{L}; \qquad \rho_s \sim L$$

Biermann in laser-plasma interactions

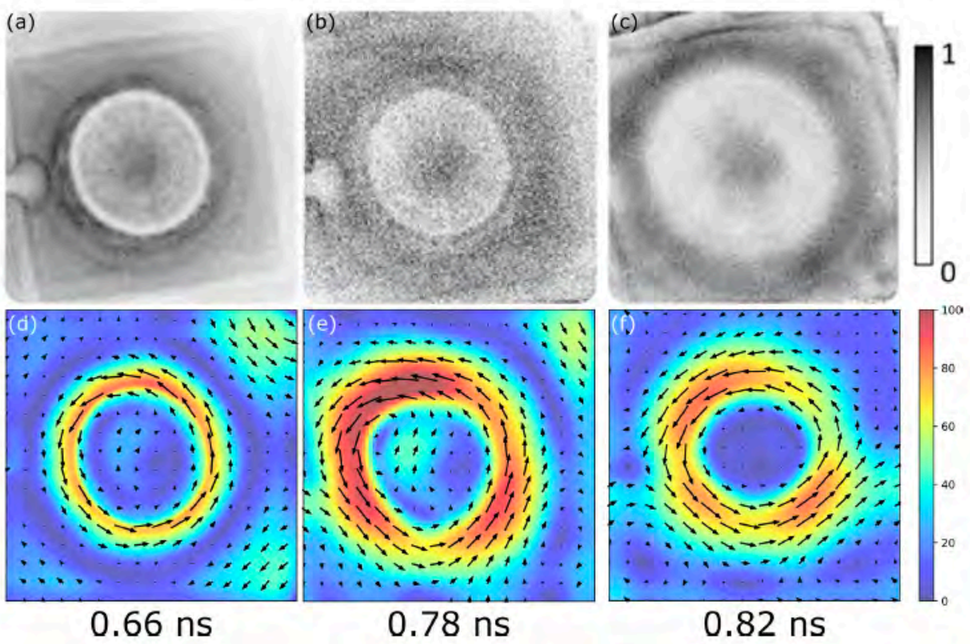


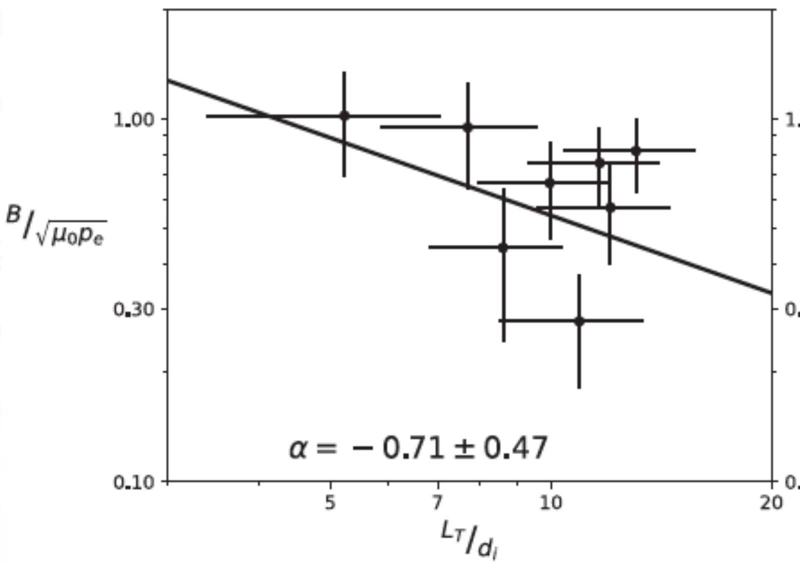


Nilson ’06,
Willingale ’10

(Stamper ‘71)

Biermann in laser-plasma interactions





Sutcliffe *et al.* PRE 2022

[But see recent work by M. Sherlock and C. Walsh on Biermann suppression in ICF-relevant conditions]

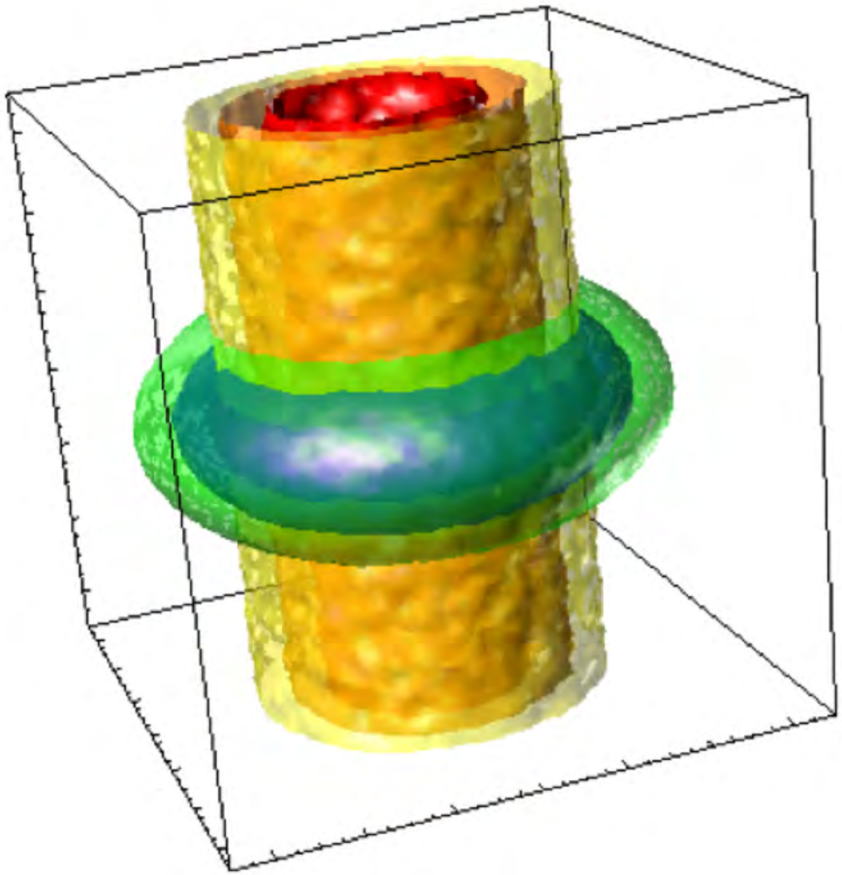
Biermann in weakly collisional plasmas

- Weak collisions + no background magnetic field imply no fluid closure is applicable

- Does the fluid-based understanding of the Biermann battery survive then?

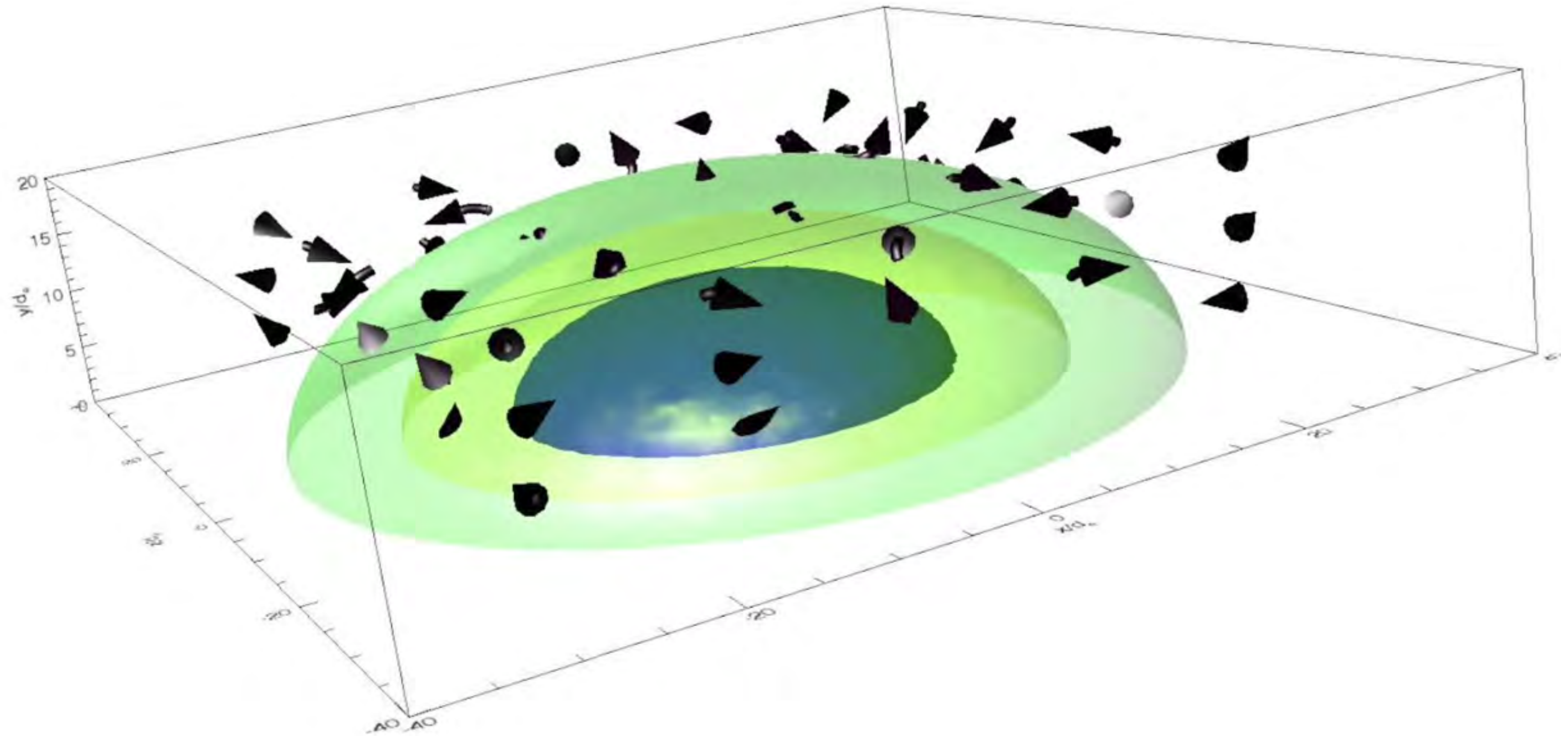
Schoeffler *et al.*, Phys. Rev. Lett. '14, Phys. Plasmas '16, Phys. Rev. E 2018
Sherlock & Bissell, Phys. Rev. Lett. '18

Computational Setup



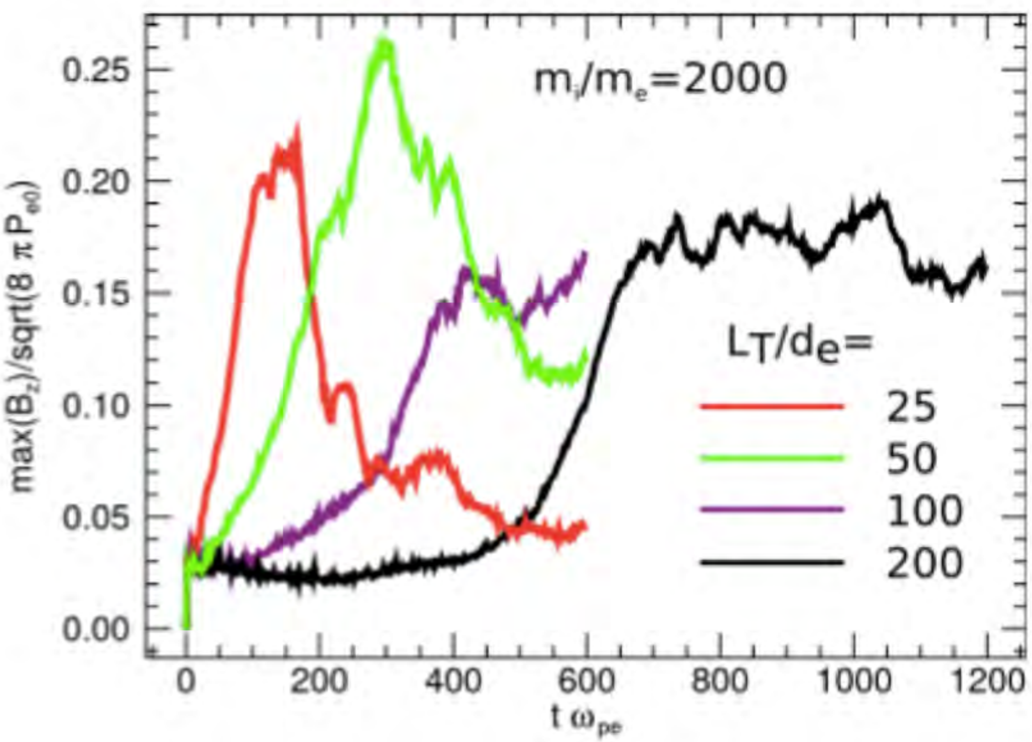
- 2D and 3D PIC simulations with OSIRIS
- **No initial magnetic fields**
- Spheroid density profile
- Cylindrical temperature profile

Schoeffler *et al.*, Phys. Rev. Lett. '14, Phys. Plasmas '16



Schoeffler *et al.*, Phys. Plasmas 2016

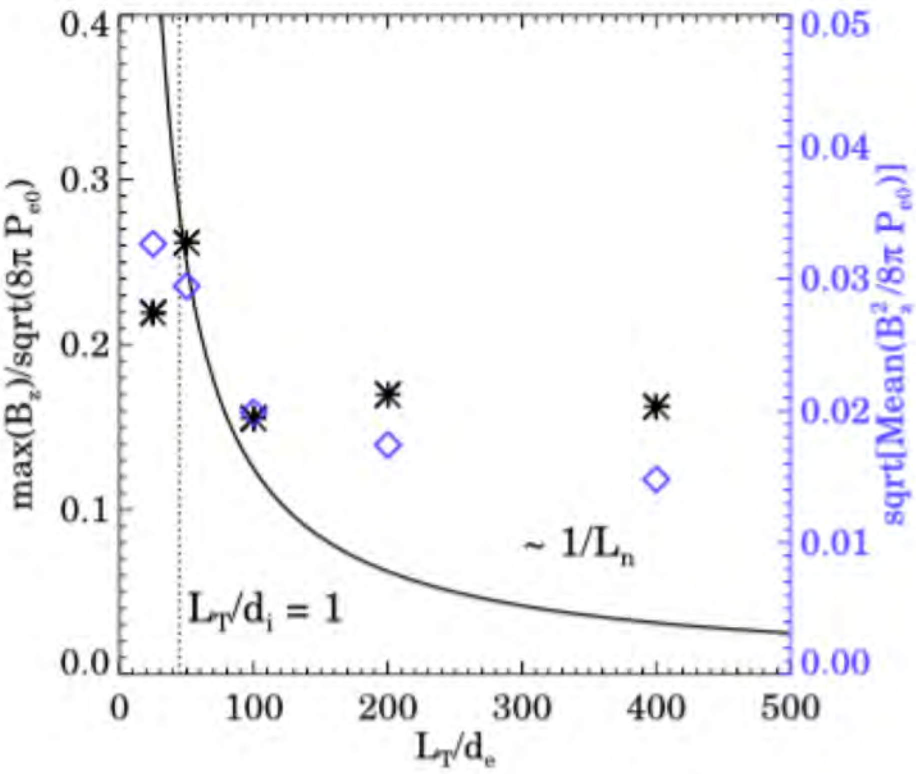
Magnetic field development



For small system sizes, magnetic field decays at late times due to ion-acoustic instability (predicted by Haines 97).

Larger systems show steady fields.

Scaling with system size



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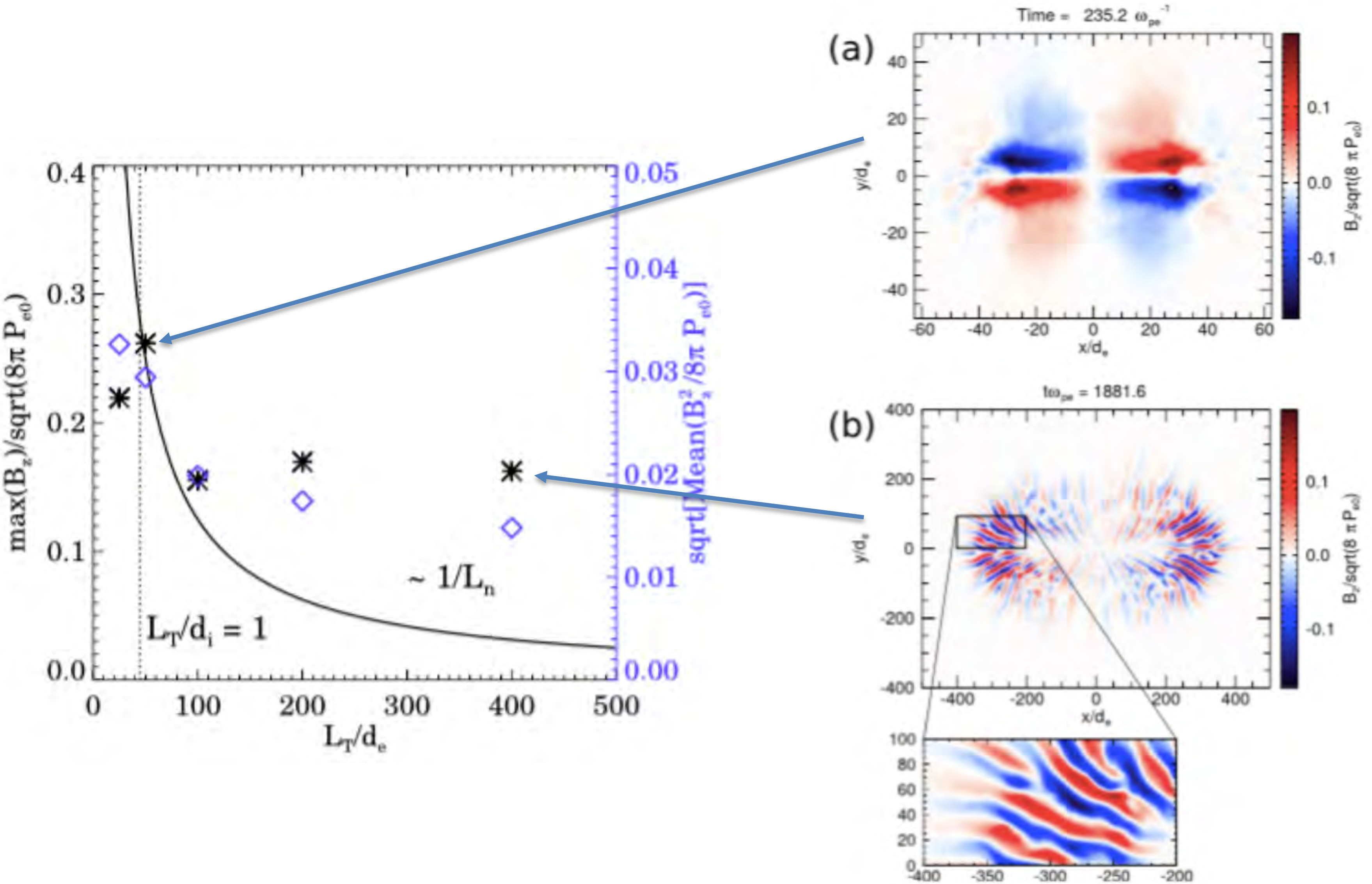

Weibel instability

$$k_\perp^2 c^2 - \omega^2 - \sum_\alpha \omega_{p\alpha}^2 A_\alpha - \sum_\alpha \omega_{p\alpha}^2 (A_\alpha + 1)\xi_\alpha Z(\xi_\alpha) = 0$$

$$A_\alpha = T_{\parallel\alpha}/T_{\perp\alpha} - 1$$

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Measure the anisotropy A in the simulations, and find fastest growing mode (k) and corresponding growth rate (γ)

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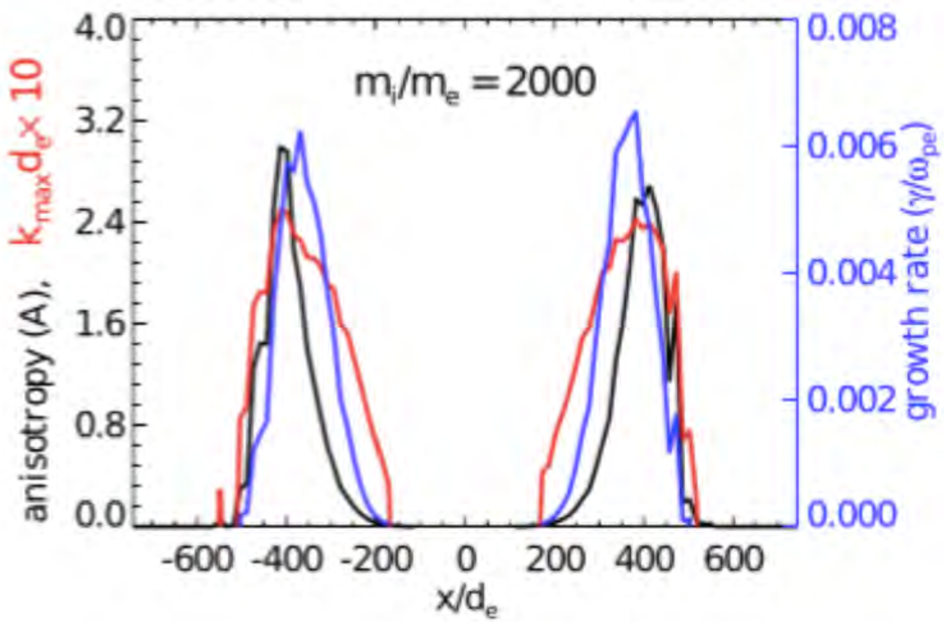

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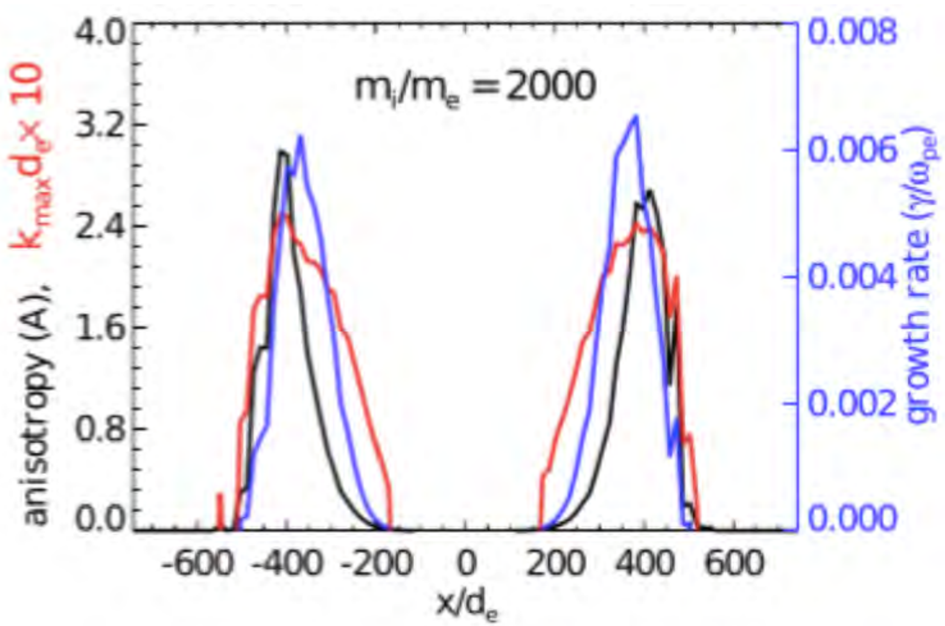

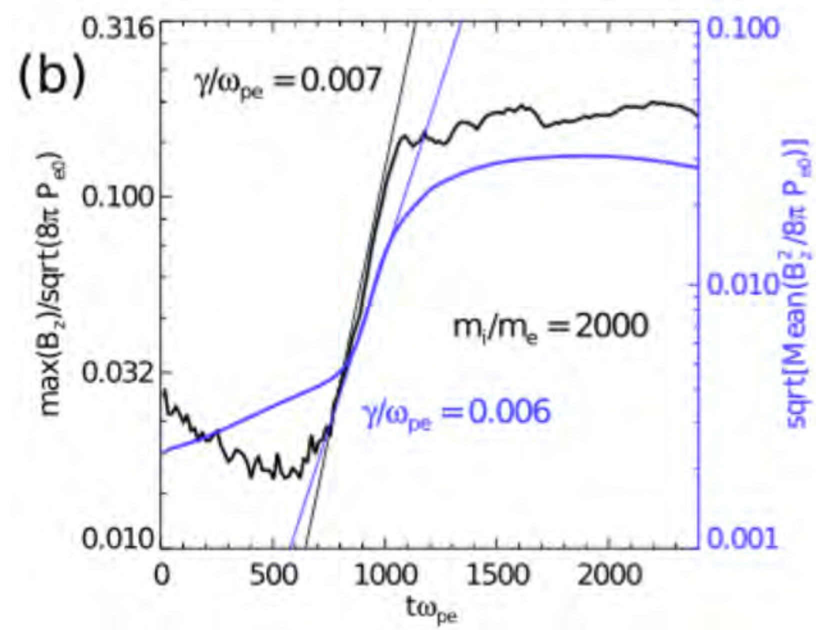

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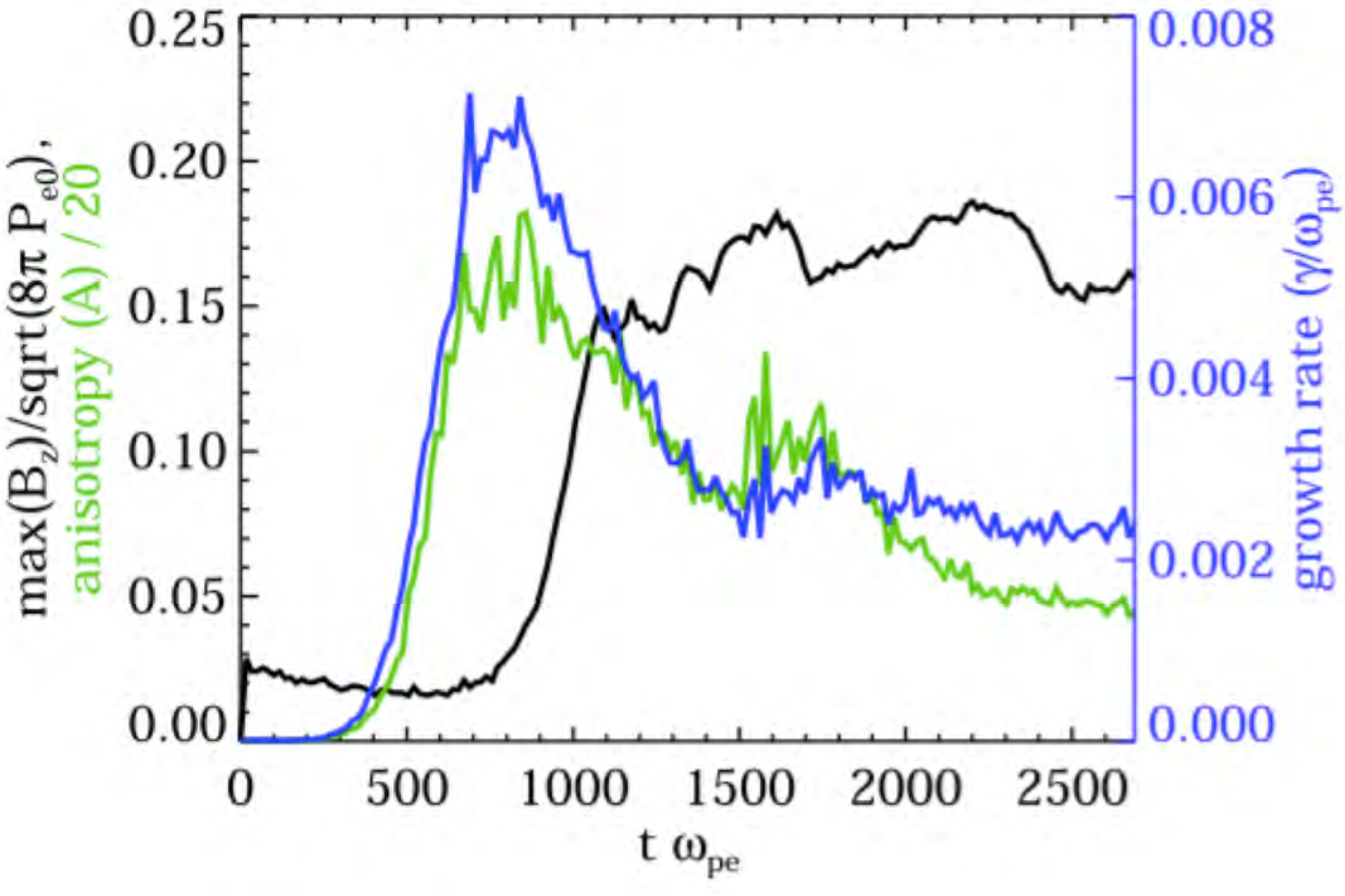

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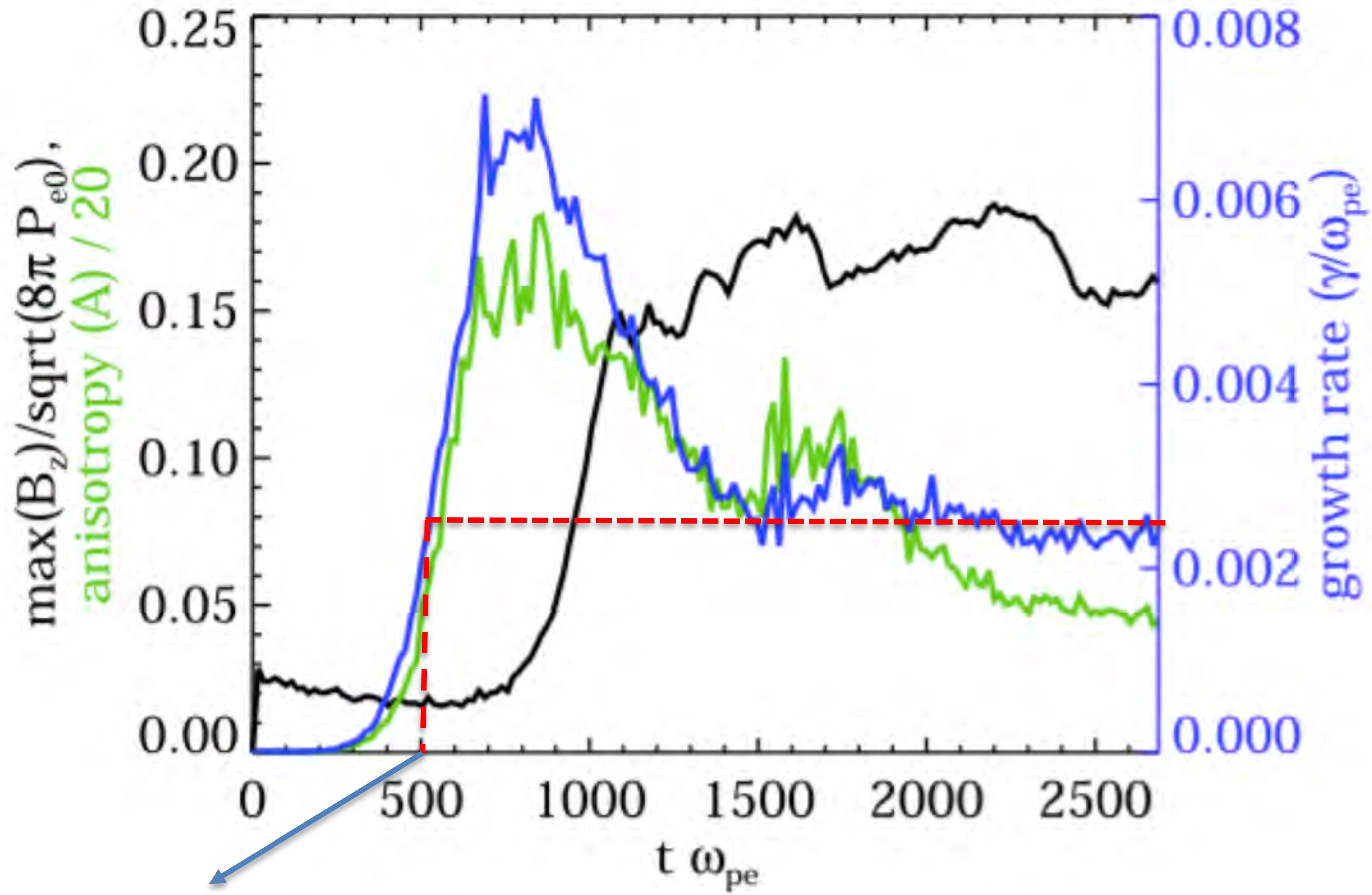
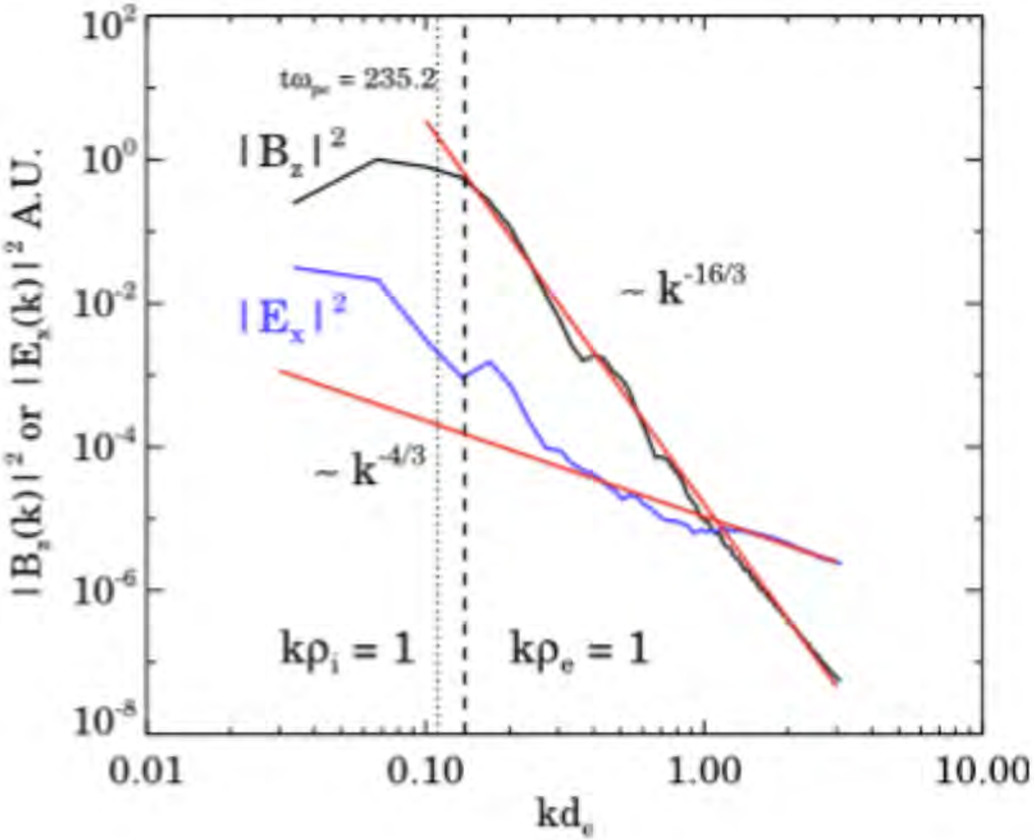
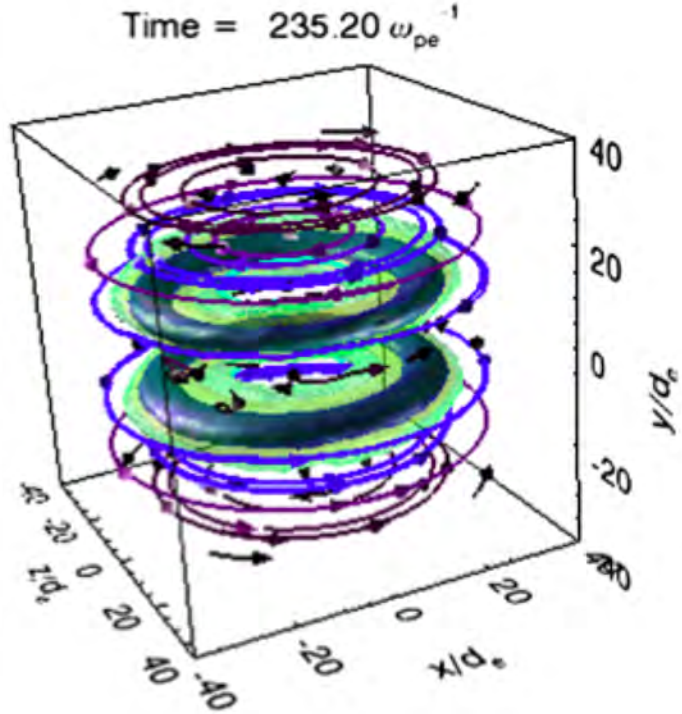


$$\gamma_W(t) L / v_{th,\mathrm{e}} = 1$$

Energy Spectra



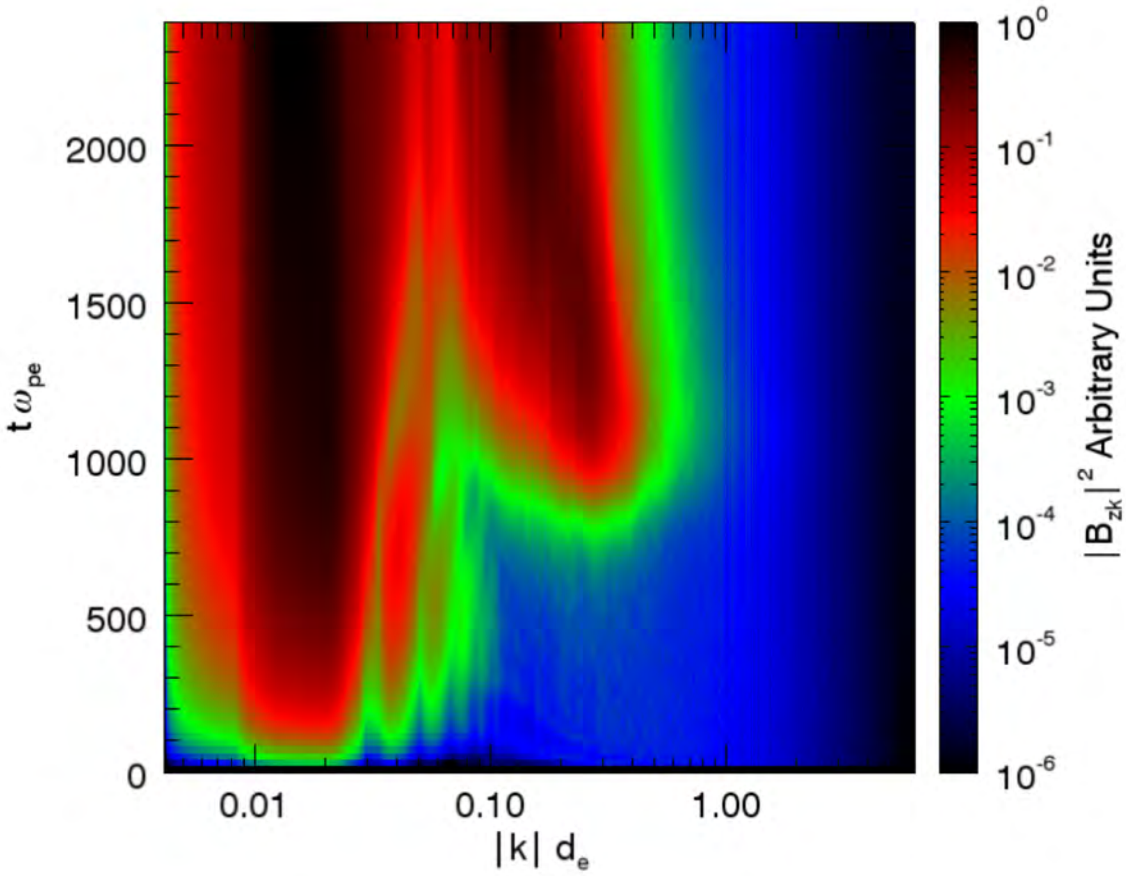


3D simulation with reduced mass ratio = 25

[see also Camporeale & Burgess 2011 (2D)]

Consistent with theoretical predictions using GK (Schekochihin '09)

Energy Spectra (2D runs)

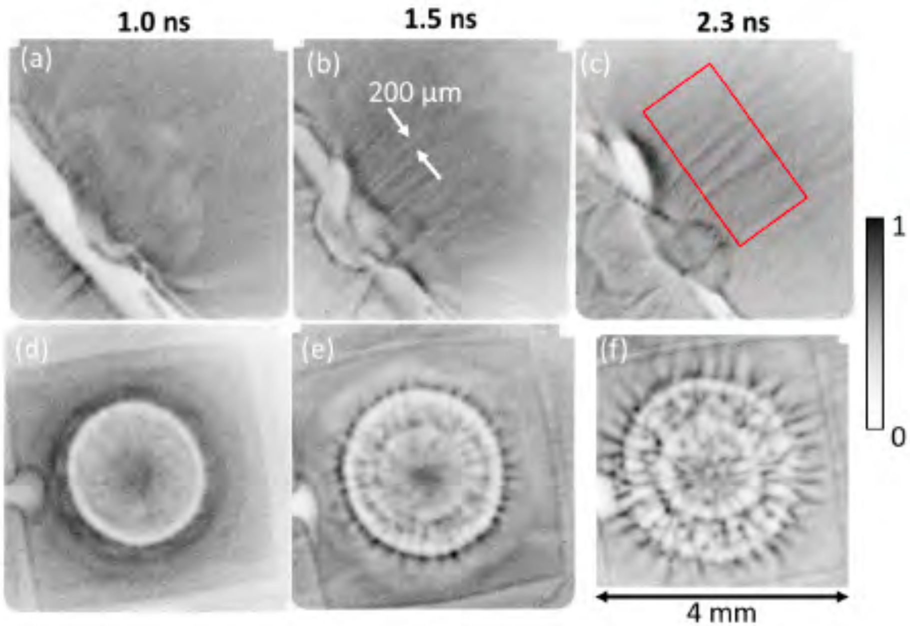


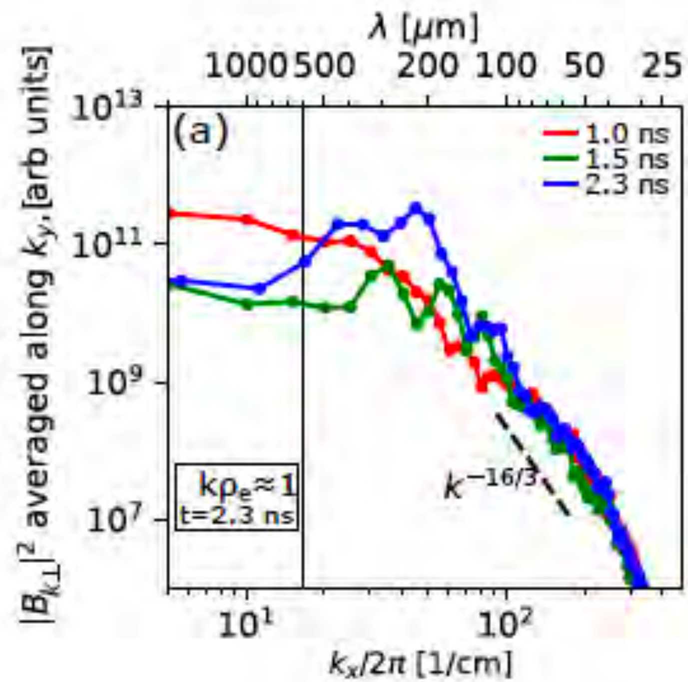
- Two peaks: Biermann at small k, Weibel at large k.
- Some Weibel inverse cascade

Electron Weibel fields on OMEGA

- Analysis of experiments on OMEGA shows good agreement with Weibel analytical dispersion relation

Sutcliffe *et al.*, arXiv:2209.02565





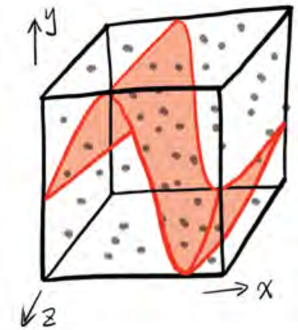
$$k_{max} d_e = \sqrt{A},$$

$$\gamma_{max} \frac{d_e}{v_{th}} = \sqrt{\frac{8}{27\pi}} A^{3/2},$$

Measure k and γ independently.
Use formulas to compute anisotropy (A)
Gives consistent values.

Large-scale shear-flow-driven Weibel

- Weibel instability is very general: even a large-scale shear flow suffices to trigger it.



$$\boldsymbol{F}_{ext}(x) = m\, a_0 \sin\left(\frac{2\pi x}{L}\right)\hat{\boldsymbol{y}} \Rightarrow \quad \frac{\partial f_s}{\partial t} + v_x \frac{\partial f_s}{\partial x} + a_0 \sin\left(\frac{2\pi}{L}x\right)\frac{\partial f_s}{\partial v_y} = 0.$$

$$f_s(t, x, \boldsymbol{v}) = f_{\mathrm{M},s}\left(\sqrt{v_x^2 + v_z^2 + \widetilde{v_y}^2}\right),$$

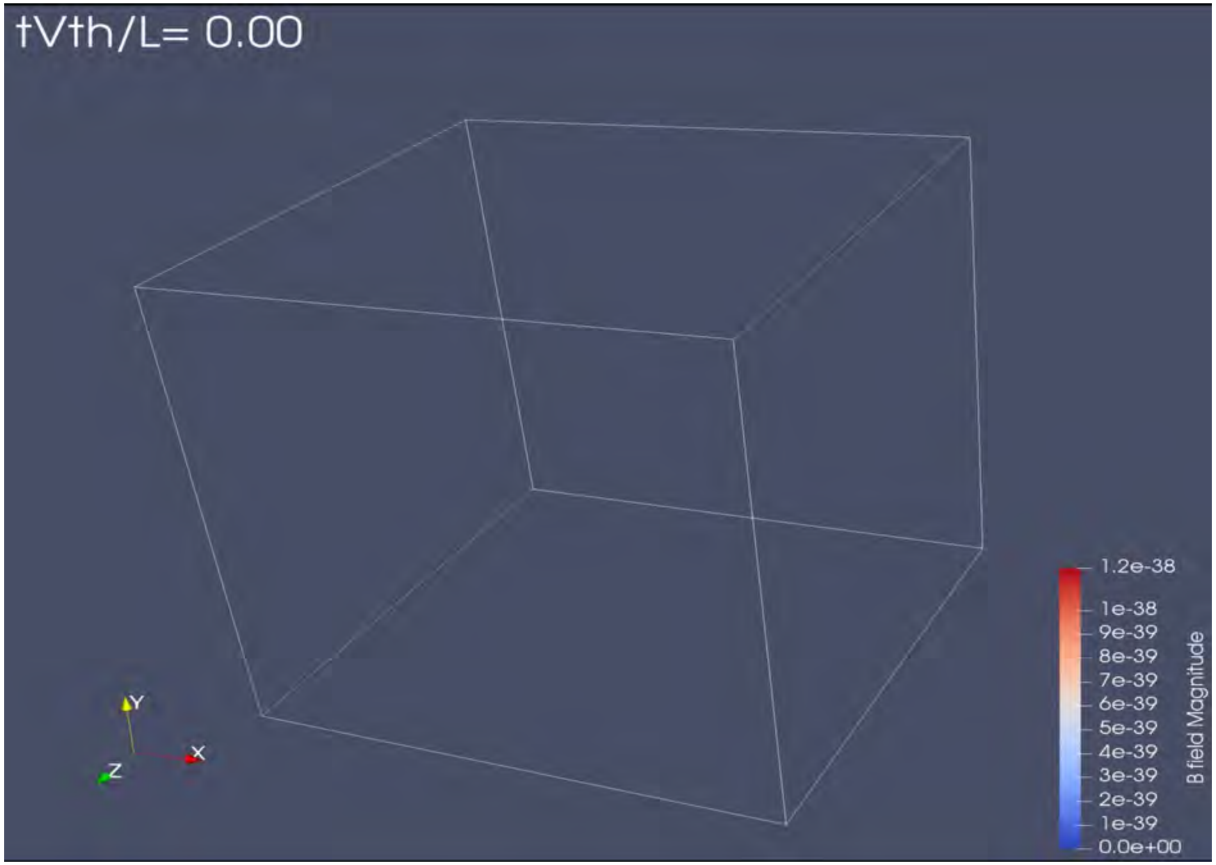
$$\widetilde{v_y} \equiv v_y + \frac{La_0}{2\pi v_x}\left[\cos\left(\frac{2\pi}{L}x\right) - \cos\left(\frac{2\pi}{L}(x - v_x t)\right)\right].$$

The transport of non-uniform y-momentum in the x-direction causes phase-mixing and thermal pressure anisotropy

$$\Delta_s(t) = \frac{3\pi}{2\sqrt{2}}\hat{a}_0\left(\frac{tv_{\mathrm{th}s}}{L}\right)^2 + \mathcal{O}(\epsilon^3).$$

M. Zhou *et al.*, Proc. Nat. Acad. Sci. (2022)

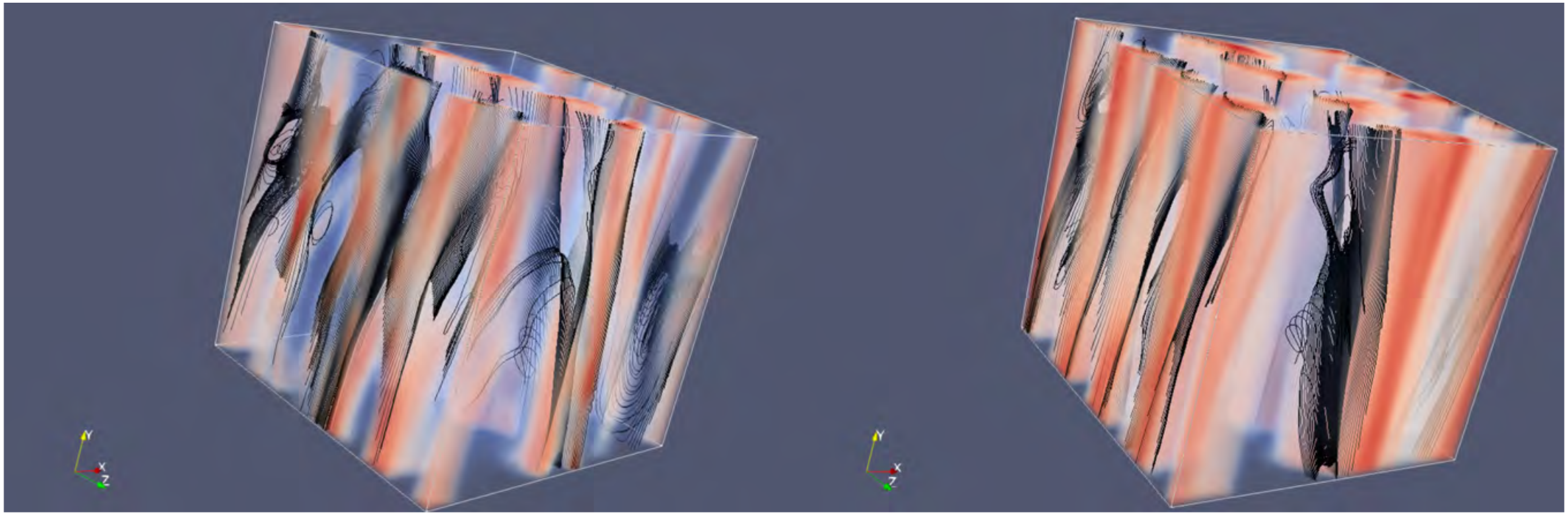
Large-scale shear-flow-driven Weibel



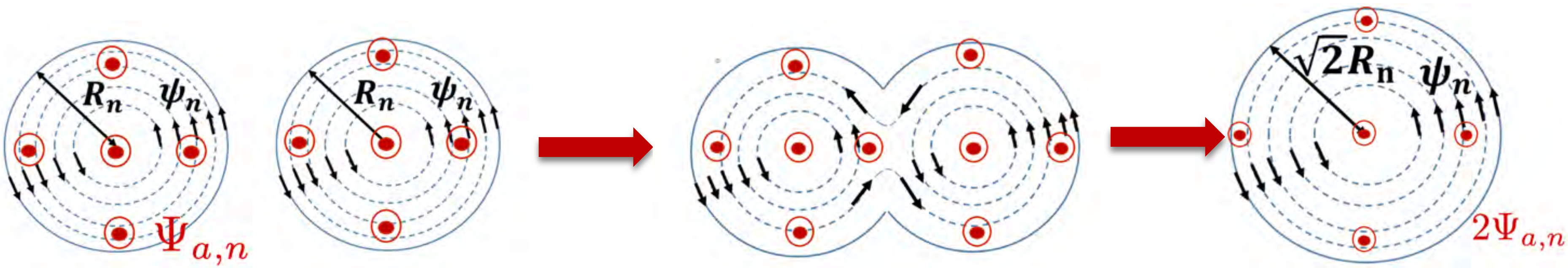
M. Zhou *et al.*, Proc. Nat. Acad. Sci. (2022)

From small to large scales

- Consider magnetic filaments (Weibel generated) at electron scales
- What's their long time evolution?
- Current filaments merge (via reconnection), cascading magnetic energy to larger scales.



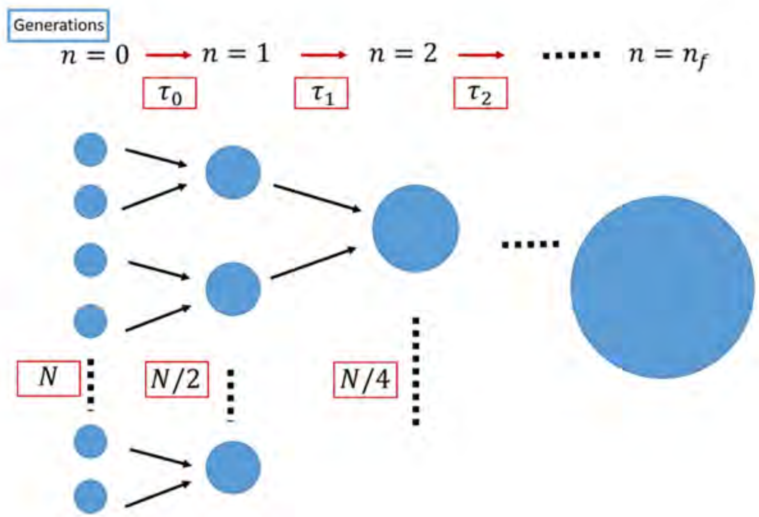
Filament mergers and inverse cascade



Can construct a hierarchical model based on conservation of:

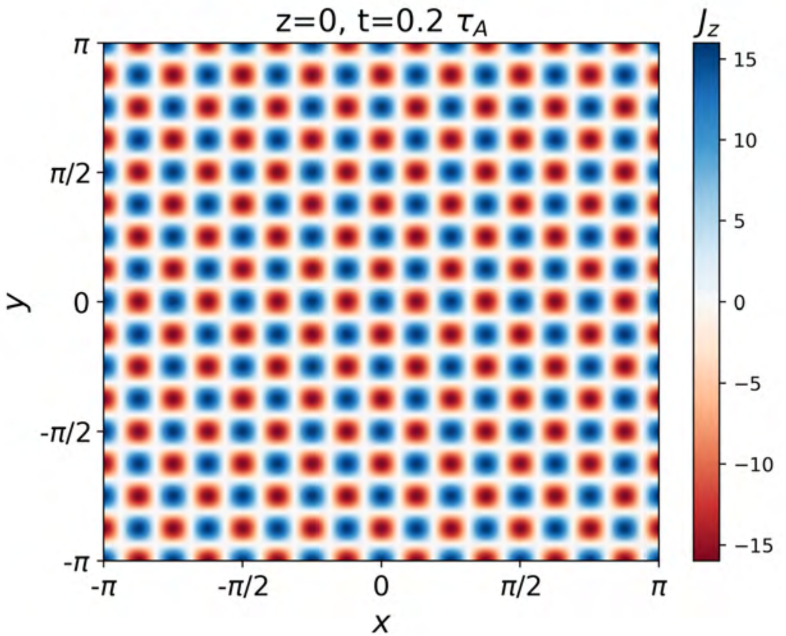
- Cross section area:
  - $\pi R_{n+1}^2 = 2\pi R_n^2 \rightarrow R_{n+1} = \sqrt{2}R_n$
- Poloidal flux:
  - $\psi_{n+1} = \psi_n \rightarrow B_{\perp,n+1} = B_{\perp,n}/\sqrt{2}$

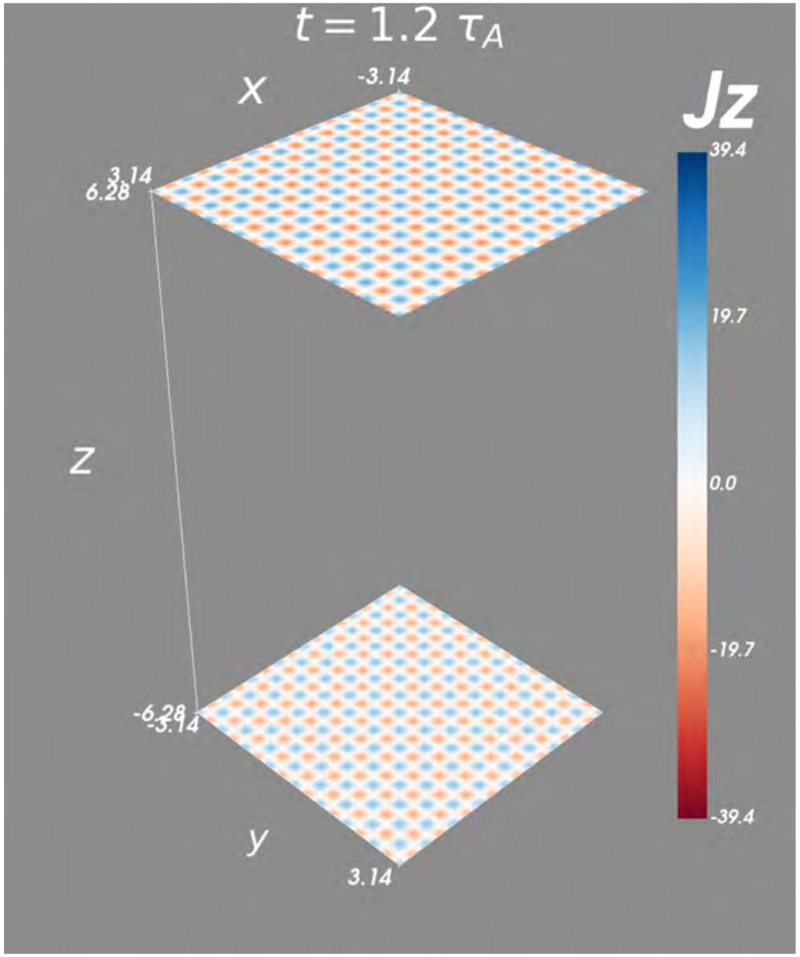
$$k_\perp = k_{\perp,0}\tilde{t}^{-1/2}, \quad B_\perp = B_{\perp,0}\tilde{t}^{-1/2}$$



Muni Zhou *et al.*, Phys. Rev. Res. 2019,
J. Plasma Phys. 2020, 2021

Inverse cascade of magnetic filaments





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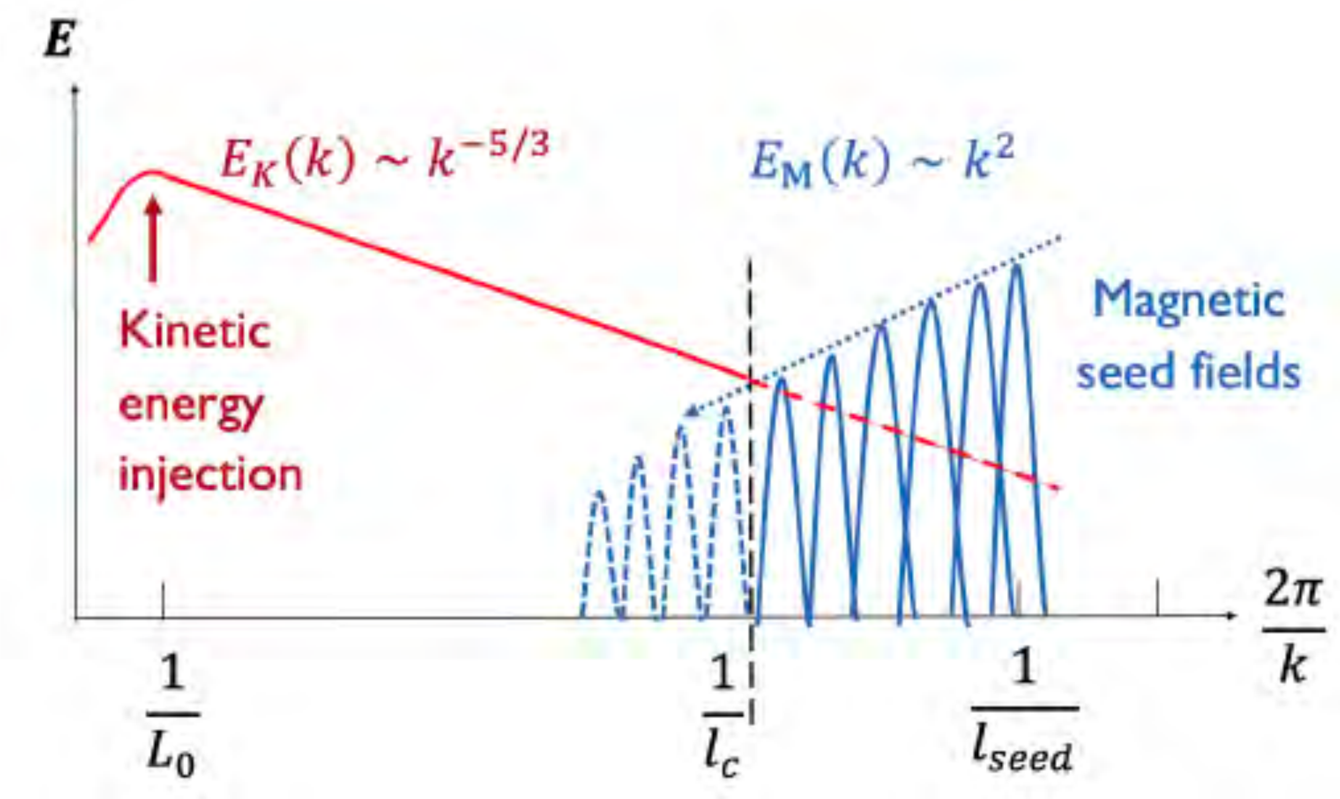


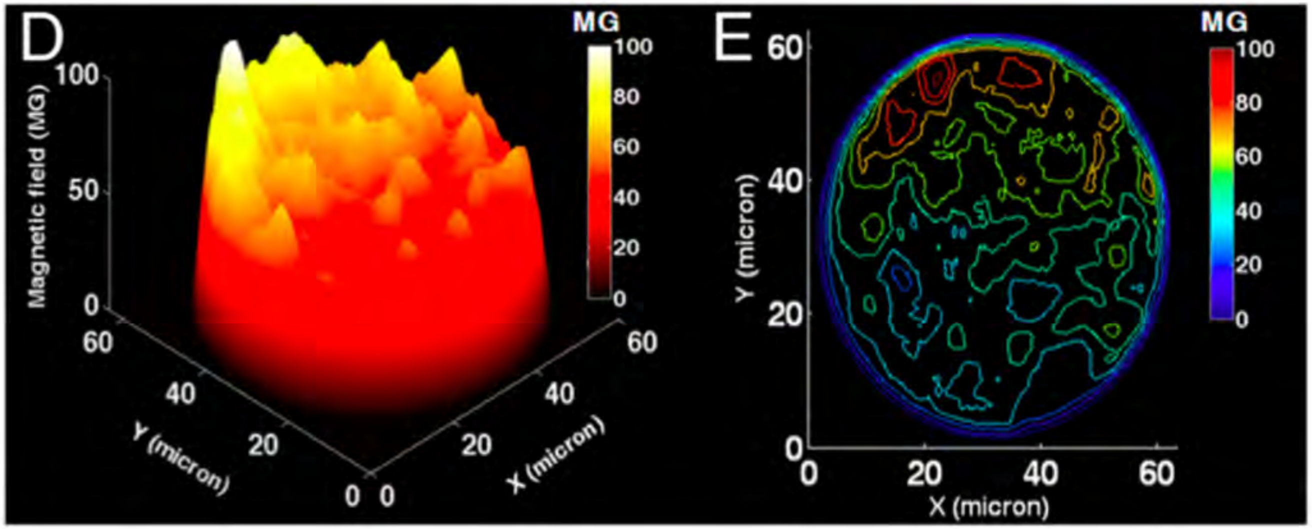
M. Zhou *et al.*, JPP 2020

Conclusions

- Biermann fields (system size, $1/L$ magnitude) are superseded by Weibel fields (electron scale, magnitude independent of L) for large, weakly collisional systems
- Weibel is very easy to drive: even a large-scale shear flow suffices.
- Can Weibel fields be efficient seeds for dynamos?
- Early stages of evolution might be the coalescence of filaments (via reconnection), transferring magnetic energy to larger scales.

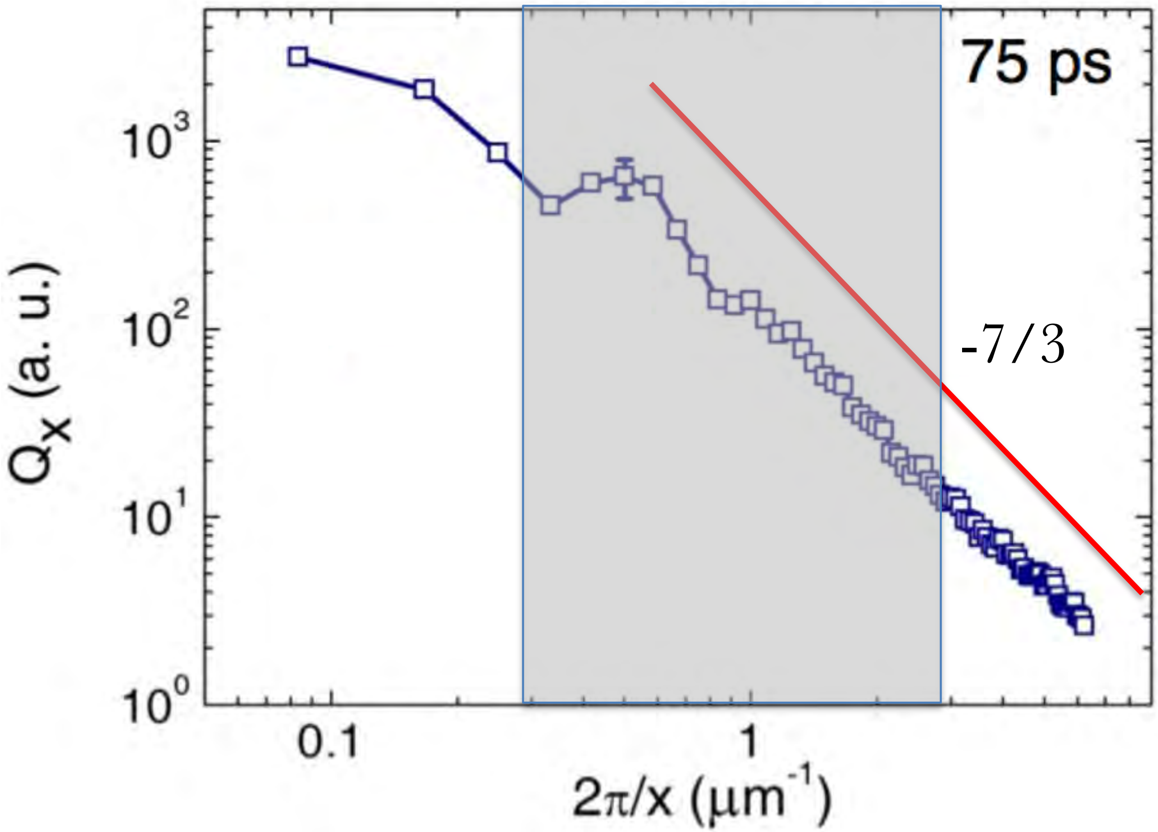
Extras

Ion-scale turbulence in laser-solid interactions



Mondal, PNAS '12

Ion-scale turbulence in laser-solid interactions



G. Chatterjee *et al.*
HEDLA '14

$$\rho_{i,\mathrm{Bier}} \sim L\sqrt{\frac{T_i}{T_e}}$$

$$\Rightarrow \quad 2\pi/\rho_{i,\mathrm{Bier}} \sim 0.3 - 3$$

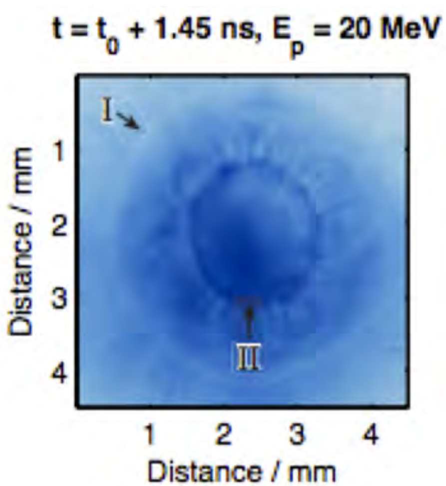
$$T_i/T_e \sim 0.001 - 0.1 \ (?)$$

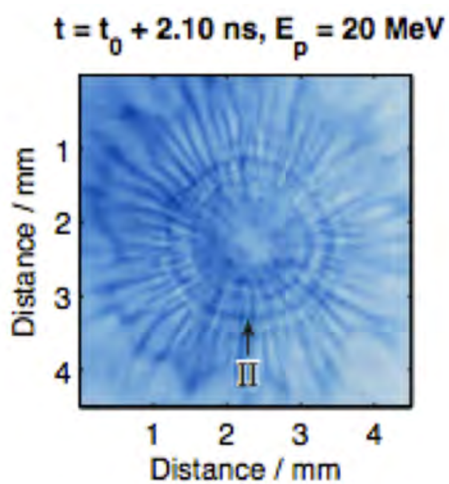
Electron Weibel in laser-solid interactions?

Need $\nu_e L / v_{th,\mathrm{e}} \ll 1$ for the temperature anisotropy to survive.

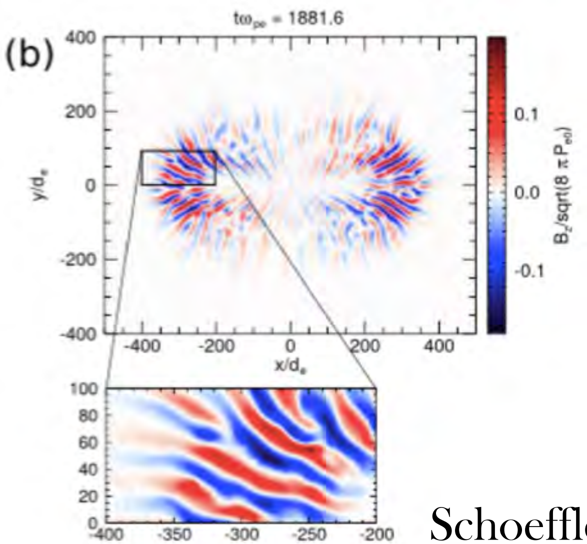
Can rewrite as:

$$1 \times 10^{-2} \left(\frac{n}{1 \times 10^{19}\,\mathrm{cm}^{-3}} \right) \left(\frac{\ln \Lambda}{10} \right) \left(\frac{L_T}{400\,\mu\mathrm{m}} \right) \left(\frac{T_e}{1\,\mathrm{keV}} \right)^{-2} \ll 1$$





L Gao 2014



Schoeffler '16